C_19 Ch 8 Hand-in 2022 with Natural Log

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Chapter 8 Hand-in Assignment – Logarithms

Name:

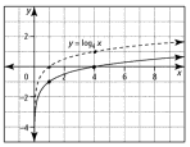
1. Describe how the graph of each logarithmic function can be obtained from the graph of $y = \log_2 x$.

a) $y = \log_2(-5(x+4)) - 1$

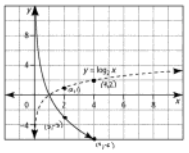
b) $y = 2\log_2(x-7) + 4$

2. For each graph, the solid curve is a transformation of the dashed curve. Write the equation of each solid graph.

a)



b)

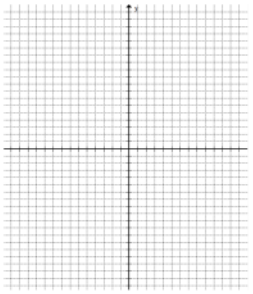


3. Given the base function $y = \log_2 x$ and its transformed function, $y = -2\log_2(x+3) - 1$ do the following:

- List the transformations that will occur and give the mapping notation
- Complete the tables, showing 5 key points for each table. Include mapping notation on the table heading.

original

x	y



- On the provided grid, sketch and label the original and transformed graphs.
- For the transformed function, give
  - Domain
  - Range
  - The equation of its asymptote
  - The coordinates of its x-intercept and y-intercept

Chapter 8 Hand-in Assignment – Logarithms

Name: Key

1. Describe how the graph of each logarithmic function can be obtained from the graph of $y = \log_2 x$.

a) $y = \log_2(-5(x+4)) - 1$

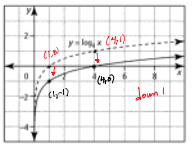
reflect across y-axis
HC by 1/5
4 left
1 down

b) $y = 2\log_2(x-7) + 4$

VE by 2
7 right
4 up

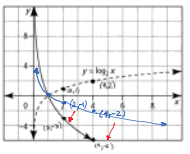
2. For each graph, the solid curve is a transformation of the dashed curve. Write the equation of each solid graph.

a)



$y = \log_4(x) - 1$

b)



reflect across x-axis
VE by 3

$y = -3\log_2(x)$

3. Given the base function $y = \log_2 x$ and its transformed function, $y = -2\log_2(x+3) - 1$ do the following:

$y = -2\log_2(x+4) - 1$

- List the transformations that will occur and give the mapping notation

VE 2
reflect across x-axis
4 left
1 down

$(x, y) \longrightarrow (x-4, -2y-1)$

- Complete the tables, showing 5 key points for each table. Include mapping notation on the table heading.

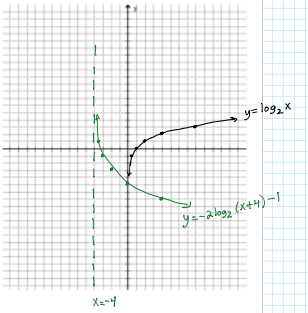
$y = \log_2 x$ original

x	y	
1/2	-1	$y = \log_2(1/2)$
1	0	$y = \log_2 1$
2	1	$y = \log_2 2$
4	2	$y = \log_2 4$
8	3	$y = \log_2 8$

Choose these pts?

$y = -2\log_2(x+4) - 1$

x-4	-2y-1
-3 1/2	1
-3	-1
-2	-3
0	-5
4	-7



- On the provided grid, sketch and label the original and transformed graphs.
- For the transformed function, give
  - Domain $\{x \mid x > -4, x \in \mathbb{R}\}$
  - Range $\{y \mid y \in \mathbb{R}\}$
  - The equation of its asymptote $x = -4$
  - The coordinates of its x-intercept and y-intercept

x-int
let $y = 0$

$0 = -2\log_2(x+4) - 1$

$\frac{1}{-2} = \frac{-2\log_2(x+4)}{-2}$

$-\frac{1}{2} = \log_2(x+4)$

change form $2^{-1/2} = x + 4$

$2^{-1/2} - 4 = x$

$x = \frac{1}{\sqrt{2}} - 4 \doteq -3.29$

$(\frac{1}{\sqrt{2}} - 4, 0)$
$\doteq (-3.29, 0)$

y-int
let $x = 0$

$y = -2\log_2(0+4) - 1$

$y = -2\log_2(4) - 1$

$y = -2(2) - 1$

$y = -4 - 1$

$y = -5$

$(0, -5)$

4. The graph of $y = \log_3 x$ has been transformed as described below. Give the equation of each transformed function:

a) $y = \log_3 x$ is expanded vertically by a factor of 4, expanded horizontally by a factor of 3, reflected across the y-axis, and is translated 2 left and 5 down.

b) $y = \log_3 x$ is compressed vertically by a factor of ½, stretched horizontally by a factor of 8, reflected across the x-axis, and is translated 3 right and 6 up.

5. Write in logarithmic form.

a) $4^{-2} = 0.0625$ b) $5^t = r+6$ c) $e^x = 8$

6. Write in exponential form.

a) $\log_2 512 = 9$ b) $\ln(16) = t$ c) $\log_2(a-4) = b$

7. Use the definition of logarithms to find the value of each expression below.

a) $\log_3 81$ b) $\log_4\left(\frac{1}{64}\right)$

8. Solve the following equations for x. If answer is not exact, give it correct to **2 decimal places.**

a) $\log_4(x-8) = 5$ b) $\log_x(18) = \frac{3}{4}$

c) $\ln(x) + \ln(8) = \ln 32$ d) $\log_6(3x^7) - \log_6(x^6) = 2$

4. The graph of $y = \log_3 x$ has been transformed as described below. Give the equation of each transformed function:

a) $y = \log_3 x$ is expanded vertically by a factor of 4, expanded horizontally by a factor of 3, reflected across the y-axis, and is translated 2 left and 5 down.

$y = 4\log_3\left(-\frac{1}{3}(x+2)\right) - 5$

b) $y = \log_3 x$ is compressed vertically by a factor of ½, stretched horizontally by a factor of 8, reflected across the x-axis, and is translated 3 right and 6 up.

$y = -\frac{1}{2}\log_3\left(\frac{1}{8}(x-3)\right) + 6$

5. Write in logarithmic form.

a) $4^{-2} = 0.0625$ b) $5^t = r+6$ c) $e^x = 8$

$\log_4(0.0625) = -2$ $\log_5(r+6) = 3$ $\ln 8 = x$

6. Write in exponential form.

a) $\log_2 512 = 9$ b) $\ln(16) = t$ c) $\log_2(a-4) = b$

$2^9 = 512$ $e^t = 16$ $2^b = a-4$

7. Use the definition of logarithms to find the value of each expression below.

a) $\log_3 81$ b) $\log_4\left(\frac{1}{64}\right)$

a) $= 4$ (since $3^4 = 81$)

b) $= \log_4\left(\frac{1}{4^3}\right) = \log_4(4^{-3}) = -3$

8. Solve the following equations for x. If answer is not exact, give it correct to **2 decimal places.**

a) $\log_4(x-8) = 5$ b) $\log_x(18) = \frac{3}{4}$

a) change form: $4^5 = x-8$; $1024 = x-8$; $x = 1032$

b) change form: $x^{3/4} = 18$; raise both sides to reciprocal exponent: $(x^{3/4})^{4/3} = (18)^{4/3}$; $x = 18^{4/3}$; $x = \sqrt[3]{18^4}$; $x \doteq 47.17$

c) $\ln(x) + \ln(8) = \ln 32$ d) $\log_6(3x^7) - \log_6(x^6) = 2$

c) use multiplication law: $\ln(8x) = \ln 32$; $8x = 32$; $x = 4$

d) division law: $\log_6\left(\frac{3x^7}{x^6}\right) = 2$; $\log_6(3x) = 2$; change form: $6^2 = 3x$; $\frac{36}{3} = \frac{3x}{3}$; $x = 12$

$\log_6\left(\frac{3x^7}{x^6}\right) = 2$

using exponent laws, this argument becomes: $7-6$

$\frac{3x}{1}$

$= \frac{3x^1}{1}$

$= 3x$

9. Evaluate each of the following on your calculator. Give answers correct to **4 decimal places.**

a) $\log_7 18$ b) $4\log_3 29$ c) $2\ln 53$

a) $= \frac{\log 18}{\log 7} \doteq 1.4854$

b) $= 4\left(\frac{\log 29}{\log 3}\right) \doteq 12.2602$

c) $= 7.9406$

10. Evaluate each of the following.

a) $\log_8 8$ b) $\log_\infty 1$ c) $\log_3 3^5$

d) $7^{\log_{7.3} 13}$ e) $\ln e^3$ f) $\ln(1)$

a) $= 1$ b) $= 0$ c) $= 5$

d) $= 13$ e) $= 3$ f) $= 0$

11. Use logarithm laws to expand the following logarithms completely.

a) $\log_2\left(\frac{8x^4\sqrt{y}}{w^3}\right)$ b) $\log_3\left(\frac{\sqrt[4]{x}}{yz^2}\right)$

a) $= \log_2 8 + \log_2 x^4 + \log_2 y^{1/2} - \log_2 w^3$

$= 3 + 4\log_2 x + \frac{1}{2}\log_2 y - 3\log_2 w$

b) $= \log_3 x^{1/4} - \log_3(yz^2)$

$= \frac{1}{4}\log_3 x - [\log_3 y + \log_3 z^2]$

$= \frac{1}{4}\log_3 x - \log_3 y - 2\log_3 z$

12. Use logarithm laws to condense each expression into a single logarithm.

a) $3\log 2x - 5\log x - \log 2 + 8\log x$ b) $\frac{\log(x-7)}{\log 5}$

a) $= \log(2x)^3 - \log x^5 - \log 2 + \log x^8$

$= \log\left(\frac{8x^3}{x^5}\right)\left(\frac{x^8}{2}\right)$

$= \log\left(\frac{8x^{11}}{2x^5}\right)$

$= \log(4x^6)$

b) $= \log_5(x-7)$

13. Determine the value of $\log_2(16a^3b^2)$ if $\log_2 a = 5$ and $\log_2 b = 4$.

$\log_2(16a^3b^2) = \log_2 16 + \log_2 a^3 + \log_2 b^2$

$= 4 + 3\log_2 a + 2\log_2 b$

$= 4 + 3(5) + 2(4)$

$= 4 + 15 + 8$

$= 27$

14. Solve each equation for x. Reject extraneous solutions. Give answers correct to **2 decimal places.**

a) $\log_4(2x+1)-\log_4 3=\log_4 11$

b) $\log_2 x=3-\log_2(x+2)$

c) $\log_5(3x+1)+\log_5(x-3)=3$

15. Solve each equation for x. Give answers correct to **2 decimal places.**

a) $e^{x+3}=45$

b) $2^{x+5}=5^{3x-4}$

c) $6^{2x+5}=3\left(7^{8-x}\right)$

16. An investment of $2000 pays interest at a rate of 1.5% per year. Determine how long it takes for the investment to grow to $3000, if the interest is compounded quarterly. **Solve algebraically, not graphically.**

17. A type of bacteria doubles every 3 days. How long would it take a sample of 400 cells to grow to 9,000 cells?

18. A sample of a radioactive substance decays from 390 mg to 80 mg in 76 days. What is the half-life of this substance? ***Solve algebraically.*** Express your answer correct to two decimal places.

14. Solve each equation for x. Reject extraneous solutions. Give answers correct to **2 decimal places.**

a) $\log_4(2x+1)-\log_4 3=\log_4 11$

$\log_4\left(\frac{2x+1}{3}\right)=\log_4(11)$

$\Rightarrow \frac{2x+1}{3}=11$

$2x+1=33$

$2x=32$

$\boxed{x=16}$

b) $\log_2 x=3-\log_2(x+2)$

$\log_2 x+\log_2(x+2)=3$

$\log_2[x(x+2)]=3$

$\log_2(x^2+2x)=3$

$2^3=x^2+2x$

$8=x^2+2x$

$0=x^2+2x-8$

$(x-2)(x+4)=0$

$x-2=0$ → $\boxed{x=2}$

$x+4=0$ → $x=-4$ extraneous, makes argument negative

c) $\log_5(3x+1)+\log_5(x-3)=3$

$\log_5(3x+1)(x-3)=3$

$\log_5(3x^2-9x+1x-3)=3$

$\log_5(3x^2-8x-3)=3$

$5^3=3x^2-8x-3$

$125=3x^2-8x-3$

$0=3x^2-8x-128$

$AC=-384$, $B=-8$ → $-24, 16$

$3x^2-24x+16x-128=0$

$3x(x-8)+16(x-8)=0$

$(3x+16)(x-8)=0$

$3x+16=0$

$3x=-16$

$x=-\frac{16}{3}$ extraneous

$x-8=0$

$\boxed{x=8}$

OR

$3x^2-8x-128=0$

Factoring this one might be hard. If you prefer, use the quadratic formula to solve.

$A=3$, $B=-8$, $C=-128$

$x=\frac{-(-8)\pm\sqrt{(-8)^2-(4)(3)(-128)}}{2(3)}$

$x=\frac{8\pm\sqrt{64+1536}}{6}$

$x=\frac{8\pm\sqrt{1600}}{6}$

$x=\frac{8\pm 40}{6}$

$x=\frac{8+40}{6}=\frac{48}{6}=\boxed{8}$

$x=\frac{8-40}{6}=\frac{-32}{6}=-\frac{16}{3}$

15. Solve each equation for x. Give answers correct to **2 decimal places.**

a) $e^{x+3}=45$

$\log e^{x+3}=\log 45$

$(x+3)\frac{\log e}{\log e}=\frac{\log 45}{\log e}$

$x+3=\frac{\log 45}{\log e}$

$x=\frac{\log 45}{\log e}-3$

$\boxed{x\doteq 0.81}$

OR

$\ln e^{x+3}=\ln 45$

$(x+3)\ln e^{1}=\ln 45$

$x+3=\ln 45$

$x=\ln 45-3$

$\boxed{x\doteq 0.81}$

b) $2^{x+5}=5^{3x-4}$

$\log 2^{x+5}=\log 5^{3x-4}$

$(x+5)\log 2=(3x-4)\log 5$

$x\log 2+5\log 2=3x\log 5-4\log 5$

$x\log 2-3x\log 5=-4\log 5-5\log 2$

$x(\log 2-3\log 5)=-4\log 5-5\log 2$

$x=\frac{-4\log 5-5\log 2}{(\log 2-3\log 5)}$ ← This is the exact answer.

$\boxed{x\doteq 2.39}$ ← approximate answer

c) $6^{2x+5}=3\left(7^{8-x}\right)$

$\log 6^{2x+5}=\log\left(3\left(7^{8-x}\right)\right)$ ← use multiplication law, because argument is a product

$(2x+5)\log 6=\log 3+\log 7^{8-x}$

$2x\log 6+5\log 6=\log 3+(8-x)\log 7$

$2x\log 6+5\log 6=\log 3+8\log 7-x\log 7$

$2x\log 6+x\log 7=\log 3+8\log 7-5\log 6$

$x(2\log 6+\log 7)=\log 3+8\log 7-5\log 6$

$x=\frac{\log 3+8\log 7-5\log 6}{(2\log 6+\log 7)}$

$\boxed{x\doteq 1.39}$

16. An investment of $2000 pays interest at a rate of 1.5% per year. Determine how long it takes for the investment to grow to $3000, if the interest is compounded quarterly. **Solve algebraically, not graphically.**

$\frac{3000}{2000}=\frac{2000}{2000}\left(1+\frac{0.015}{4}\right)^n$ ← n = number of compounding periods

$1.5=1.00375^n$

$\log 1.5=\log 1.00375^n$

$\log 1.5=n\log 1.00375$

$\frac{\log 1.5}{\log 1.00375}=n$

compounding periods ⇒ round to log periods

19. Suppose that solution A is 500 times as acidic as than solution B. If we know the pH of solution B is 9.1, what is the pH of solution A?

20. An earthquake in California measured 4.8 on the Richter scale. Another earthquake near Japan was 300 times more intense. What was the Richter scale reading for the earthquake near Japan?

21. The loudness level of a vacuum cleaner is 75 dB and that of a chainsaw is 110 dB. How many times as loud as a vacuum cleaner is a chainsaw?

22. The population of Toronto is given by $P(t) = 4{,}000{,}000e^{0.012t}$, where $t = 0$ corresponds to the year 2000. In what year will the population reach 5,800,000?

$1.5 = 1.00375^n$

$\log 1.5 = \log 1.00375^n$

$\log 1.5 = n \log 1.00375$

$\frac{\log 1.5}{\log 1.00375} = n$

$n = 108.3266\ldots$ compounding periods $\Rightarrow$ round to 109 periods

To get years, divide by 4; $\frac{109}{4} = 27.25$ years

17. A type of bacteria doubles every 3 days. How long would it take a sample of 400 cells to grow to 9,000 cells?

$\frac{9000}{400} = \frac{400}{400}(2)^{t/3}$

$22.5 = 2^{t/3}$

$\log 22.5 = \log 2^{t/3}$

$3 \times (\log 22.5) = \left(\frac{t}{3} \log 2\right) \times 3$

$3 \log 22.5 = t \log 2$

$\frac{3 \log 22.5}{\log 2} = t$

$t \approx 13.5$ days

18. A sample of a radioactive substance decays from 400 mg to 80 mg in 76 days. What is the half-life of this substance? ***Solve algebraically.*** Express your answer correct to two decimal places.

$\frac{80}{400} = \frac{400}{400}(0.5)^{\frac{76}{t}}$

$0.2 = 0.5^{76/t}$

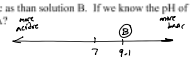
$\log 0.2 = \log 0.5^{76/t}$

$t \times (\log 0.2) = \left(\frac{76}{t} \log 0.5\right) \times t$

$t \log 0.2 = 76 \log 0.5$

$t = \frac{76 \log 0.5}{\log 0.2}$, $t \approx 32.73$ days

19. Suppose that solution A is 500 times as acidic as than solution B. If we know the pH of solution B is 9.1, what is the pH of solution A?



$I = I_0(10)^{9-p}$

$\frac{500I_0}{I_0} = \frac{I_0}{I_0}(10)^{9.1-p}$

$500 = 10^{9.1-p}$

$\log 500 = \log 10^{9.1-p}$

$\log 500 = (9.1-p)\log 10$ ($\log 10 = 1$)

$\log 500 = 9.1 - p$

$p = 9.1 - \log 500$

$p \approx 6.4$

20. An earthquake in California measured 4.8 on the Richter scale. Another earthquake near Japan was 300 times more intense. What was the Richter scale reading for the earthquake near Japan?

$I = I_0(10)^{R-r}$

$\frac{300I_0}{I_0} = \frac{I_0}{I_0}(10)^{R-4.8}$

$300 = 10^{R-4.8}$

$\log 300 = \log 10^{R-4.8}$

$\log 300 = (R-4.8)\log 10$ ($\log 10 = 1$)

$R - 4.8 = \log 300$

$R = \log 300 + 4.8$

$R \approx 7.3$

21. The loudness level of a vacuum cleaner is 75 dB and that of a chainsaw is 110 dB. How many times as loud as a vacuum cleaner is a chainsaw?

$I = I_0(10)^{\frac{D-d}{10}}$

$I = I_0(10)^{\frac{110-75}{10}}$

$I = I_0(10)^{\frac{35}{10}}$

$I \approx I_0(3162.28)$

About 3162 times as loud.

22. The population of Toronto is given by $P(t) = 4{,}000{,}000e^{0.012t}$, where $t = 0$ corresponds to the year 2000. In what year will the population reach 5,800,000?

$\frac{5\,800\,000}{4\,000\,000} = \frac{4\,000\,000}{4\,000\,000}e^{0.012t}$

$1.45 = e^{0.012t}$

$\log 1.45 = \log e^{0.012t}$

$\log 1.45 = 0.012t \log e$

$\frac{\log 1.45}{(0.012 \log e)} = t$

$t = 30.9636\ldots$

$t \approx 31$ years

So in 2031