C_19 WB Log Patterns

Sunday, November 3, 2019 3:12 PM

Logarithms – Investigation

Part I:

Evaluate the expressions on the left, using your understanding of logs.
Re-write each of your answers as a single logarithm, as shown in the example.

Example: $\log_4(16)+\log_4(4)=2+1=3$ This answer, 3, is equal to $\log_4(64)$

We've shown that: $\log_4(16)+\log_4(4)=\log_4(64)$

1) $\log_2(8)+\log_2(4)=$ _____ $\log_2(\quad)$

2) $\log_3(9)+\log_3(81)=$ _____ $\log_3(\quad)$

these can be written as...

3) $\log_3\left(\frac{1}{9}\right)+\log_3(81)=$ _____ $\log_3(\quad)$

4) $\log_5(5)+\log_5(1)=$ _____ $\log_5(\quad)$

5) What pattern seems to hold? Write a rule:

$$\boxed{\log_c X+\log_c Y=\log_c(\qquad)}$$

Below are some questions that we can't answer directly with the definition of logarithms. Use the pattern discovered above to write each one as a single logarithm, then evaluate it.

6) $\log_6 12+\log_6 3$

7) $\log 250+\log 40$

8) $\log_8\left(\frac{3}{64}\right)+\log_8\left(\frac{1}{3}\right)$

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We've shown that: $\log_4(16)+\log_4(4)=\log_4(64)$

1) $\log_2(8)+\log_2(4)=$ 5 $\log_2(32)=5$

$3+2$

2) $\log_3(9)+\log_3(81)=$ 6 $\log_3(729)=6$

$2+4$

these can be written as...

3) $\log_3\left(\frac{1}{9}\right)+\log_3(81)=$ 2 $\log_3(9)=2$

$-2+4$

4) $\log_5(5)+\log_5(1)=$ 1 $\log_5(5)=1$

$1+0$

5) What pattern seems to hold? Write a rule:

$$\boxed{\log_c X+\log_c Y=\log_c(XY)}$$

Below are some questions that we can't answer directly with the definition of logarithms. Use the pattern discovered above to write each one as a single logarithm, then evaluate it.

6) $\log_6 12+\log_6 3 = \log_6(12\cdot 3) = \log_6(36) = 2$

7) $\log 250+\log 40 = \log(250\cdot 40) = \log(10000) = 4$

8) $\log_8\left(\frac{3}{64}\right)+\log_8\left(\frac{1}{3}\right) = \log_8\left(\frac{\cancel{3}}{64}\cdot\frac{1}{\cancel{3}}\right) = \log_8\left(\frac{1}{64}\right) = \log_8\left(\frac{1}{8^2}\right) = \log_8(8^{-2}) = -2$

This result links to an exponent law we already know:

$\left.\begin{array}{l}\log_c X=a \text{ means } c^a=x\\ \log_c Y=b \text{ means } c^b=y\end{array}\right\}$ definition of logarithm

substituting from above

$\log_c(XY)=\log_c(c^a c^b)$

$=\log_c(c^{a+b})$ using exponent law here

$=a+b$ } definition of logarithm

$=\log_c X+\log_c Y$

Part II:

Evaluate the expressions on the left, using your understanding of logs.
Re-write each of your answers as a single logarithm, as shown in the example.

Example: $\log_4(64) - \log_4(16) = 3 - 2 = 1$ This answer, 1, is equal to $\log_4(4)$

We've shown that: $\log_4(64) - \log_4(16) = \log_4(4)$

9) $\log_5 625 - \log_5 5 =$ ______ $\log_5(\quad)$

10) $\log_6 36 - \log_6 6 =$ ______ $\log_6(\quad)$

which is the same as...

11) $\log_3 9 - \log_3 1 =$ ______ $\log_3(\quad)$

12) $\log_2 16 - \log_2 32 =$ ______ $\log_2(\quad)$

13) What pattern seems to hold? Write a rule:

$$\boxed{\log_c X - \log_c Y = \log_c(\qquad)}$$

Below are more questions that we can't answer directly with the definition of logarithms. Use the new pattern discovered above to write each one as a single logarithm, then evaluate it.

14) $\log_6 72 - \log_6 2$

15) $\log 12 - \log 0.12$

16) $\log_{12} 2 - \log_{12} 288$

Part II:

Evaluate the expressions on the left, using your understanding of logs.
Re-write each of your answers as a single logarithm, as shown in the example.

Example: $\log_4(64) - \log_4(16) = 3 - 2 = 1$ This answer, 1, is equal to $\log_4(4)$

We've shown that: $\log_4(64) - \log_4(16) = \log_4(4)$

9) $\log_5 625 - \log_5 5 = \underline{3}$ $\log_5(125) = 3$

$4 - 1$

10) $\log_6 36 - \log_6 6 = \underline{1}$ $\log_6(6) = 1$

$2 - 1$

which is the same as...

11) $\log_3 9 - \log_3 1 = \underline{2}$ $\log_3(9) = 2$

$2 - 0$

12) $\log_2 16 - \log_2 32 = \underline{-1}$ $\log_2(\frac{1}{2}) = -1$

$4 - 5$

13) What pattern seems to hold? Write a rule:

$$\boxed{\log_c X - \log_c Y = \log_c\left(\frac{X}{Y}\right)}$$

Below are more questions that we can't answer directly with the definition of logarithms. Use the new pattern discovered above to write each one as a single logarithm, then evaluate it.

14) $\log_6 72 - \log_6 2 = \log_6\left(\frac{72}{2}\right)$

$= \log_6(36) = 2$

15) $\log 12 - \log 0.12 = \log\left(\frac{12}{0{,}12}\right)$

$= \log(100) = 2$

16) $\log_{12} 2 - \log_{12} 288 = \log_{12}\left(\frac{2}{288}\right)$

$= \log_{12}\left(\frac{1}{144}\right)$

$= \log_{12}\left(\frac{1}{12^2}\right)$

$= \log_{12}(12^{-2}) = -2$

This result also links to an exponent law we already know:

$\left.\begin{array}{l}\log_c X = a \text{ means } c^a = x \\ \log_c Y = b \text{ means } c^b = y\end{array}\right\}$ definition of logarithm

substituting from above

$\log_c\left(\frac{X}{Y}\right) = \log_c\left(\frac{c^a}{c^b}\right)$

$= \log_c(c^{a-b})$ using exponent law here

$= a - b$ } definition of logarithm

$= \log_c X - \log_c Y$

Your Turn

Use the laws of logarithms to simplify and evaluate each expression.

b) $\log_5 1000 - \log_5 4 - \log_5 2$

$= \log_5\left(\frac{1000}{4}\right) - \log_5 2$

$= \log_5(250) - \log_5 2$

$= \log_5\left(\frac{250}{2}\right)$

$= \log_5 125 = 3$

c) $2 \log_3 6 - \frac{1}{2} \log_3 64 + \log_3 2$

$= \log_3 6^2 - \log_3 64^{1/2} + \log_3 2$

$= \log_3 36 - \log_3 8 + \log_3 2$

$= \log_3\left(\frac{36}{8}\right) + \log_3 2$

$= \log_3\left(\frac{36 \times 2}{8}\right)$

$= \log_3\left(\frac{72}{8}\right) = \log_3 9 = 2$