C_10 Key Cor 2022 Ch 4 Hand-in

Monday, January 31, 2022 3:38 PM

C_10 Ch 4 Hand-in 2022

Chapter 4 Hand-in Assignment – Trigonometry

Name: Key

Unless a question says differently, round to 2 decimal places when rounding is necessary.

1. Convert each angle to degree measure.

a) $\frac{7\pi}{8}$

$\frac{7\cancel{\pi}}{8} \cdot \frac{180}{\cancel{\pi}} = \boxed{157.5^\circ}$

b) 4.2 radians

$\frac{4.2}{1} \times \frac{180}{\pi} \doteq \boxed{240.64^\circ}$

2. Convert each angle to radian measure, in simplest ***exact form***. (Answers will include π)

a) -200°

$\frac{-200^\circ}{1} \times \frac{\pi}{180^\circ} = \frac{-200\pi}{180} = \frac{-20\pi}{18} = \boxed{-\frac{10\pi}{9}}$

b) 1040°

$\frac{1040^\circ}{1} \times \frac{\pi}{180^\circ} = \frac{1040\pi}{180} = \frac{104\pi}{18} = \boxed{\frac{52\pi}{9}}$

3. Convert each angle to radian measure, in ***approximate form***.

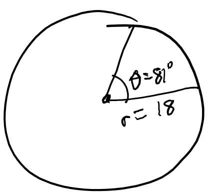
a) 258°

$\frac{258^\circ}{1} \times \frac{\pi}{180^\circ} \doteq \boxed{4.50}$

b) -95°

$\frac{-95}{1} \times \frac{\pi}{180^\circ} \doteq \boxed{-1.66}$

4. Find the arc length subtended by an angle measuring 81° in a circle with radius 18 cm.

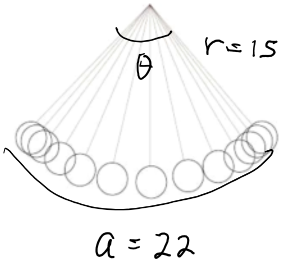


$a = r\theta$

$= \frac{18}{1} \times \frac{81^\circ}{1} \times \frac{\pi}{180}$

$\doteq \boxed{25.45 \text{ cm}}$

5. Suppose that a clock's pendulum has a length of 15 cm, and it swings back and forth, making an arc of 22 cm. What angle does the pendulum pass through in one swing, in ***degree measure***?



$a = r\theta$

$\frac{a}{r} = \theta$

$\theta = \frac{22}{15}$ radians

Convert to degrees:

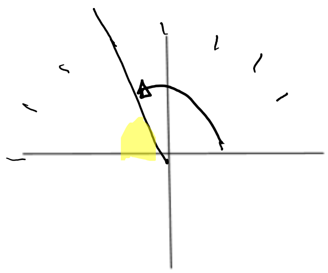
$\frac{22}{15} \times \frac{180^\circ}{\pi} \doteq \boxed{84.03^\circ}$

6. For each angle below:

- graph it in standard position
- find the measure of one angle that is ***coterminal*** to the given angle
- find the ***reference angle*** to the given angle

a) $\frac{5\pi}{8}$

(Give coterminal & reference angles in exact radian measure)

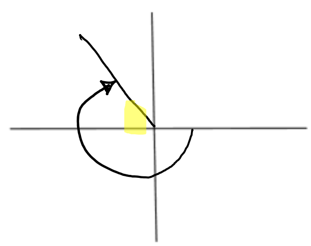


Coterminal: $\frac{5\pi}{8} + \frac{2\pi}{1}\cdot\frac{8}{8} = \frac{5\pi}{8} + \frac{16\pi}{8} = \boxed{\frac{21\pi}{8}}$

Reference: $\pi - \frac{5\pi}{8} = \frac{8}{8}\pi - \frac{5\pi}{8} = \boxed{\frac{3\pi}{8}}$

b) -220°

(Give coterminal & reference angles in degree measure)



Coterminal: $-220^\circ + 360^\circ = \boxed{140^\circ}$

Reference: $\boxed{40^\circ}$

7. Find the x-coordinate of all points on the unit circle that have a y-coordinate of $\frac{2}{5}$. Give answers in fractional form, not decimal form.

Equation of unit circle: $x^2 + y^2 = 1$

$x^2 + \left(\frac{2}{5}\right)^2 = 1$

$x^2 + \frac{4}{25} = 1$

$x^2 = 1 - \frac{4}{25}$

$x^2 = \frac{25}{25}\cdot 1 - \frac{4}{25}$

$x^2 = \frac{21}{25}$

$x = \pm\sqrt{\frac{21}{25}}$

$\boxed{x = \pm\frac{\sqrt{21}}{5}}$

8. Find each value, correct to **three decimal places**. (Use a calculator!)

a) $\csc 185^\circ$

$\csc 185^\circ = \frac{1}{\sin 185^\circ}$

(degree mode on calc!)

$\doteq \boxed{-11.474}$

b) $\cot\left(\frac{3\pi}{7}\right)$

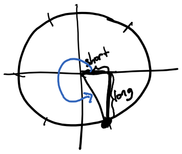
$\cot\frac{3\pi}{7} = \frac{1}{\tan\frac{3\pi}{7}}$

(radian mode on calc!)

$\doteq \boxed{0.228}$

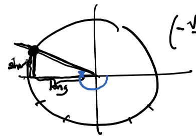
9. Find the EXACT $(x,\ y)$ coordinates where the terminal arm of each angle listed below intersects the unit circle:

a) $\frac{5\pi}{3}$



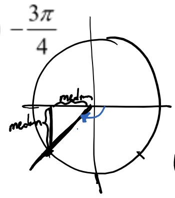
$\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

b) $-\frac{7\pi}{6}$



$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

c) $-\frac{3\pi}{4}$

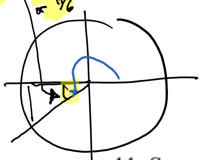


$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

10. Find the angle measure, in BOTH radians and degrees, that corresponds with each point on the unit circle:

a) $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

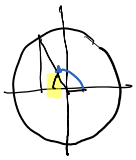
reference angle is small $= 30°$ or $\pi/6$



$\boxed{\text{angle: } \frac{7\pi}{6} \text{ OR } 210°}$

b) $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

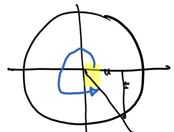
$\boxed{\text{angle: } \frac{2\pi}{3} \text{ or } 120°}$



ref. angle is BIG $= 60°$ or $\pi/3$

c) $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

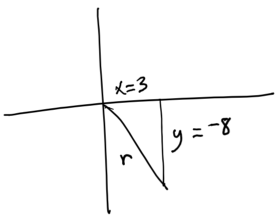
$\boxed{\text{angle} = \frac{7\pi}{4} \text{ or } 315°}$



ref angle is MEDIUM $= 45°$ or $\pi/4$

11. Suppose the terminal arm of a standard position angle θ passes through the point $(3,-8)$.
Find the exact value of all six trigonometric ratios for angle θ, in fractional form.

x y



$x^2+y^2=r^2$
$3^2+(-8)^2=r^2$
$9+64=r^2$
$73=r^2$
$r=\sqrt{73}$
(radius is always positive)

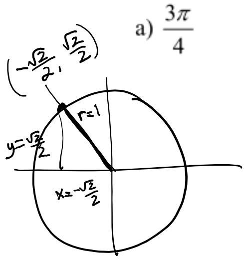
$\sin\theta = \frac{y}{r} = \frac{-8}{\sqrt{73}}$ $\qquad \csc\theta = \frac{r}{y} = \frac{\sqrt{73}}{-8}$

$\cos\theta = \frac{x}{r} = \frac{3}{\sqrt{73}}$ $\qquad \sec\theta = \frac{r}{x} = \frac{\sqrt{73}}{3}$

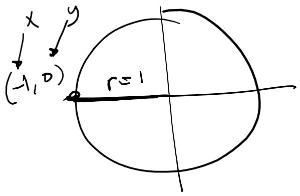
$\tan\theta = \frac{y}{x} = \frac{-8}{3}$ $\qquad \cot\theta = \frac{x}{y} = \frac{3}{-8}$

12. Find the ***exact value*** of all six trigonometric ratios for each angle θ. Give answers in simple form (no complex fractions).

a) $\frac{3\pi}{4}$



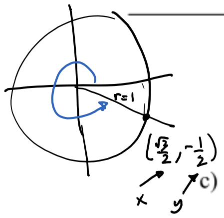
$\sin\theta = \frac{\sqrt{2}}{2}$	$\cos\theta = -\frac{\sqrt{2}}{2}$	$\tan\theta = \frac{y}{x} = \frac{\frac{\sqrt{2}}{2}}{\frac{-\sqrt{2}}{2}} = \frac{\sqrt{2}}{2}\cdot\frac{-2}{\sqrt{2}} = \frac{-2\sqrt{2}}{2\sqrt{2}} = -1$
$\csc\theta = \frac{2}{\sqrt{2}}$ (OR $\frac{2\sqrt{2}}{2} = \sqrt{2}$)	$\sec\theta = -\frac{2}{\sqrt{2}}$ (OR $-\frac{2\sqrt{2}}{2} = -\sqrt{2}$)	$\cot\theta = \frac{x}{y} = \frac{\frac{\sqrt{2}}{2}}{\frac{-\sqrt{2}}{2}} = \frac{\sqrt{2}}{2}\cdot\frac{-2}{\sqrt{2}} = \frac{-2\sqrt{2}}{2\sqrt{2}} = -1$



b) $-\pi$

$\sin\theta = \frac{y}{r} = \frac{0}{1} = \boxed{0}$ | $\cos\theta = \frac{x}{r} = \frac{-1}{1} = \boxed{-1}$ | $\tan\theta = \frac{y}{x} = \frac{0}{-1} = \boxed{0}$

$\csc\theta = \frac{r}{y} = \frac{1}{0}$ $\boxed{\text{undefined}}$ | $\sec\theta = \frac{r}{x} = \frac{1}{-1} = \boxed{-1}$ | $\cot\theta = \frac{x}{y} = \frac{-1}{0} = \boxed{\text{undefined}}$



c) 330°

$\sin\theta = \boxed{-\frac{1}{2}}$ $\cos\theta = \boxed{\frac{\sqrt{3}}{2}}$ $\tan\theta = \frac{-1/2}{\sqrt{3}/2} = -\frac{1}{2}\cdot\frac{2}{\sqrt{3}} = \frac{-\cancel{2}}{\cancel{2}\sqrt{3}} = \boxed{\frac{-1}{\sqrt{3}}}$ (OR $-\frac{\sqrt{3}}{3}$)

$\csc\theta = \frac{-2}{1} = \boxed{-2}$ $\sec\theta = \boxed{\frac{2}{\sqrt{3}}}$ (OR $\frac{2\sqrt{3}}{3}$) $\cot\theta = \frac{\frac{\sqrt{3}}{2}}{-1/2} = \frac{\sqrt{3}}{2}\cdot\frac{-2}{1} = \frac{-\cancel{2}\sqrt{3}}{\cancel{2}} = \boxed{-\sqrt{3}}$

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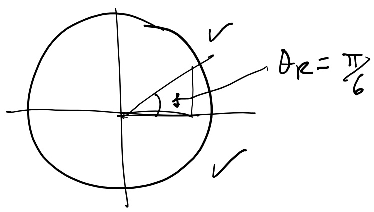
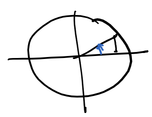
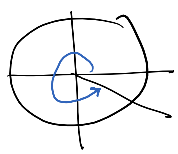
14. Solve these trigonometric equations algebraically.
- Give answers in EXACT form when possible.
- If domain is in radians, give answers in radian measure

a) $\cos\theta = \frac{\sqrt{3}}{2}$, $0 \le \theta < 2\pi$



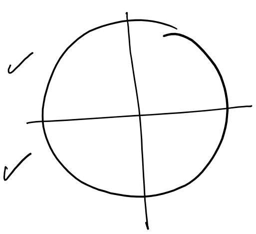


Q_1 answer $= \boxed{\pi/6}$

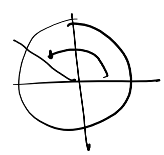


Q_4 answer $= 2\pi - \theta_R$
$= 2\pi\left(\frac{6}{6}\right) - \pi/6$
$= \frac{12\pi}{6} - \pi/6 = \boxed{\frac{11\pi}{6}}$

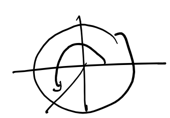
b) $\cos\theta = -0.813$, for $0 \le \theta < 2\pi$ radian mode on calc



$\theta_R = \cos^{-1}(+0.813)$
$= 0.62151\ldots$ radians



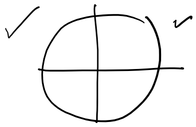
Q_2 answer $= \pi - \theta_R$
$\doteq \boxed{2.52}$



Q_3 answer $= \pi + \theta_R$
$\doteq \boxed{3.76}$

c) $\sin\theta = 0.247$, for $0^\circ \le \theta \le 720^\circ$

degree mode on calc



$\theta_R = \sin^{-1}(0.247)$

$= 14.300058...^\circ$

Q_1 answer $\doteq \boxed{14.30^\circ}$

Q_2 answer $= 180^\circ - \theta_R$

$= \boxed{165.70^\circ}$

We need coterminals to each of these answers, because we need all answers from 0° to 720°.

other Q_1 answer $= 14.30^\circ + 360^\circ$

$= \boxed{374.30^\circ}$

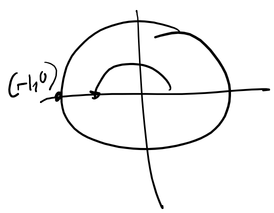
other Q_2 answer $= 165.70^\circ + 360^\circ$

$= \boxed{525.70^\circ}$

d) $2\cos\theta + 1 = -1$, $0 \le \theta < 2\pi$

$\frac{2\cos\theta}{2} = \frac{-2}{2}$

$\cos\theta = -1$



Only one solution between 0 and 2π:

$\boxed{\theta = \pi}$

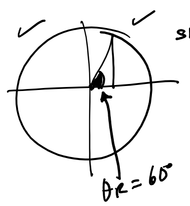
e) $4\sin^2\theta - 3 = 0$, $0^\circ \le \theta < 360^\circ$

$\frac{4\sin^2\theta}{4} = \frac{3}{4}$

$\sin^2\theta = \frac{3}{4}$

$\sin\theta = \pm\sqrt{\frac{3}{4}}$

$\sin\theta = \pm\frac{\sqrt{3}}{2}$

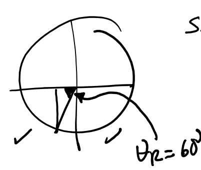


$\sin\theta = \frac{\sqrt{3}}{2}$

Q_1 answer $= \boxed{60^\circ}$

Q_2 answer $= 180^\circ - \theta_R$

$= \boxed{120^\circ}$



$\sin\theta = -\frac{\sqrt{3}}{2}$

Q_3 answer $= 180^\circ + \theta_R$

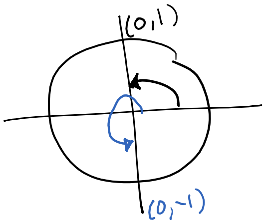
$= \boxed{240^\circ}$

Q_4 answer $= 360^\circ - \theta_R$

$= \boxed{300^\circ}$

f) $\sqrt{2}\cos^2\theta - \cos\theta = 0,\ 0 \le \theta < 2\pi$

Factor out common factor: $\cos\theta\,(\sqrt{2}\cos\theta - 1) = 0$



$\cos\theta = 0$

On unit circle, where does x-coordinate = 0?

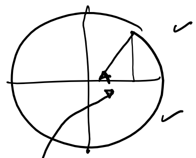
Two answers: $\boxed{\frac{\pi}{2},\ \frac{3\pi}{2}}$

$\sqrt{2}\cos\theta - 1 = 0$

$\sqrt{2}\cos\theta = 1$

$\cos\theta = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\cos\theta = \frac{\sqrt{2}}{2}$



$\theta_R = \pi/4$

Q_1 answer = $\boxed{\pi/4}$

Q_4 answer $= 2\pi - \theta_R = \boxed{\frac{7\pi}{4}}$

Mistake in the question!! Sorry!!

g) $2\tan^2\theta - 7\tan\theta + 3 = 0,\ 0^\circ \le \theta < 720^\circ$

$AC = -6$

mult to -6, add to -7 $\}$ $-6, -1$

$2\tan^2\theta - 6\tan\theta - \tan\theta + 3 = 0$

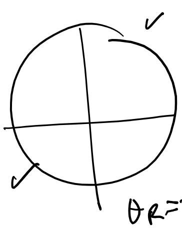
$2\tan\theta(\tan\theta - 3) - 1(\tan\theta - 3) = 0$

$(2\tan\theta - 1)(\tan\theta - 3) = 0$

$2\tan\theta - 1 = 0$

$2\tan\theta = 1$

$\tan\theta = \frac{1}{2}$



$\theta_R = \tan^{-1}(\frac{1}{2})$

$= 26.565...^\circ$

Q_1 answer = $\boxed{26.57^\circ}$

Q_3 answer = $180^\circ + \theta_R$

$= \boxed{206.57^\circ}$

and, we need coterminals:

other Q_1 answer = $26.57^\circ + 360^\circ$

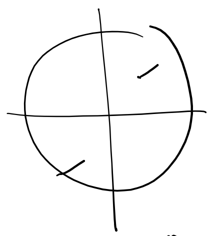
$= \boxed{386.57^\circ}$

other Q_3 answer = $206.57^\circ + 360^\circ$

$= \boxed{566.57^\circ}$

$\tan\theta - 3 = 0$

$\tan\theta = 3$



$\theta_R = \tan^{-1}(3)$

$= 71.565...^\circ$

Q_1 answer = $\boxed{71.57^\circ}$

Q_3 answer = $180^\circ + \theta_R$

$= \boxed{251.57^\circ}$

coterminals needed:

other Q_1 answer = $71.57^\circ + 360^\circ$

$= \boxed{431.57^\circ}$

other Q_3 answer = $251.57^\circ + 360^\circ$

$= \boxed{611.57^\circ}$