C_16 Ch 7 Key 2022 Hand-in

Tuesday, March 1, 2022 4:11 PM

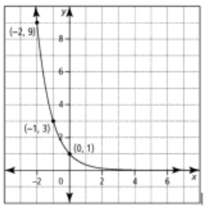
C_16 Ch 7 Hand-in 2022

Chapter 7 Hand-in Assignment – Exponential Functions

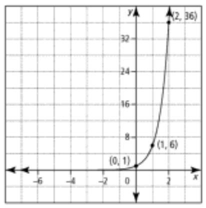
Name: ______________________________

1. Fill in the characteristics for each graph shown below.

a)



b)



	Graph (a)	Graph (b)
Domain		
Range		
Equation of its horizontal asymptote		
Coordinates of its y-intercept		
Whether graph is *increasing* or *decreasing*		
Equation of the graph, in the form $y = c^x$		

2. Label each of the following exponential equations as either exponential ***growth*** or ***decay***.

a) $y = 2^x$ b) $y = 5\left(\frac{1}{3}\right)^{4x}$ c) $y = 4(0.2)^{3x+2}$ d) $y = \left(\frac{7}{3}\right)^{x-1}$

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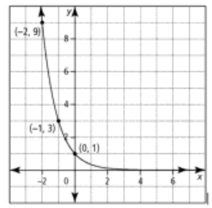
Chapter 7 Hand-in Assignment – Exponential Functions

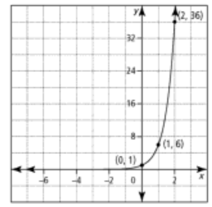
Name: Key

1. Fill in the characteristics for each graph shown below.

a)



b)



	Graph (a)	Graph (b)
Domain	$\{x \mid x \in \mathbb{R}\}$	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y > 0, y \in \mathbb{R}\}$	$\{y \mid y > 0, y \in \mathbb{R}\}$
Equation of its horizontal asymptote	$y = 0$	$y = 0$
Coordinates of its y-intercept	$(0, 1)$	$(0, 1)$
Whether graph is *increasing* or *decreasing*	decreasing	increasing
Equation of the graph, in the form $y = c^x$	$y = \left(\frac{1}{3}\right)^x$	$y = 6^x$

base > 1 0 < base < 1

2. Label each of the following exponential equations as either exponential ***growth*** or ***decay***.

a) $y = 2^x$ b) $y = 5\left(\frac{1}{3}\right)^{4x}$ c) $y = 4(0.2)^{3x+2}$ d) $y = \left(\frac{7}{3}\right)^{x-1}$

growth decay decay growth

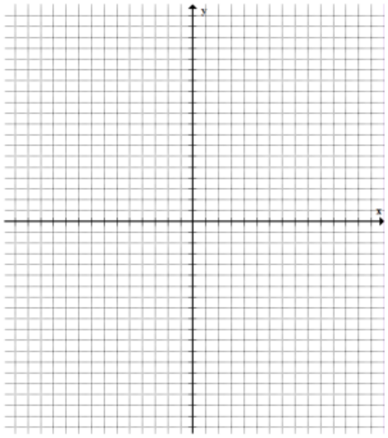
3a) Give the ***mapping*** that shows what happens to points on the base function, $y = 3^x$, when the equation is changed to $y = 6(3)^{x-4} - 2$.

b) Give the equation of the asymptote for the transformed equation's graph.

c) Fill in the table of key points for the base function. Then, use the mapping to create the table of image points that result when those original points are transformed. If any points are fractional, leave them in fraction form (not decimal form).

Sketch the transformed graph on the grid. Include the horizontal asymptote on the graph, using a dotted line.

x	y



d) State the domain and range for the final, transformed graph.

3a) Give the ***mapping*** that shows what happens to points on the base function, $y = 3^x$, when the equation is changed to $y = 6(3)^{x-4} - 2$.

VE 6 4 right 2 down

$(x, y) \rightarrow (x+4,\ 6y-2)$

b) Give the equation of the asymptote for the transformed equation's graph.

$y = -2$

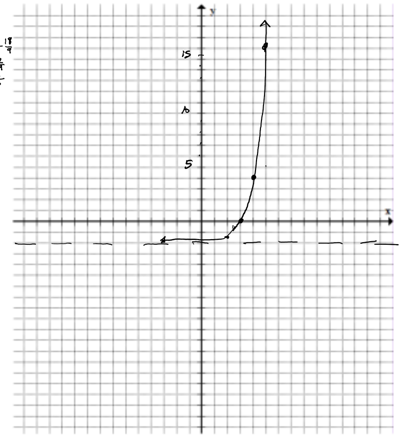
c) Fill in the table of key points for the base function. Then, use the mapping to create the table of image points that result when those original points are transformed. If any points are fractional, leave them in fraction form (not decimal form).

Sketch the transformed graph on the grid. Include the horizontal asymptote on the graph, using a dotted line.

$y = 3^x$

x	y
-2	$3^{-2} = \frac{1}{9}$
-1	$3^{-1} = \frac{1}{3}$
0	$3^0 = 1$
1	$3^1 = 3$
2	$3^2 = 9$

$x+4$	$6y-2$
2	$\frac{6}{1}\left(\frac{1}{9}\right)-2$ $= \frac{6}{9} - \frac{18}{9}$ $= -\frac{12}{9}$ $= -\frac{4}{3}$
3	$\frac{6}{1}\left(\frac{1}{3}\right)-2 = 0$
4	$6(1)-2 = 4$
5	$6(3)-2 = 16$
6	$6(9)-2 = 52$



d) State the domain and range for the final, transformed graph.

$\{x \mid x \in \mathbb{R}\}$

$\{y \mid y > -2,\ y \in \mathbb{R}\}$

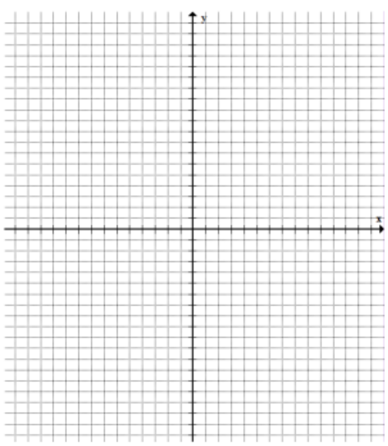
4a) Give the ***mapping*** that shows what happens to points on the base function, $y = \left(\frac{1}{3}\right)^x$, when the equation is changed to $y = -\left(\frac{1}{3}\right)^{\frac{1}{2}(x-3)} + 2$.

b) Give the equation of the asymptote for the transformed equation's graph.

c) Fill in the table of key points for the base function. Then, use the mapping to create the table of image points that result when those original points are transformed. If any points are fractional, leave them in fraction form (not decimal form).

Sketch the transformed graph on the grid. Include the horizontal asymptote on the graph, using a dotted line.

x	y



d) State the domain and range for the final, transformed graph.

4a) Give the ***mapping*** that shows what happens to points on the base function, $y = \left(\frac{1}{3}\right)^x$, when the equation is changed to $y = -\left(\frac{1}{3}\right)^{\frac{1}{2}(x-3)} + 2$.

HE 2 reflect across x-axis right 3 up 2

$(x, y) \rightarrow (2x+3,\ -y+2)$

b) Give the equation of the asymptote for the transformed equation's graph.

$y = 2$

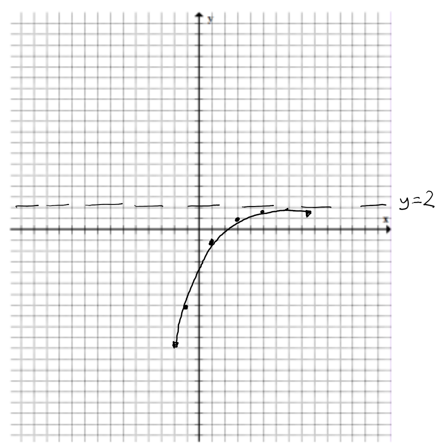
c) Fill in the table of key points for the base function. Then, use the mapping to create the table of image points that result when those original points are transformed. If any points are fractional, leave them in fraction form (not decimal form).

Sketch the transformed graph on the grid. Include the horizontal asymptote on the graph, using a dotted line.

$y = \left(\frac{1}{3}\right)^x$

x	y
-2	$\left(\frac{1}{3}\right)^{-2} = 9$
-1	$\left(\frac{1}{3}\right)^{-1} = 3$
0	$\left(\frac{1}{3}\right)^0 = 1$
1	$\left(\frac{1}{3}\right)^1 = \frac{1}{3}$
2	$\left(\frac{1}{3}\right)^2 = \frac{1}{9}$

$2x+3$	$-y+2$
-1	-7
1	-1
3	1
5	$5/3$
7	$17/9$



d) State the domain and range for the final, transformed graph.

$\{x \mid x \in \mathbb{R}\}$

$\{y \mid y < 2,\ y \in \mathbb{R}\}$

5. The function $y = -2(3^x)$ is transformed by translating 6 units left and 1 unit up.

a) What is the equation of the transformed function?

b) The transformed function passes through the point $(x, -17)$. Determine the value of x.

6. For each part, create an exponential equation that describes the growth or decay.

a) A sample of cells contains 450 cells. It doubles every 3 days.

b) The population of rabbits in a park is increasing by 15% every year. Presently there are 28 rabbits in the park.

c) A 65-gram sample of a radioactive substance decays, with half-life 1820 years.

d) The intensity of light seen when scuba diving is reduced by 12% for each meter one descends into the lake. (Remember, at the surface of the water one can see 100% of the available light.)

e) Jane invests \$1200 in an account that is compounded quarterly, at a rate of 3% per year.

5. The function $y = -2(3^x)$ is transformed by translating 6 units left and 1 unit up.

a) What is the equation of the transformed function?

$$y = -2(3^{x+6}) + 1$$

b) The transformed function passes through the point $(x, -17)$. Determine the value of x.

$$-17 = -2(3^{x+6}) + 1$$

$$\frac{-18}{-2} = \frac{-2}{-2}(3^{x+6})$$

$$9 = 3^{x+6}$$

$$3^2 = 3^{x+6}$$

$$\Rightarrow \quad 2 = x + 6$$

$$-4 = x$$

$$x = -4$$

6. For each part, create an exponential equation that describes the growth or decay.

a) A sample of cells contains 450 cells. It doubles every 3 days.

base = 2

$$A = 450(2)^{t/3}$$

b) The population of rabbits in a park is increasing by 15% every year. Presently there are 28 rabbits in the park.

base = 100% + 15% = 115% = 1.15

$$P = 28(1.15)^{t/1}$$

c) A 65-gram sample of a radioactive substance decays, with half-life 1820 years.

base = $\frac{1}{2}$ or 0.5

$$A = 65(0.5)^{t/1820}$$

d) The intensity of light seen when scuba diving is reduced by 12% for each meter one descends into the lake. (Remember, at the surface of the water one can see 100% of the available light.)

→ initial amount, as %

base = 100% − 12% = 88% = 0.88

$$I = 100(0.88)^{d/1}$$

e) Jane invests \$1200 in an account that is compounded quarterly, at a rate of 3% per year.

$$A = 1200(1.0075)^n,$$

where n = number of compounding periods

base $= 1 + \frac{0.03}{4} = 1.0075$

7. Solve for x, algebraically. (Hint: re-write equation in terms of a common base)

a) $16^{2x-1} = 32^{x+4}$

b) $\left(\sqrt{5}\right)^{3x+1} = \left(5^2\right)^{2x-5}$

c) $\left(\frac{1}{81}\right)^x = 3^{x+3}(27)^{2x+1}$

7. Solve for x, algebraically. (Hint: re-write equation in terms of a common base)

a) $16^{2x-1} = 32^{x+4}$

$$(2^4)^{2x-1} = (2^5)^{x+4}$$

$$2^{8x-4} = 2^{5x+20}$$

$$\Rightarrow \quad 8x - 4 = 5x + 20$$

$$3x = 24$$

$$x = 8$$

b) $\left(\sqrt{5}\right)^{3x+1} = \left(5^2\right)^{2x-5}$

$$\left(5^{1/2}\right)^{3x+1} = \left(5^2\right)^{2x-5}$$

$$5^{\frac{3}{2}x + \frac{1}{2}} = 5^{4x-10}$$

$$\Rightarrow \quad \frac{3}{2}x + \frac{1}{2} = 4x - 10$$

$$2\left(\frac{1}{2} + 10\right) = 2\left(4x - \frac{3}{2}x\right)$$

This step clears the fractions.

$$1 + 20 = 8x - 3x$$

$$\frac{21}{5} = \frac{5x}{5}$$

$$x = 21/5 \text{ or } 4.2$$

c) $\left(\frac{1}{81}\right)^x = 3^{x+3}(27)^{2x+1}$

$$\left(\frac{1}{3^4}\right)^x = 3^{x+3}\left(3^3\right)^{2x+1}$$

$$\left(3^{-4}\right)^x = 3^{x+3}\, 3^{6x+3}$$

$$3^{-4x} = 3^{x+3+6x+3}$$

Multiplying two terms with the same base. Keep the base, add the exponents: $a^m a^n = a^{m+n}$

$$3^{-4x} = 3^{7x+6}$$

$$\Rightarrow \quad -4x = 7x + 6$$

$$\frac{-11x}{-11} = \frac{6}{-11}$$

$$x = 6/-11$$

8. The trout population in a lake has been doubling every 4 years. There were 300 trout at the initial count.

a) Write an exponential equation giving the number of trout, N, at time t.

b) Determine the number of trout this equation predicts there will be 7 years after the initial count. *(Round down to the nearest whole trout)*

9. A town had a population of 9700 people in 2005. Each year since then the population has decreased by 8%.

a) Write an exponential equation to represent the population, P, of the town at time t.

b) What will the population be in the year 2030? (*Round down to the nearest whole person.*)

base = 2

8. The trout population in a lake has been doubling every 4 years. There were 300 trout at the initial count.

a) Write an exponential equation giving the number of trout, N, at time t.

$$N(t) = 300(2)^{t/4}$$

b) Determine the number of trout this equation predicts there will be 7 years after the initial count. *(Round down to the nearest whole trout)*

N(7) does NOT mean we are multiplying by 7. We are evaluating the function when t = 7.

$$N(7) = 300(2)^{7/4}$$

$$= 1009.075...$$

$$\Rightarrow \doteq \boxed{1009 \text{ trout}}$$

9. A town had a population of 9700 people in 2005. Each year since then the population has decreased by 8%.

base = 100% − 8%
= 92% = 0.92

a) Write an exponential equation to represent the population, P, of the town at time t.

$$P(t) = 9700(0.92)^{t/1}$$

b) What will the population be in the year 2030? (*Round down to the nearest whole person.*)

This is 25 years after the initial count.

P(25) means we are evaluating the function when t = 25

$$P(25) = 9700(0.92)^{25}$$

$$= 1206.3335...$$

$$\doteq \boxed{1206 \text{ people}}$$