

Chapter 6 Hand-in Assignment – Trigonometric Identities

Name: _____

1. Simplify each expression.

a) $\frac{\tan x}{\sec x}$

b) $\frac{1}{\tan x \csc x}$

c) $\frac{1 - \cot x}{\tan x - 1} = \frac{1 - \frac{\cos x}{\sin x}}{\frac{\sin x}{\cos x} - 1} = \frac{\frac{\sin x - \cos x}{\sin x}}{\frac{\sin x - \cos x}{\cos x}} = \frac{\sin x - \cos x}{\sin x} \cdot \frac{\cos x}{\sin x - \cos x} = \frac{\cos x}{\sin x} = \cot x$

To get rid of the fractions, we can do the denominator first for the ENTIRE expression
Denominator: $\cos x \sin x$
 2) multiply by CD "top" and "bottom"

d) $\frac{1 + \cot^2 x}{\cot^2 x} = \frac{1 + \frac{\cos^2 x}{\sin^2 x}}{\frac{\cos^2 x}{\sin^2 x}} = \frac{\frac{\sin^2 x + \cos^2 x}{\sin^2 x}}{\frac{\cos^2 x}{\sin^2 x}} = \frac{\sin^2 x + \cos^2 x}{\sin^2 x} \cdot \frac{\sin^2 x}{\cos^2 x} = \frac{\sin^2 x + \cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$

e) $\sec x \cos x + \frac{\cos^2 x}{\sin^2 x} = \frac{\cos x}{\sin x} \cos x + \frac{\cos^2 x}{\sin^2 x} = \frac{\cos^2 x}{\sin x} + \frac{\cos^2 x}{\sin^2 x} = \frac{\cos^2 x \sin x + \cos^2 x}{\sin^2 x} = \frac{\cos^2 x (\sin x + 1)}{\sin^2 x}$

f) $\frac{1}{\sec^2 x} + \frac{1}{\csc^2 x} = \cos^2 x + \sin^2 x = 1$

2. Verify the following identity algebraically, for $x = \frac{\pi}{4}$

$\frac{1 - \sin x}{\cos x} = \frac{\cos x}{1 + \sin x}$

Chapter 6 Hand-in Assignment – Trigonometric Identities

Name: Key

1. Simplify each expression.

a) $\frac{\tan x}{\sec x} = \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}} = \sin x$

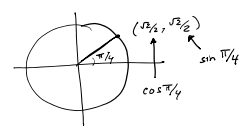
b) $\frac{1}{\tan x \csc x} = \frac{1}{\frac{\sin x}{\cos x} \cdot \frac{1}{\sin x}} = \frac{1}{\frac{1}{\cos x}} = \cos x$

c) $\frac{1 - \cot x}{\tan x - 1} = \frac{1 - \frac{\cos x}{\sin x}}{\frac{\sin x}{\cos x} - 1} = \frac{\frac{\sin x - \cos x}{\sin x}}{\frac{\sin x - \cos x}{\cos x}} = \frac{\sin x - \cos x}{\sin x} \cdot \frac{\cos x}{\sin x - \cos x} = \frac{\cos x}{\sin x} = \cot x$

d) $\frac{1 + \cot^2 x}{\cot^2 x} = \frac{1 + \frac{\cos^2 x}{\sin^2 x}}{\frac{\cos^2 x}{\sin^2 x}} = \frac{\frac{\sin^2 x + \cos^2 x}{\sin^2 x}}{\frac{\cos^2 x}{\sin^2 x}} = \frac{\sin^2 x + \cos^2 x}{\sin^2 x} \cdot \frac{\sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$

e) $\sec x \cos x + \frac{\cos^2 x}{\sin^2 x} = \frac{\cos x}{\sin x} \cos x + \frac{\cos^2 x}{\sin^2 x} = \frac{\cos^2 x}{\sin x} + \frac{\cos^2 x}{\sin^2 x} = \frac{\cos^2 x \sin x + \cos^2 x}{\sin^2 x} = \frac{\cos^2 x (\sin x + 1)}{\sin^2 x}$

f) $\frac{1}{\sec^2 x} + \frac{1}{\csc^2 x} = \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} = \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} = \frac{1}{\sin^2 x \cos^2 x}$



2. Verify the following identity algebraically, for $x = \frac{\pi}{4}$

$\frac{1 - \sin x}{\cos x} = \frac{\cos x}{1 + \sin x}$

$\left(\frac{1 - \frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} \right) = \left(\frac{\frac{\sqrt{2}}{2}}{1 + \frac{\sqrt{2}}{2}} \right)$

0.414... = 0.414...

Same value \Rightarrow Verified

3. Write each expression as a single trigonometric function.

a) $\cos 20^\circ \cos 5^\circ + \sin 20^\circ \sin 5^\circ$

b) $2 \cos^2\left(\frac{\pi}{5}\right) - 1$

c) $2 \sin 7x \cos 7x$

3. Write each expression as a single trigonometric function.

a) $\cos 20^\circ \cos 5^\circ + \sin 20^\circ \sin 5^\circ$

$= \cos(20^\circ - 5^\circ)$
 $= \cos 15^\circ$

Using $\cos A \cos B + \sin A \sin B = \cos(A - B)$

b) $2 \cos^2\left(\frac{\pi}{5}\right) - 1$
 $= \cos\left(2 \cdot \frac{\pi}{5}\right)$
 $= \cos\left(\frac{2\pi}{5}\right)$

Using $\cos 2\theta = 2\cos^2\theta - 1$

c) $2 \sin 7x \cos 7x$
 $= \sin(2 \cdot 7x)$
 $= \sin(14x)$

Using $\sin 2\theta = 2\sin\theta \cos\theta$

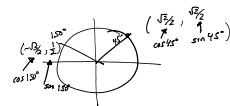
4. Use identities and special angle values to determine the exact value of each trigonometric expression.

a) $\cos 195^\circ$

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a) $\cos 195^\circ$

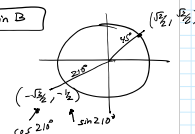
$$\begin{aligned} & \text{Using } \cos(A+B) = \cos A \cos B - \sin A \sin B \\ & = \cos(150^\circ + 45^\circ) \\ & = \cos 150^\circ \cos 45^\circ - \sin 150^\circ \sin 45^\circ \\ & = \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) \\ & = -\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{-\sqrt{6}-\sqrt{2}}{4} \end{aligned}$$



b) $\sin 255^\circ$

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$$\begin{aligned} & \text{Using } \sin(A+B) = \sin A \cos B + \cos A \sin B \\ & = \sin(210^\circ + 45^\circ) \\ & = \sin 210^\circ \cos 45^\circ + \cos 210^\circ \sin 45^\circ \\ & = \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) \\ & = -\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{-\sqrt{2}-\sqrt{6}}{4} \end{aligned}$$

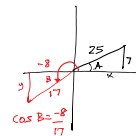


5. If $\angle A$ is in quadrant I, $\angle B$ is in quadrant III, and $\sin A = \frac{7}{25}$ and $\cos B = -\frac{8}{17}$, use identities to evaluate each of the following:

a) $\sin(A-B)$

5. If $\angle A$ is in quadrant I, $\angle B$ is in quadrant III, and $\sin A = \frac{7}{25}$ and $\cos B = -\frac{8}{17}$, use identities to evaluate each of the following:

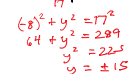
$$\begin{aligned} & \sin(A-B) = \sin A \cos B - \cos A \sin B \\ & = \left(\frac{7}{25}\right)\left(-\frac{8}{17}\right) - \left(\frac{24}{25}\right)\left(-\frac{15}{17}\right) \\ & = \frac{-56}{425} - \left(\frac{-360}{425}\right) = \frac{304}{425} \end{aligned}$$



$$\begin{aligned} \sin A &= \frac{7}{25} \\ x^2 + 7^2 &= 25^2 \\ x^2 &= 625 - 49 \\ x^2 &= 576 \\ x &= \pm 24, \text{ use } x=24 \\ \cos A &= \frac{24}{25} \end{aligned}$$

b) $\cos 2B$

$$\begin{aligned} & \cos 2B = \cos^2 B - \sin^2 B \\ & = (\cos B)^2 - (\sin B)^2 \\ & = \left(-\frac{8}{17}\right)^2 - \left(-\frac{15}{17}\right)^2 \\ & = \frac{64}{289} - \frac{225}{289} = \frac{-161}{289} \end{aligned}$$



$$\begin{aligned} \cos B &= -\frac{8}{17} \\ (-8)^2 + y^2 &= 17^2 \\ 64 + y^2 &= 289 \\ y^2 &= 225 \\ y &= \pm 15, \text{ use } y=15 \\ \sin B &= \frac{-15}{17} \end{aligned}$$

c) $\sin 2A$

$$\begin{aligned} & \sin 2A = 2 \sin A \cos A \\ & = 2\left(\frac{7}{25}\right)\left(\frac{24}{25}\right) \\ & = \frac{336}{625} \end{aligned}$$

6. Prove the following identities:

a) $\frac{1 + \cos 2x}{\sin 2x} = \cot x$

6. Prove the following identities:

a) $\frac{1 + \cos 2x}{\sin 2x} = \cot x$

$$\begin{aligned} & \frac{1 + (2\cos^2 x - 1)}{2 \sin x \cos x} \\ & = \frac{\sqrt{2\cos^2 x - 1}}{2 \sin x \cos x} \\ & = \frac{2\cos^2 x}{2 \sin x \cos x} \\ & = \frac{\cos x}{\sin x} \\ & = \cot x \end{aligned}$$

b) $\frac{1 - \sin^2 x - 2 \cos x}{\cos^2 x - \cos x - 2} = \frac{1}{1 + \sec x}$

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$$\begin{aligned} & \frac{\cos^2 x - 2 \cos x}{(\cos x + 1)(\cos x - 2)} \\ & = \frac{\cos x (\cos x - 2)}{(\cos x + 1)(\cos x - 2)} \\ & = \frac{\cos x}{\cos x + 1} \end{aligned}$$

$$\begin{aligned} & \left(1 + \frac{1}{\cos x}\right) \frac{\cos x}{\cos x} \\ & = \frac{\cos x}{\cos x + \frac{\cos x}{\cos x}} \\ & = \frac{\cos x}{\cos x + 1} \end{aligned}$$

c) $\frac{\csc x + \cot x}{\tan x + \sin x} = \cot x \csc x$



d) $\sec x + \tan x = \frac{\cos x}{1 - \sin x}$



7. Solve each equation algebraically, over the given domain. You will need to use identities!

a) ~~$\sin 2x + \cos x = 0$~~ for $0 \leq x < 360^\circ$.

$\sin 2x + \cos x = 0$

b) $\sin^2 x = \cos x - \cos 2x$, for $0 \leq x < 2\pi$

c) $\frac{\csc x + \cot x}{\tan x + \sin x} = \cot x \csc x$

$$\frac{\frac{1}{\sin x} + \frac{\cos x}{\sin x}}{\frac{\sin x}{\cos x} + \sin x} = \frac{\cos x}{\sin x} \cdot \frac{1}{\sin x}$$

$$\frac{\frac{1 + \cos x}{\sin x}}{\frac{\sin x + \sin^2 x}{\cos x}} = \frac{\cos x}{\sin^2 x}$$

$$\frac{1 + \cos x}{\sin x} \cdot \frac{\cos x}{\sin x + \sin^2 x} = \frac{\cos x}{\sin^2 x}$$

$$\frac{\cos x (1 + \cos x)}{\sin^2 x (1 + \sin x)} = \frac{\cos x}{\sin^2 x}$$

$$\frac{\cos x (1 + \cos x)}{\sin^2 x (1 + \sin x)} \cdot \frac{1 + \sin x}{1 + \sin x} = \frac{\cos x}{\sin^2 x}$$

$$\frac{\cos x (1 + \cos x)(1 + \sin x)}{\sin^2 x (1 + \sin x)^2} = \frac{\cos x}{\sin^2 x}$$

$$\frac{\cos x (1 + \cos x)}{\sin^2 x (1 + \sin x)} = \frac{\cos x}{\sin^2 x}$$

$$\frac{\cos x}{\sin x} \cdot \frac{1}{\sin x}$$

$$\frac{\cos x}{\sin^2 x}$$

d) $\sec x + \tan x = \frac{\cos x}{1 - \sin x}$

$$\frac{1}{\cos x} + \frac{\sin x}{\cos x}$$

$$\frac{1 + \sin x}{\cos x}$$

$$\frac{\cos x}{1 - \sin x} \cdot \frac{(1 + \sin x)}{(1 + \sin x)}$$

$$\frac{\cos x (1 + \sin x)}{1 + \sin x - \sin x - \sin^2 x}$$

$$\frac{\cos x (1 + \sin x)}{1 - \sin^2 x}$$

$$\frac{\cos x (1 + \sin x)}{\cos^2 x}$$

$$\frac{1 + \sin x}{\cos x}$$

Multiply by conjugate.

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7. Solve each equation algebraically, over the given domain. You will need to use identities!

a) ~~$\sin 2x + \cos x = 0$~~ for $0 \leq x < 360^\circ$ (radians/degrees)

$\sin 2x + \cos x = 0$

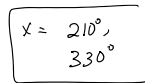
$2 \sin x \cos x + \cos x = 0$

$\cos x (2 \sin x + 1) = 0$

$\cos x = 0$



$2 \sin x + 1 = 0$
 $2 \sin x = -1$
 $\sin x = -\frac{1}{2}$



b) $\sin^2 x = \cos x - \cos 2x$, for $0 \leq x < 2\pi$ (Use radians for answers)

$1 - \cos^2 x = \cos x - (2 \cos^2 x - 1)$

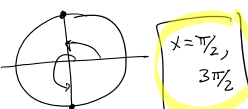
$1 - \cos^2 x = \cos x - 2 \cos^2 x + 1$

$2 \cos^2 x - \cos^2 x - \cos x = 0$

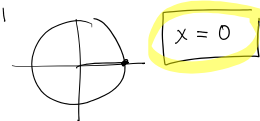
$\cos^2 x - \cos x = 0$

$\cos x (\cos x - 1) = 0$

$\cos x = 0$



$\cos x - 1 = 0$
 $\cos x = 1$



Use identities to get equations all in terms of cos(x).