C_11 Key 2022 Ch 5 Hand-in

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C_11 Ch 5 Hand-in 2022

Chapter 5 Hand-in Assignment – Trigonometric Graphs and Applications

Name:

1. Fill in the table below, stating each function's amplitude, period, phase shift and vertical displacement. (If there is none, say so.)

Equation	Amplitude	Period	Phase Shift (how much, and in which direction)	Vertical displacement (how much, and in which direction)	Maximum	Minimum
$y=4\cos\left(x+\frac{\pi}{4}\right)-3$						
$y=-7\sin(2x-\pi)+5$						
$y=2\cos\left(\frac{1}{2}(x+30^{\circ})\right)-1$						

2. Write an equation for a function with the given characteristics:

a) A **sine** function with amplitude 5, period π, phase shift $\frac{\pi}{3}$ left, and vertical displacement 2 down

b) A **cosine** function with amplitude 2, period 3π, phase shift 5 right, and vertical displacement 4 up

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Chapter 5 Hand-in Assignment – Trigonometric Graphs and Applications

Name: Key

1. Fill in the table below, stating each function's amplitude, period, phase shift and vertical displacement. (If there is none, say so.)

Equation	Amplitude	Period	Phase Shift (how much, and in which direction)	Vertical displacement (how much, and in which direction)	Maximum	Minimum
$y=4\cos\left(x+\frac{\pi}{4}\right)-3$	4	2π	$\pi/4$ left	down 3	1	-7
$y=-7\sin(2x-\pi)+5$ $2(x-\pi/2)$	7	π	$\pi/2$ right	up 5	12	-2
$y=2\cos\left(\frac{1}{2}(x+30^{\circ})\right)-1$	2	720°	30° left	down 1	1	-3

2. Write an equation for a function with the given characteristics:

a) A **sine** function with amplitude 5, period π, phase shift $\frac{\pi}{3}$ left, and vertical displacement 2 down

$$y = 5\sin\left(2\left(x+\pi/3\right)\right) - 2$$

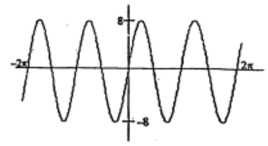
$$b = \frac{\text{normal period}}{\text{graph's period}} = \frac{2\pi}{\pi} = 2$$

b) A **cosine** function with amplitude 2, period 3π, phase shift 5 right, and vertical displacement 4 up

$$y = 2\cos\left(\frac{2}{3}(x-5)\right) + 4$$

$$b = \frac{\text{normal period}}{\text{graph's period}} = \frac{2\cancel{\pi}}{3\cancel{\pi}} = \frac{2}{3}$$

3. For the graph shown below, find:



Amplitude ________

Period ________

A correct equation for it.

____

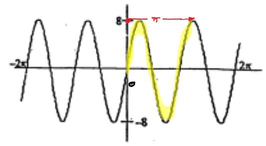
4a) State the ***range*** of the graph for the function $y = 8\cos\left(x - 45^\circ\right) + 5$

b) State the ***range*** of the graph for the function $y = a\sin\left(x + \frac{\pi}{6}\right) + b$, where $a > 0$ and $b > 0$.

5a) A cosine graph has range $-2 \le y \le 12$, a maximum occurs at $(\pi, 12)$, and the period of the function is 8π. What is a possible equation for this?

b) A sine graph has a maximum point at $(4,10)$ and the nearest minimum point to the right of this point is $(14,6)$. What is a possible equation for this?

3. For the graph shown below, find:



Amplitude 8

Period π

A correct equation for it.

$y = 8\sin(2x)$

4a) State the ***range*** of the graph for the function $y = 8\cos\left(x - 45^\circ\right) + 5$

amp = 8

center line at y = 5

max = 5 + 8 = 13

min = 5 − 8 = −3

$\boxed{\text{range: } \{y \mid -3 \le y \le 13\}}$

b) State the ***range*** of the graph for the function $y = a\sin\left(x + \frac{\pi}{6}\right) + b$, where $a > 0$ and $b > 0$.

amp = a

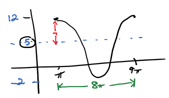
center line at y = b

max = b + a

min = b − a

$\boxed{\text{range: } \{y \mid b-a \le y \le b+a\}}$

5a) A cosine graph has range $-2 \le y \le 12$, a maximum occurs at $(\pi, 12)$, and the period of the function is 8π. What is a possible equation for this?



$\frac{\text{max} + \text{min}}{2}$ = v. disp.

$\frac{-2 + 12}{2} = 5$ up

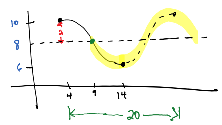
amplitude = max − center
= 12 − 5
= 7

$b = \frac{2\pi}{\text{period}} = \frac{2\pi}{8\pi} = \frac{1}{4}$

p. shift = π right

$\boxed{y = 7\cos\left[\frac{1}{4}(x - \pi)\right] + 5}$

b) A sine graph has a maximum point at $(4,10)$ and the nearest minimum point to the right of this point is $(14,6)$. What is a possible equation for this?



$\frac{\text{max} + \text{min}}{2}$ = v disp.

$\frac{6 + 10}{2} = 8$ up

amplitude = max − center
= 10 − 8
= 2

$b = \frac{2\pi}{\text{period}} = \frac{2\pi}{20} = \frac{\pi}{10}$

p. shift = 9 right, for upside-down sine

$\boxed{y = -2\sin\left(\frac{\pi}{10}(x - 9)\right) + 8}$

6. Consider this sinusoidal equation: $y = 2\cos\left(\frac{2\pi}{16}(x+9)\right) + 1$

a) Identify its key features:

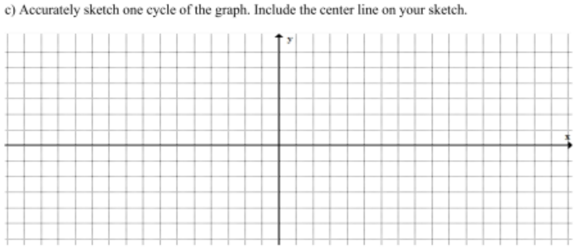
Sketch base graph's shape	vertical displacement	equation of center line
amplitude	**maximum**	**minimum**
period	**spacing**	**phase shift**

b) Use either method shown in class to fill in the table with coordinates of 5 key points on the graph.

Know how to do this without using a graphing calculator. (You can use the calculator to check your results, though.)

x	y

c) Accurately sketch one cycle of the graph. Include the center line on your sketch.



VE 2

HE $\frac{16}{2\pi} = \frac{8}{\pi}$

9 left

1 up

6. Consider this sinusoidal equation: $y = 2\cos\left(\frac{2\pi}{16}(x+9)\right) + 1$

a) Identify its key features:

Sketch base graph's shape	vertical displacement	equation of center line
	up 1	$y = 1$
amplitude	**maximum**	**minimum**
2	CL + amp = 1 + 2 = 3	CL − amp = 1 − 2 = −1
period	**spacing**	**phase shift**
$\frac{2\pi}{\frac{2\pi}{16}} = 2\pi \cdot \frac{16}{2\pi} = 16$	$16 \div 4 = 4$	9 left

b) Use either method shown in class to fill in the table with coordinates of 5 key points on the graph.

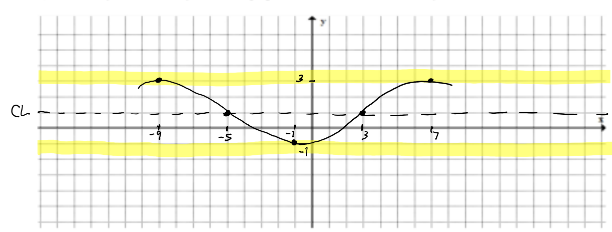
Know how to do this without using a graphing calculator. (You can use the calculator to check your results, though.)

x	y	
−9	3	Max
−5	1	CL
−1	−1	min
3	1	CL
7	3	Max

OR $(x, y) \rightarrow \left(\frac{8}{\pi}x - 9,\ 2y + 1\right)$

x	y	$\frac{8}{\pi}x - 9$	$2y + 1$
0	1	$\frac{8}{\pi}(0) - 9 = -9$	3
$\frac{\pi}{2}$	0	$\frac{8}{\pi}\left(\frac{\pi}{2}\right) - 9 = -5$	1
π	−1	$\frac{8}{\pi}\left(\frac{\pi}{1}\right) - 9 = -1$	−1
$\frac{3\pi}{2}$	0	$\frac{8}{\pi}\left(\frac{3\pi}{2}\right) - 9 = 3$	1
2π	1	$\frac{8}{\pi}\left(\frac{2\pi}{1}\right) - 9 = 7$	3

c) Accurately sketch one cycle of the graph. Include the center line on your sketch.



7. Consider this sinusoidal equation: $y = 4\sin(2x - 60^\circ) + 1$

a) Identify its key features:

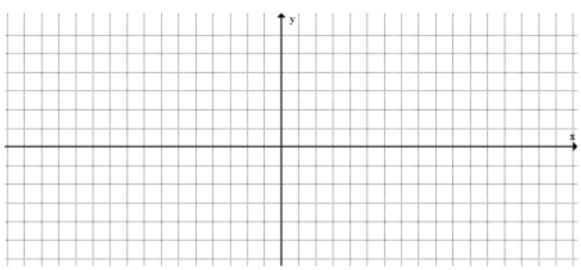
Sketch base graph's shape	vertical displacement	equation of center line
		
amplitude	maximum	minimum
period	spacing	phase shift

b) Use either method shown in class to fill in the table with coordinates of 5 key points on the graph.

Know how to do this without using a graphing calculator. (You can use the calculator to check your results, though.)

x	y

c) Accurately sketch one cycle of the graph. Include the center line on your sketch.

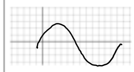


7. Consider this sinusoidal equation: $y = 4\sin(2x - 60^\circ) + 1$

Factor!

$y = 4\sin(2(x - 30^\circ)) + 1$

a) Identify its key features:

Sketch base graph's shape	vertical displacement	equation of center line
	up 1	$y = 1$
amplitude	maximum	minimum
4	CL + amp $= 1 + 4 = 5$	CL − amp $= 1 - 4 = -3$
period	spacing	phase shift
$\frac{360^\circ}{2} = 180^\circ$	$\frac{180^\circ}{4} = 45^\circ$	30° right

b) Use either method shown in class to fill in the table with coordinates of 5 key points on the graph.

Know how to do this without using a graphing calculator. (You can use the calculator to check your results, though.)

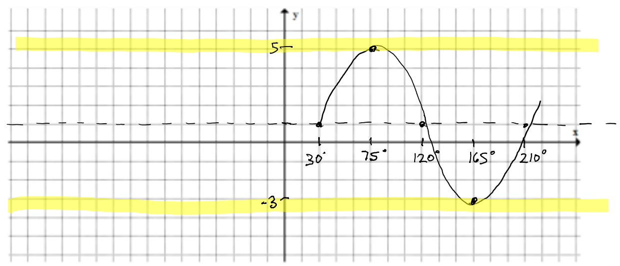
x	y	
30°	1	CL
75°	5	max
120°	1	CL
165°	−3	min
210°	1	CL

OR:

$(x, y) \to \left(\frac{1}{2}x + 30,\ 4y + 1\right)$

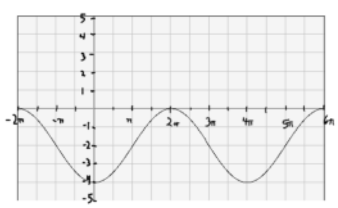
x	y	$\frac{1}{2}x + 30$	$4y + 1$
0°	0	$\frac{1}{2}(0) + 30 = 30^\circ$	1
90°	1	$\frac{1}{2}(90) + 30 = 75^\circ$	5
180°	0	$\frac{1}{2}(180) + 30 = 120^\circ$	1
270°	−1	$\frac{1}{2}(270) + 30 = 165^\circ$	−3
360°	0	$\frac{1}{2}(360) + 30 = 210^\circ$	1

c) Accurately sketch one cycle of the graph. Include the center line on your sketch.



8. For each graph, find two different equations that create it. Give one equation using sine and a different one using cosine.

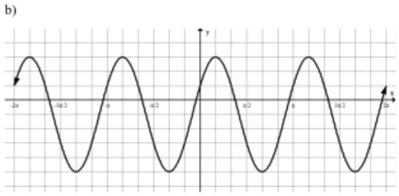
a)



Possible sine equation:

Possible cosine equation:

b)



Possible sine equation:

Possible cosine equation:

9. State the domain and period for each tangent equation:

a) $y = \tan x$

b) $y = \tan(5x)$

10. The depth of water, h meters, at a certain ocean port, at time t hours, is given by the equation

$$h(t) = 1.8\sin\left(\frac{2\pi}{12.4}(t - 1.4)\right) + 2.6.$$ (Note $t = 0$ corresponds to midnight.)

a) What is the depth of the water at 7:00 A.M., correct to 2 decimal places?

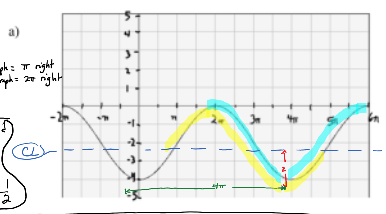
b) What is the depth of the water at 7:00 P.M., correct to 2 decimal places?

8. For each graph, find two different equations that create it. Give one equation using sine and a different one using cosine.

a)

amp = 2
vdisp = down 2
period = 4π
p.s. for sine graph = π right
p.s. for cosine graph = 2π right

To get period 4π, we need $b = \frac{2\pi}{4\pi} = \frac{2}{4} = \frac{1}{2}$



Possible sine equation:

$y = 2\sin\left(\frac{1}{2}(x - \pi)\right) - 2$

Possible cosine equation:

$y = 2\cos\left(\frac{1}{2}(x - 2\pi)\right) - 2$

OR

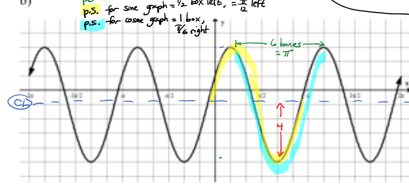
$y = -2\cos\left(\frac{1}{2}x\right) - 2$

(no p.shift needed, for an upside-down cosine graph)

b)

amp = 4
vdisp = down 1
period = π
p.s. for sine graph = ½ box left, $= \frac{\pi}{12}$ left
p.s. for cosine graph = 1 box, $\frac{\pi}{6}$ right

To get period of π, we need $b = \frac{2\pi}{\pi}$, so $b = 2$



Possible sine equation:

$y = 4\sin\left(2\left(x + \frac{\pi}{12}\right)\right) - 1$

Possible cosine equation:

$y = 4\cos\left(2\left(x - \frac{\pi}{6}\right)\right) - 1$

9. State the domain and period for each tangent equation:

a) $y = \tan x$

$x \neq \frac{\pi}{2} + n\pi,\ n \in I$

period $= \pi$

b) $y = \tan(5x)$

HC ⅕

$x \neq \frac{1}{5}\left(\frac{\pi}{2} + n\pi\right)$

$x \neq \frac{\pi}{10} + \frac{n\pi}{5},\ n \in I$

period $= \pi/5$

10. The depth of water, h meters, at a certain ocean port, at time t hours, is given by the equation

$$h(t) = 1.8\sin\left(\frac{2\pi}{12.4}(t - 1.4)\right) + 2.6.$$ (Note $t = 0$ corresponds to midnight.)

a) What is the depth of the water at 7:00 A.M.?

Use $t = 7$ in equation

$h(7) = 1.8\sin\left(\frac{2\pi}{12.4}(7 - 1.4)\right) + 2.6$

$\doteq \boxed{3.14\text{ m}}$

b) What is the depth of the water at 7:00 P.M.?

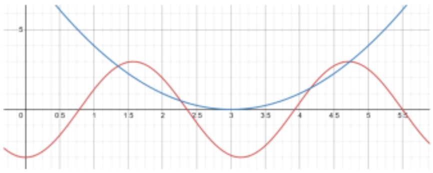
use $t = 19$ in equation, this is 19 hours after midnight

$h(19) = 1.8\sin\left(\frac{2\pi}{12.4}(19 - 1.4)\right) + 2.6$

$\doteq \boxed{3.47\text{ m}}$

11. We want to solve this equation ***graphically***: $2\sin\left(2\left(x-\frac{\pi}{4}\right)\right)=(x-3)^2$

Use the graphs of $y=3\sin\left(2\left(x-\frac{\pi}{4}\right)\right)$ and $y=(x-3)^2$, shown on the grid below, to solve the above equation, correct to the nearest tenth.



Solutions are:

12. A Ferris wheel with radius of 64 m has its center 65 m above the ground. It rotates once every 50 seconds. Suppose a rider gets on at the lowest point at $t = 0$.

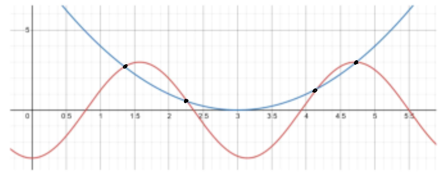
a) Write an equation to model the height of a passenger above the ground, as a function of time.

b) To the nearest tenth of a second, at what time will the passenger first be 35 m above the ground? Solve this algebraically. (If you like, check your answer by solving graphically.)

11. We want to solve this equation ***graphically***: $2\sin\left(2\left(x-\frac{\pi}{4}\right)\right)=(x-3)^2$

Use the graphs of $y=3\sin\left(2\left(x-\frac{\pi}{4}\right)\right)$ and $y=(x-3)^2$, shown on the grid below, to solve the above equation, correct to the nearest tenth.

x-values of the intersections are the solutions

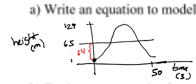


Solutions are:

$x \doteq 1.4,\ 2.2,\ 4.1,\ 4.7$

12. A Ferris wheel with radius of 64 m has its center 65 m above the ground. It rotates once every 50 seconds. Suppose a rider gets on at the lowest point at $t = 0$.

a) Write an equation to model the height of a passenger above the ground, as a function of time.



$h(t) = -64\cos\left(\frac{2\pi}{50}t\right) + 65$

OR

$h(t) = -64\cos\left(\frac{\pi}{25}t\right) + 65$

b) To the nearest tenth of a second, at what time will the passenger first be 35 m above the ground? Solve this algebraically. (If you like, check your answer by solving graphically.)

$35 = -64\cos\left(\frac{\pi}{25}t\right) + 65$

$64\cos\left(\frac{\pi}{25}t\right) = 30$

$\cos\left(\frac{\pi}{25}t\right) = \frac{30}{64}$

$\left(\frac{\pi}{25}t\right) = \cos^{-1}\left(\frac{30}{64}\right)$

$\frac{25}{\pi}\left(\frac{\pi}{25}t\right) = (1.082921179)\frac{25}{\pi}$

$t = 8.6176\ldots$

$\Rightarrow \boxed{t \doteq 8.6 \text{ seconds}}$