C_06 Key 2022 Ch 3 Hand-in

Sunday, January 23, 2022 7:22 PM

C_06 Chapter 3 Hand-in 2022

Chapter 3 Hand-in Assignment – Polynomials

Name: ______________________________

1. Complete the following table of polynomial characteristics.

Characteristic	$f(x)=4x^4-7x^3-3x^2+x-4$	$f(x)=-3x^5+2x^4-3x^2+2x+1$
Sign of leading coefficient		
Degree (is it odd or even?)		
End behavior		
Number of x-intercepts an equation of this degree can have	between _____ and _____	between _____ and _____
Coordinates of y-intercept		
Domain		
Range		

2. Use **long division** to find the quotient of the following polynomial division question:

$$\left(2x^4-5x^3+3x^2-12\right)\div(x-2)$$

Write the final answer as a division statement in this form: $\frac{P(x)}{x-a}=Q(x)+\frac{R}{x-a}$

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Chapter 3 Hand-in Assignment – Polynomials

Name: Key

1. Complete the following table of polynomial characteristics.

Characteristic	$f(x)=4x^4-7x^3-3x^2+x-4$	$f(x)=-3x^5+2x^4-3x^2+2x+1$
Sign of leading coefficient	+	−
Degree (is it odd or even?)	even	odd
End behavior	↑Q2 ↑Q1	↑Q2 ↓Q4
Number of x-intercepts an equation of this degree can have	between 0 and 4	between 1 and 5
Coordinates of y-intercept	$(0,-4)$	$(0,1)$
Domain	$\{x \mid x\in\mathbb{R}\}$	$\{x \mid x\in\mathbb{R}\}$
Range	$\{y \mid y\geq -12.6, y\in\mathbb{R}\}$	$\{y \mid y\in\mathbb{R}\}$

* use technology to find this value

2. Use **long division** to find the quotient of the following polynomial division question:

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Write the final answer as a division statement in this form: $\frac{P(x)}{x-a}=Q(x)+\frac{R}{x-a}$

$$
\begin{array}{r|l}
 & 2x^3 - x^2 + x + 2 \\
\hline
x-2 & 2x^4 - 5x^3 + 3x^2 + 0x - 12 \\
 & -(2x^4 - 4x^3) \\
 & -x^3 + 3x^2 \\
 & -(-x^3 + 2x^2) \\
 & x^2 + 0x \\
 & -(x^2 - 2x) \\
 & 2x - 12 \\
 & -(2x - 4) \\
 & -8
\end{array}
$$

$$\frac{2x^4-5x^3+3x^2-12}{x-2} = 2x^3-x^2+x+2+\frac{-8}{x-2}$$

3. Use **synthetic division** to find the quotient of the following polynomial division question:
$$\left(3x^3-6x^2+3x+7\right)\div\left(x+1\right)$$

Write the final answer as a division statement in this form: $\frac{P(x)}{x-a}=Q(x)+\frac{R}{x-a}$

4. Show the use of the remainder theorem to find the remainder for the following:

a) $\left(2x^4-5x^3+3x^2+2x-5\right)\div\left(x-3\right)$

b) $\left(-5x^2+15x-6\right)\div\left(x+2\right)$

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3. Use **synthetic division** to find the quotient of the following polynomial division question:
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Write the final answer as a division statement in this form: $\frac{P(x)}{x-a}=Q(x)+\frac{R}{x-a}$

x+1

-1	3	-6	3	7
−	↓	3	-9	12
×	3	-9	12	-5

remainder

$$\frac{3x^3-6x^2+3x+7}{x+1}=3x^2-9x+12+\frac{-5}{x+1}$$

4. Show the use of the remainder theorem to find the remainder for the following:

a) $\left(2x^4-5x^3+3x^2+2x-5\right)\div\left(x-3\right)$

Remainder when we divide by x−3 is equal to P(3).

$$\begin{aligned}P(3) &= 2(3)^4-5(3)^3+3(3)^2+2(3)-5\\ &= 2(81)-5(27)+3(9)+6-5\\ &= 162-135+27+1\\ &= \boxed{55}\end{aligned}$$

b) $\left(-5x^2+15x-6\right)\div\left(x+2\right)$

Remainder when we divide by x+2 is equal to P(−2)

$$\begin{aligned}P(-2) &= -5(-2)^2+15(-2)-6\\ &= -5(4)+-30-6\\ &= -20-36\\ &= \boxed{-56}\end{aligned}$$

5. Determine the value of k if we know that the remainder for this division question is 11.
$(4x^3 - kx + 8) \div (x - 3)$

6. Find the value of k if we know that $x - 2$ is a factor of $-3x^4 + kx^3 - 20x^2 + 5x + 62$.

7. List all possible integral zeros for the polynomial: $f(x) = x^3 - 8x^2 + 20$

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5. Determine the value of k if we know that the remainder for this division question is 11.
$(4x^3 - kx + 8) \div (x - 3)$

If remainder when we divide by $x-3$ is 11, we know that

$$P(3) = 11$$
$$P(3) = 4(3)^3 - k(3) + 8 = 11$$
$$4(27) - 3k + 8 = 11$$
$$108 - 3k + 8 = 11$$
$$-3k = 11 - 108 - 8$$
$$\frac{-3k}{-3} = \frac{-105}{-3}$$
$$\boxed{k = 35}$$

6. Find the value of k if we know that $x - 2$ is a factor of $-3x^4 + kx^3 - 20x^2 + 5x + 62$.

If $x-2$ is a factor of the polynomial $P(x)$, we know that

$$P(2) = 0$$
$$P(2) = -3(2)^4 + k(2)^3 - 20(2)^2 + 5(2) + 62 = 0$$
$$-3(16) + k(8) - 20(4) + 10 + 62 = 0$$
$$-48 + 8k - 80 + 72 = 0$$
$$8k - 56 = 0$$
$$\frac{8k}{8} = \frac{56}{8}$$
$$\boxed{k = 7}$$

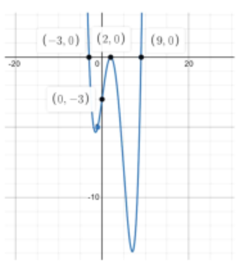
7. List all possible integral zeros for the polynomial: $f(x) = x^3 - 8x^2 + 20$

Integral zeros are divisors of the constant.

All possible integral zeros for $f(x)$ are: $\boxed{\pm 1, \pm 20 \quad \pm 2, \pm 10 \quad \pm 4, \pm 5}$

8. Fully factor the following polynomial: $2x^4 - 3x^3 - 23x^2 + 27x + 45$

9. Determine the equation of the function shown in the graph.



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8. Fully factor the following polynomial: $2x^4 - 3x^3 - 23x^2 + 27x + 45$

possible integral zeros are divisors of 45:

$\pm 1, \pm 45$
$\pm 3, \pm 15$
$\pm 5, \pm 9$

$$P(1) = 2(1)^4 - 3(1)^3 - 23(1)^2 + 27(1) + 45$$
$$= 48 \neq 0, \quad x-1 \text{ is } \underline{\text{NOT}} \text{ a factor}$$
$$P(-1) = 2(-1)^4 - 3(-1)^3 - 23(-1)^2 + 27(-1) + 45$$
$$= 0 \Rightarrow x+1 \text{ is a factor}$$

$x+1$

$$\begin{array}{c|ccccc}
1 & 2 & -3 & -23 & 27 & 45 \\
- & \downarrow & 2 & -5 & -18 & 45 \\
\hline
\times & 2 & -5 & -18 & 45 & 0
\end{array}$$

$\underbrace{2 \quad -5 \quad -18 \quad 45}_{2x^3 - 5x^2 - 18x + 45}$ remainder

So far, $P(x) = (x+1)(2x^3 - 5x^2 - 18x + 45)$

now, we try to find another factor:

$$P(3) = 2(3)^3 - 5(3)^2 - 18(3) + 45$$
$$= 0 \Rightarrow x-3 \text{ is a factor}$$

$x-3$

$$\begin{array}{c|cccc}
-3 & 2 & -5 & -18 & 45 \\
- & \downarrow & -6 & -3 & 45 \\
\hline
\times & 2 & 1 & -15 & 0
\end{array}$$

$\underbrace{2 \quad 1 \quad -15}_{2x^2 + x - 15}$ remainder

So far, $P(x) = (x+1)(x-3)(2x^2 + x - 15)$

Now we can either keep using factor theorem & division, or we can just factor:

$$\underset{A}{2}x^2 + x - \underset{C}{15}$$

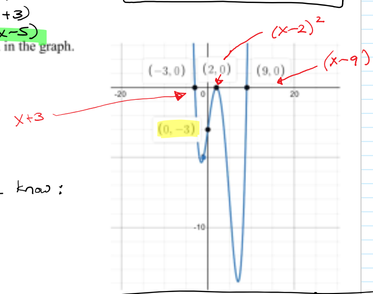
$AC = -30$
$\left.\begin{array}{l}\text{product} = -30 \\ \text{sum} = 1\end{array}\right\} 6, -5$

$$2x^2 + 6x - 5x - 15$$
$$= 2x(x+3) - 5(x+3)$$
$$= (x+3)(2x-5)$$

$$\boxed{\text{Polynomial, fully factored} = (x+1)(x-3)(x+3)(2x-5)}$$

9. Determine the equation of the function shown in the graph.

$$y = a(x+3)(x-2)^2(x-9)$$



Since $f(x)$ goes through $(0,-3)$, we know:

$$-3 = a(0+3)(0-2)^2(0-9)$$
$$-3 = a(3)(-2)^2(-9)$$
$$-3 = a(3)(4)(-9)$$
$$\frac{-3}{-108} = \frac{a(-108)}{-108}$$
$$a = \frac{3}{108} = \frac{1}{36}$$

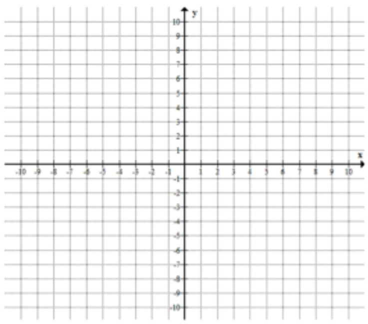
$$\boxed{y = \frac{1}{36}(x+3)(x-2)^2(x-9)}$$

10. Determine the equation of the function that has the following:
Roots at -2, -1 (multiplicity 3) and 4 (multiplicity 2) and a y-intercept at $(0, -8)$.
You do NOT need to expand the equation – just leave it in factored form.

11a) Give the mapping that shows what happens to points on the base function, $y = x^3$, when the equation is changed to $y = \frac{1}{2}\left(-\frac{1}{2}x - 1\right)^3 - 2$

b) Fill in the table of key points for the base function. Using the mapping, create the table of image points that result when the original points are transformed. Sketch the transformed graph on the grid.

x	y



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10. Determine the equation of the function that has the following:
Roots at -2, -1 (multiplicity 3) and 4 (multiplicity 2) and a y-intercept at $(0, -8)$.
You do NOT need to expand the equation – just leave it in factored form.

$y = a(x+2)(x+1)^3(x-4)^2$ must go through $(0,-8)$

$\Rightarrow$
$-8 = a(0+2)(0+1)^3(0-4)^2$
$-8 = a(2)(1)^3(-4)^2$
$-8 = a(2)(1)(16)$
$\frac{-8}{32} = \frac{32a}{32}$, $a = \frac{-8}{32} = -\frac{1}{4}$

$\boxed{y = -\frac{1}{4}(x+2)(x+1)^3(x-4)^2}$

11a) Give the mapping that shows what happens to points on the base function, $y = x^3$, when the equation is changed to $y = \frac{1}{2}\left(-\frac{1}{2}x - 1\right)^3 - 2$

$y = \frac{1}{2}\left(-\frac{1}{2}(x+2)\right)^3 - 2$

2 down

2 left

VC $\frac{1}{2}$

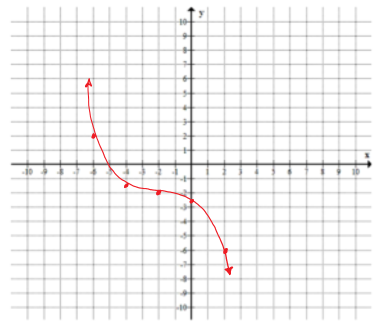
HE 2, reflect across y-axis

$(x, y) \rightarrow (-2x-2, \frac{1}{2}y - 2)$

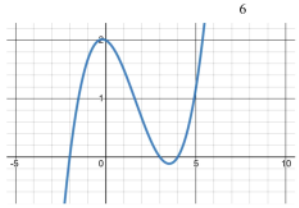
b) Fill in the table of key points for the base function. Using the mapping, create the table of image points that result when the original points are transformed. Sketch the transformed graph on the grid.

x	y
-2	-8
-1	-1
0	0
1	1
2	8

$-2x-2$	$\frac{1}{2}y-2$
2	-6
0	$-2\frac{1}{2}$
-2	-2
-4	$-1\frac{1}{2}$
-6	2



12. Use the graph of the given function to fill in the table below.



Characteristic	
Roots of the equation $f(x) = 0$	
End Behavior	
Intervals where $y = f(x)$ is greater than or equal to 0	
Intervals where $y = f(x)$ is less than 0	

13. If a box has a volume represented by $V(x) = 2x^3 - 17x^2 + 27x + 18$, determine the dimensions of the box, as expressions in terms of x.

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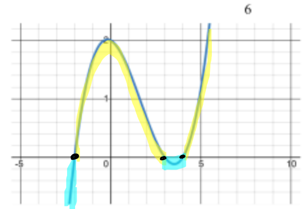
12. Use the graph of the given function to fill in the table below.



Characteristic	
Roots of the equation $f(x) = 0$	$x = -2$, $x = 3$, $x = 4$
End Behavior	↓ Q3 ↑ Q1
Intervals where $y = f(x)$ is greater than or equal to 0	$-2 \le x \le 3$, $x \ge 4$
Intervals where $y = f(x)$ is less than 0	$x < -2$, $3 < x < 4$

13. If a box has a volume represented by $V(x) = 2x^3 - 17x^2 + 27x + 18$, determine the dimensions of the box, as expressions in terms of x.

We have to factor $V(x)$.

possible integral zeros are: $\pm 1, \pm 18$, $\pm 2, \pm 9$, $\pm 3, \pm 6$

$V(1) = 2(1)^3 - 17(1)^2 + 27(1) + 18 = 30$
$V(3) = 2(3)^3 - 17(3)^2 + 27(3) + 18 = 0$
$\Rightarrow$ $x-3$ is a factor

$$\begin{array}{c|cccc}
x-3 \quad -3 & 2 & -17 & 27 & 18 \\
- & \downarrow & -6 & 33 & 18 \\
\hline
\times & 2 & -11 & -6 & 0
\end{array}$$

remainder

represents $2x^2 - 11x - 6$

So far, $V(x) = (x-3)(2x^2 - 11x - 6)$

Now, we factor $2x^2 - 11x - 6$

A C

$AC = 2(-6) = -12$

numbers product $= -12$
sum $= -11$
$-12, 1$

$2x^2 - 12x + x - 6$
$= 2x(x-6) + 1(x-6)$
$= (x-6)(2x+1)$

So,
$V(x) = (x-3)(x-6)(2x+1)$

$\Rightarrow$ box dimensions are
$x-6$
$x-3$
$2x+1$