Part 1: Working with Fractions

In this course you need to know how to work with fractions without relying on a calculator.

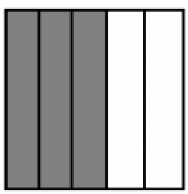
Adding and Subtracting Fractions

Recall from earlier math classes that a fraction is made up of two parts. The bottom part, called the ***denominator***, tells you how many parts the whole is divided into. The top part, called the ***numerator***, tells you how many of those parts you have.

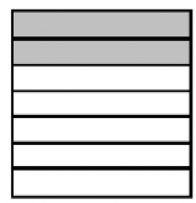
Let's figure this out: $\frac{3}{5}+\frac{2}{7}=$

Many people have forgotten how to do that and so either reach for their calculators, getting a very ugly decimal that they round to 0.89, or they guess – maybe coming up with $\frac{5}{12}$.

$\frac{3}{5}$ means 3 out of 5 parts

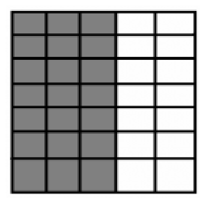


and $\frac{2}{7}$ means 2 out of 7 parts.

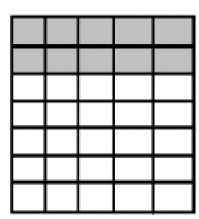


Before we can add these together, we need the pieces we are adding to be the ***same size***. The shaded rectangles in the $\frac{3}{5}$ diagram are NOT the same size as the ones in the $\frac{2}{7}$ diagram.

At right we still have shaded $\frac{3}{5}$ of the whole box, but by subdividing it into seven rows, we see that the shaded amount is also equal to $\frac{21}{35}$ of the box.



In the box at the right, the portion that is shaded is $\frac{2}{7}$. We subdivide this box into five columns and find that the shaded amount is also equal to $\frac{10}{35}$.



$\frac{3}{5}+\frac{2}{7}=\qquad \frac{21}{35}+\frac{10}{35}=\qquad \frac{31}{35}$

Example 1

Evaluate each of the following.

a) $\frac{9}{4}-\frac{2}{3}$

Solution

$$\frac{9}{4}-\frac{2}{3} = \left(\frac{9}{4}\right)\cdot\left(\frac{3}{3}\right)-\left(\frac{2}{3}\right)\cdot\left(\frac{4}{4}\right)$$

Multiplying by $\frac{3}{3}$ or by $\frac{4}{4}$ is like multiplying by "1."

$$= \frac{27}{12}-\frac{8}{12}$$

$$= \frac{19}{12}$$

Doing this does not change the value of the original fractions, but it makes them have a ***common denominator*** so they can be combined.

This answer can also be written as a mixed fraction: $1\frac{7}{12}$

b) $\frac{1}{4}+2$

Solution

$$\frac{1}{4}+2 = \frac{1}{4}+\frac{2}{1}$$

When one of the terms is not written as a fraction, we make it look like one by putting "1" underneath it as its denominator.

$$= \frac{1}{4}+\left(\frac{2}{1}\right)\cdot\left(\frac{4}{4}\right)$$

Get a common denominator.

$$= \frac{1}{4}+\frac{8}{4}$$

Add.

$$= \frac{9}{4}$$

Usually we leave fractions in improper form.

Multiplying and Dividing Fractions

Multiplying fractions and dividing fractions are both easier processes than adding fractions or subtracting fractions. The reason is that **<u>we do NOT need to get a common denominator when multiplying or dividing fractions</u>**.

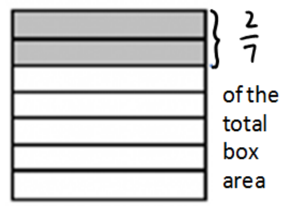
Let's figure this out: $\frac{3}{5} \times \frac{2}{7} =$

You may remember that all we need to do is multiply directly across. The answer is: $\frac{6}{35}$.

Why does this method work?

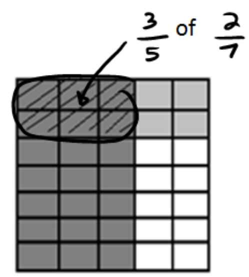
$\frac{3}{5} \times \frac{2}{7}$ means $\frac{3}{5}$ of the region with a size that is $\frac{2}{7}$ of the size of the entire box.

The shaded portion at right is $\frac{2}{7}$ of the entire box.



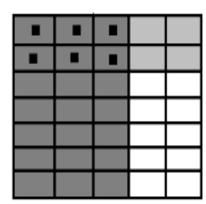
Now we split the entire box into 5 equal pieces (up-and-down columns), and shade 3 of those 5 pieces.

The overlap of the two shaded areas, circled at right, is $\frac{3}{5}$ of $\frac{2}{7}$.



How many of the 35 little boxes are in the overlap?

6 of them, so the answer is $\frac{6}{35}$



What about this question? $\frac{3}{5} \div \frac{2}{7} =$

We change this to a multiplication question by ***inverting the second fraction***:

$$\frac{3}{5} \times \frac{7}{2} = \frac{21}{10}, \text{ or } 2\frac{1}{10}$$

Example 2

Evaluate each of the following.

a) $\frac{21}{4}\times\frac{2}{7}$

Solution

$$\frac{21}{4}\times\frac{2}{7} = \frac{21}{4}\times\frac{2}{7}$$

We can multiply directly across and then reduce. Sometimes this gives us quite large numbers to reduce.

$$= \frac{7\times3}{2\times2}\times\frac{2}{7}$$

A way to avoid large numbers is to ***reduce first,*** before multiplying. We factor first and then cancel identical factors to reduce.

$$= \frac{\cancel{7}\times3}{2\times\cancel{2}}\times\frac{\cancel{2}}{\cancel{7}}$$

$$= \frac{3}{2}$$

This method is also useful for working with rational expressions involving variables.

b) $\frac{5}{7}\div\frac{2}{21}$

Solution

$$\frac{5}{7}\div\frac{2}{21} = \frac{5}{7}\times\frac{21}{2}$$

Invert the second fraction at the first step.

$$= \frac{5}{7}\times\frac{7\times3}{2}$$

Factor completely.

$$= \frac{5}{\cancel{7}}\times\frac{\cancel{7}\times3}{2}$$

Reduce. A factor on the top divided by an identical factor on the bottom can be cancelled, since it is simply equal to “1.”

$$= \frac{15}{2}$$

Part 2: Solving Linear Equations

"Linear" means that if the equation is graphed, you get a straight line. Variables in linear equations are not raised to any exponent other than 1.

Here are some steps to guide you when solving linear equations:

1) If there are brackets, distribute.

2) If there are fractions, eliminate them by multiplying each term by the least common denominator.

3) Collect all the terms containing the variable on one side of the equation and all terms that are constants on the other side of the equation.

4) Combine all like terms.

5) If the variable has a coefficient other than 1, divide to eliminate it.

Example 1

Solve for x.

$$\begin{aligned} 3(x-5) &= 21 \\ 3x - 15 &= 21 \\ 3x - 15 + 15 &= 21 + 15 \\ 3x &= 36 \\ x &= 12 \end{aligned}$$

Example 2

Solve for x.

$$\frac{3x}{8}+\frac{2}{5}=-2$$

$$40\left(\frac{3x}{8}+\frac{2}{5}\right)=40(-2)$$

$$\frac{40}{1}\left(\frac{3x}{8}\right)+\frac{40}{1}\left(\frac{2}{5}\right)=\frac{40}{1}\left(\frac{-2}{1}\right)$$

$$\frac{\overset{5}{\cancel{40}}}{1}\left(\frac{3x}{\underset{1}{\cancel{8}}}\right)+\frac{\overset{8}{\cancel{40}}}{1}\left(\frac{2}{\underset{1}{\cancel{5}}}\right)=\frac{40}{1}\left(\frac{-2}{1}\right)$$

$$5(3x)+8(2)=40(-2)$$

$$15x+16=-80$$

$$15x+16-16=-80-16$$

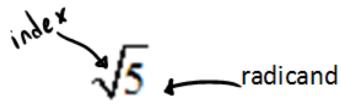
$$15x=-96$$

$$x=-\frac{96}{15}$$

Part 3: Radicals and Exponent Laws

1) Radicals

Here is a radical:



The expression underneath the radical sign is called the ***radicand*** and the tiny number tucked in to the left of the radical sign is the ***index***. Often there is no number visible, in which case we know we are dealing with a square root – this means the index is equal to 2. Sometimes we need to simplify radicals, making the radicand as small as possible.

Example 1

Simplify the radical $\sqrt{32}$.

$$\sqrt{32} = \sqrt{16\times 2}$$
$$= \sqrt{16}\times\sqrt{2}$$
$$= 4\sqrt{2}$$

Knowing a few of the smaller perfect squares and perfect cubes really helps make simplifying radicals easier:

Perfect squares: 4, 9, 16, 25, 36, 49

Perfect cubes: 8, 27, 64, 125

Example 2

Simplify the radical $\sqrt[3]{40}$.

$$\sqrt[3]{40} = \sqrt[3]{8\times 5}$$
$$= \sqrt[3]{8}\times\sqrt[3]{5}$$
$$= 2\sqrt[3]{5}$$

Example 1 (again)

Simplify the radical $\sqrt{32}$.

$$\sqrt{32} = \sqrt{2\times 2\times 2\times 2\times 2}$$
$$= 2\times 2\times\sqrt{2}$$
$$= 4\sqrt{2}$$

Many students prefer to completely factor the radicand and look for repeated factors. This approach also works well.

For a square-root, any PAIR of identical factors allows us to take its square-root out in front. For a cube-root, we need THREE identical factors to let us take the cube-root out in front.

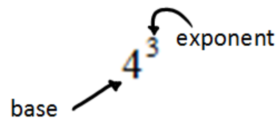
Example 2 (again)

Simplify the radical $\sqrt[3]{40}$.

$$\sqrt[3]{40} = \sqrt[3]{2\times 2\times 2\times 5} = 2\times\sqrt[3]{5} = 2\sqrt[3]{5}$$

2) Laws of Exponents

Here is a power:



Powers with positive exponents are a shorthand way of writing a ***multiplication*** of identical factors:

$$(4)^3 = (4)(4)(4) = 64$$

$$(2x)^5 = (2x)(2x)(2x)(2x)(2x) = 2^5 x^5 = 32x^5$$

$$5x^4 = 5(x)(x)(x)(x) = 5x^4$$

$$(-7)^2 = (-7)(-7) = 49$$

$$-7^2 = -\left[(7)(7)\right] = -49$$

Negative exponents tell us to ***divide*** by that number of factors, instead of multiplying:

$$(2)^{-3} = \frac{1}{2^3} = \frac{1}{(2)(2)(2)} = \frac{1}{8}$$

$$(4x)^{-2} = \frac{1}{(4x)^2} = \frac{1}{(4x)(4x)} = \frac{1}{16x^2}$$

$$\begin{aligned} \frac{3}{x^{-4}} &= \frac{3}{\left(\frac{1}{x^4}\right)} \\ &= 3\times\left(\frac{x^4}{1}\right) \\ &= 3x^4 \end{aligned}$$

Notice that when there is a negative exponent on an expression in the ***denominator*** of a fraction, it ends up "moving" to the top of the fraction, with a positive exponent.

If the exponent is equal to ***zero***, we get a very simple answer. Anything raised to the zero power is equal to 0 (except for 0^0, which is indeterminate).

$$\left(3xy^2\right)^0 = 1$$

$$\left(-7w\right)^0 = 1$$

Some Exponent Laws:

$$x^m x^n = x^{m+n} \qquad (xy)^m = x^m y^m$$

$$\frac{x^m}{x^n} = x^{m-n} \qquad \left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$$

$$\left(x^m\right)^n = x^{mn} \qquad x^{\frac{m}{n}} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$$

Example 1

Simplify. Write answers with positive exponents only.

a) $\left(3m^4n^6\right)(2mn)^0(2m^2n)^3 = \left(3m^4n^6\right)(1)(8m^6n^3)$

$= 24m^{10}n^9$

b) $\dfrac{-28a^6b^{-2}c^6}{7a^{12}b^{-7}c^6} = \dfrac{-4b^{-2}b^7}{a^{-6}a^{12}}$

$= \dfrac{-4b^5}{a^6}$

c) $\dfrac{\left(-2ab^7\right)^3}{\left(-a^4b^2\right)^5} = \dfrac{(-8)\left(a^3\right)\left(b^{21}\right)}{(-1)^5\left(a^{20}\right)\left(b^{10}\right)}$

$= \dfrac{-8b^{11}}{-a^{17}}$

$= \dfrac{8b^{11}}{a^{17}}$

Here are some questions to try.

1. Simplify each of the following. Leave your answer as a fraction, not a decimal.

   a) $\frac{6}{11}+\frac{3}{5}$

   b) $-\frac{2}{3}+1-\frac{3}{4}$

   c) $\frac{15}{8}\div\frac{7}{12}$

2. Solve for *x.* Clearly show the solution process.
- If a solution is a fraction, leave it in reduced fractional form.

a) $5(6x-1)+4(3x+8)=2(3x-7)+11$

b) $\frac{2x-5}{3}+\frac{6x-1}{5}=\frac{5x+14}{15}$

c) $\frac{2}{5}x+\frac{5}{8}=8x-\frac{16}{3}$

3. Simplify each radical.

a) $\sqrt{28x^4y}$

b) $\sqrt[3]{108w^2y^5}$

4. Simplify. Write answers with positive exponents only.

a) $(7ab)\left(-a^4b^3\right)^2\left(2a^5b^6\right)^{-1}$

b) $\dfrac{\left(3z^7\right)^2}{\left(3z^{-3}\right)^2}\times\dfrac{2x^4y^3}{2xy^{-6}}$

c) $\sqrt[3]{8x^6y^{12}}$

Additional Practice Questions

Fractions: Evaluate.

1. $1\frac{2}{3}+\frac{4}{5}=$

2. $-\frac{2}{3}-\frac{4}{5}=$

3. $-\frac{2}{5}\left(\frac{7}{2}-\frac{6}{4}\right)=$

4. $-3+\frac{10}{6}\times\frac{8}{12}=$

5. $-\frac{9}{4}\div 1\frac{1}{2}=$

Solving Linear Equations:

Solve for m in each of the equations. Clearly show the solution process.

6. $4(m-1)-6m=-10(2m-1)-1$

7. $2(m+1)+4m=4(m-2)+6$

8. $\frac{5m+2}{2}=\frac{3m-1}{3}$

9. $\frac{m+5}{4}=\frac{2m+4}{5}$

10. $\frac{m}{4}+5m=\frac{1}{2}m+2$

11. $\frac{5}{2}(m-2)+2=5$

Radicals and Exponent Laws

Simplify the following radicals.

12. $\sqrt{40}$

13. $\sqrt{27}$

14. $5\sqrt{12}+2\sqrt{75}$

15. $-3\sqrt{20}-\sqrt{45}$

16. $(3m^5)(4m^6)=$

17. $\frac{m^{50}m^3}{m^{40}}=$

18. $\dfrac{m^2 m^8 (m^5)^2}{m^3 (m^2)^3} =$

19. $\dfrac{m^2 (m^8)^2 (m^5)^2}{m^3} =$

20. $\dfrac{4m^5 m^3 (m^3)^2}{6m^3 (m^2)^2} =$

Answers:

1. $\dfrac{37}{15}$ 2. $-\dfrac{22}{15}$ 3. $-\dfrac{4}{5}$ 4. $-\dfrac{17}{9}$ 5. $-\dfrac{3}{2}$

6. $\dfrac{13}{18}$ 7. -2 8. $-\dfrac{8}{9}$ 9. 3 10. $\dfrac{8}{19}$ 11. $\dfrac{16}{5}$ 12. $2\sqrt{10}$ 13. $3\sqrt{3}$

14. $20\sqrt{3}$ 15. $-9\sqrt{5}$

16. $12m^{11}$ 17. m^{13} 18. m^{11} 19. m^{25} 20. $\dfrac{2m^7}{3}$