C_20 Key and Unit 3 Revised Practice Test

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C_20 Unit 3 Practice Test

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PreCalc 12 – Unit 3 Practice Test **Name:** _____

1. Solve: $7 = 2^{x+1}$

A. -0.64 B. 1.36 C. 1.81 D. 3.81

2. The population in a community is increasing at an annual rate of 6.5%. Assume this trend will continue. In how many years will the present population of 12 000 grow to 32 000?

A. 15.5 B. 15.6 C. 15.8 D. 16.1

3. In a study which compared the pH of urine and tears, the following data was collected. On average, how many times more alkaline are tears than urine?

	Urine	Tears
Joe	6.2	7.6
Bob	6.3	7.4
Bill	5.5	7.5
Average	**6.0**	**7.5**

A. 1.3 B. 1.5
C. 15.0 D. 31.6

4. Solve $2^{3x-1} = 8^{2x+1}$.

A. $x = -\frac{4}{3}$ B. $x = -1$ C. $x = -\frac{2}{3}$ D. $x = -\frac{3}{4}$

5. Simplify fully: $\log\left(\frac{x^2}{10y^3}\right)$

A. $2\log x - 1 - 3\log y$ B. $2\log x - 1 + 3\log y$
C. $2\log x - 10 - 3\log y$ D. $2\log x - 10 + 3\log y$

6. Evaluate: $\log_3 \sqrt{27}$.

A. $\frac{2}{9}$ B. $\frac{2}{3}$ C. $\frac{3}{2}$ D. $\frac{9}{2}$

7. Determine the x-intercept for the graph of $y = 5^x - 3$.

A. -2 B. 0.008 C. 0.6 D. 0.68

8. Express as a single logarithm: $\log a - \log b - 3\log c$

A. $\log\frac{a}{bc^3}$ B. $\log\frac{a}{b^3c^3}$ C. $\log\frac{ac^3}{b}$ D. $\log\frac{ac^3}{b^3}$

9. An investment earns 2.25% per year compounded daily. How many years would be required for an investment to triple in value? (Assume all years have 365 days.)

A. 4.88 years B. 5.41 years C. 48.83 years D. 49.37 years

10. What is the value of $\log_3 30$?

A. 3.1 B. 3.4 C. 3.6 D. 3.9

11. Determine the Richter scale reading for an earthquake that is 5 times more intense than another earthquake that measures 4.0 on the Richter scale.

A. 9 B. 20 C. 4 + log 5 D. 5 + log 4

12. Determine the domain of the function $y = \log\left(4 - x^2\right)$

A. $-2 < x < 2$ B. $-2 \le x \le 2$

C. $x < -2,\ x > 2$ D. $x \le -2,\ x \ge 2$

13. Write $3^y = 73$ in logarithmic form.

A. $73 = \log_3 y$ B. $73 = \log_y 3$ C. $3 = \log_y 73$ D. $3 = \log_{73} y$ E. $y = \log_3 73$

14. Find the value of x if $\log_x 18 = 2$.

A. $2\sqrt{3}$ B. $3\sqrt{2}$ C. 9 D. $18\sqrt{2}$ E. 324

15. Which of the following is the inverse relation of $y = \log 3x$?

A. $y = 3^x$ B. $y = 3x$ C. $y = \dfrac{1}{\log 3x}$ D. $y = \dfrac{10^x}{3}$ E. $y = \log_{3x} 10$

16. Write as a single logarithm: $3\log 2 - \dfrac{1}{2}\log 16$

A. $\log 1$ B. $\log 2$ C. $\log \dfrac{3}{4}$ D. $\log \dfrac{9}{4}$ E. $\dfrac{3}{2}\log 32$

17. Simplify: $\log_a (bc) - \log_a \left(\dfrac{c}{b}\right)$

A. $\left(\log_a b\right)^2$ B. $2\log_a (b)$ C. $\log_a (2b)$ D. $a \log b^2$ E. 0

18. Which of the following is a simplified value of $\dfrac{\log x^2 + \log x^5}{\log x^5 - \log x^3}$?

A. $\dfrac{7}{2}$ B. $\dfrac{10}{2}$ C. $\dfrac{\log 7}{\log 2}$ D. $\dfrac{\log 10}{\log 2}$ E. $\dfrac{\log 7x}{\log 2x}$

19. Which of the following is equal to $\log x$ if $x = \dfrac{ab}{\sqrt[4]{c}}$?

A. $\log a + \log b - \log 4c$ B. $\log a + \log b - 4\log c$ C. $\log a + \log b - \dfrac{1}{4}\log c$

D. $\dfrac{\log a + \dfrac{1}{4}\log b}{\log c}$ E. $\dfrac{\log a + 4\log b}{\log c}$

20. Find the exact value of t given that $7^{2t} = 3^5$.

A. $5\log 3 - 2\log 7$ B. $\dfrac{3^5}{49}$ C. $\dfrac{5\log 3}{2\log 7}$

D. $3^5 - 49$ E. $3\log 5 - 7\log 2$

21. A bacteria population doubles every 9 days. Determine an expression for the number of bacteria N after t days, given an initial amount of 500 bacteria.

A. $N = 500(9)^{t/9}$ B. $N = 500(2)^{t/9}$ C. $N = 500(9)^{t/2}$

D. $N = 500(2)^{9t}$ E. $N = 500(9)^{2t}$

22. Determine the initial investment needed, P, if an interest rate of 12% per year compounded quarterly yields \$1 000 in 5 years.

A. $P = \dfrac{1000}{1.03^5}$ B. $P = \dfrac{1000}{1.04^5}$

C. $P = \dfrac{1000}{1.03^{20}}$ D. $P = \dfrac{1000}{1.04^{20}}$

23. The half-life of a radioactive substance is 14 years. Determine an expression for the mass of the substance remaining from 50 grams, after t years have gone by

A. $0.5(50)^{14/t}$ B. $50\left(\frac{1}{2}\right)^{14/t}$

C. $0.5(50)^{t/14}$ D. $50\left(\frac{1}{2}\right)^{t/14}$

24. A strain of bacteria triples every 5 days. Determine an expression for the initial number of bacteria if after t days, 200 bacteria are present.

A. $A=\frac{200}{3^{t/5}}$ B. $A=\frac{200}{3^{5/t}}$

C. $A=200(3)^{t/5}$ D. $A=200(3)^{5/t}$

Written Questions

1. Solve algebraically: $\log_{15}(3-x)+\log_{15}(1-x)=1$

2. A food sample contains 300 bacteria. The doubling time for bacteria left at room temperature is 20 minutes. How many minutes will it take to reach an unsafe level of 100 000 bacteria? Solve algebraically using logarithms. Give answer as a decimal accurate to 2 decimal places.

3. Given the function $y=\log_3(x+2)-2$.

a) Graph the function. Clearly show at least 4 points on the curve and state the equation of the asymptote.

b) Calculate the x-intercept of this function.

Answers:

1. C
2. B
3. D
4. A
5. A
6. C
7. D
8. A
9. C
10. A
11. C
12. A
13. E
14. B
15. D
16. B
17. B
18. A
19. C
20. C
21. B
22. C
23. D
24. A

1. $x=-2$
2. 167.62 minutes
3b) $x=7$

PreCalc 12 – Unit 3 Practice Test Name: Key

1. Solve: $7 = 2^{x+1}$

A. -0.64 B. 1.36 (C.) 1.81 D. 3.81

$\log 7 = \log 2^{x+1}$

$\frac{\log 7}{\log 2} = \frac{(x+1)\log 2}{\log 2}$

$x+1 = \frac{\log 7}{\log 2}$

$x = \frac{\log 7}{\log 2} - 1$

$x \doteq 1.81$

2. The population in a community is increasing at an annual rate of 6.5%. Assume this trend will continue. In how many years will the present population of 12 000 grow to 32 000?

A. 15.5 (B.) 15.6 C. 15.8 D. 16.1

every year

$\frac{32\,000}{12\,000} = \frac{12\,000(1.065)^{t/1}}{12\,000}$

0.065

rate = 1 + 0.065 = 1.065

$\frac{32}{12} = 1.065^t$

$\log\left(\frac{32}{12}\right) = t\log 1.065$

$t = \frac{\log\left(\frac{32}{12}\right)}{\log 1.065}$

$t = 15.574\ldots \doteq 15.6$

3. In a study which compared the pH of urine and tears, the following data was collected. On average, how many times more alkaline are tears than urine?

	Urine	Tears
Joe	6.2	7.6
Bob	6.3	7.4
Bill	5.5	7.5
Average	**6.0**	**7.5**

A. 1.3 B. 1.5

C. 15.0 (D.) 31.62

$I = I_0(10)^{7.5-6.0}$

$I = I_0(10)^{1.5}$

$I \doteq I_0(31.62277\ldots)$

4. Solve $2^{3x-1} = 8^{2x+1}$.

(A.) $x = -\frac{4}{3}$ B. $x = -1$ C. $x = -\frac{2}{3}$ D. $x = -\frac{3}{4}$

$2^{3x-1} = (2^3)^{2x+1}$

$2^{3x-1} = 2^{6x+3}$

$\Rightarrow 3x - 1 = 6x + 3$

$-3x = 4$

$x = \frac{4}{-3}$

1

$2^{x-1} = 2$ ✓ $\quad x = \frac{4}{-3}$

5. Simplify fully: $\log\left(\frac{x^2}{10y^3}\right)$

A. $2\log x - 1 - 3\log y$ B. $2\log x - 1 + 3\log y$

C. $2\log x - 10 - 3\log y$ D. $2\log x - 10 + 3\log y$

$\log\left(\frac{x^2}{10y^3}\right) = \log x^2 - \log 10y^3$

$= 2\log x - [\log 10 + \log y^3]$

$= 2\log x - \log 10 - 3\log y$

$= 2\log x - 1 - 3\log y$

6. Evaluate: $\log_3 \sqrt{27}$.

A. $\frac{2}{9}$ B. $\frac{2}{3}$ C. $\frac{3}{2}$ D. $\frac{9}{2}$

$\log_3 \sqrt{27} = \log_3 \sqrt{3^3}$

$= \log_3 (3^{3/2})$

$= \frac{3}{2}$

7. Determine the x-intercept for the graph of $y = 5^x - 3$.

A. -2 B. 0.008 C. 0.6 D. 0.68

For x-intercept, let $y = 0$

$0 = 5^x - 3$

$+3 \quad +3$

$3 = 5^x$

$\log 3 = \log 5^x$

$\frac{\log 3}{\log 5} = \frac{x \log 5}{\log 5}$

$x = \frac{\log 3}{\log 5}$

$x \approx 0.68$

8. Express as a single logarithm: $\log a - \log b - 3\log c$

A. $\log\frac{a}{bc^3}$ B. $\log\frac{a}{b^3c^3}$ C. $\log\frac{ac^3}{b}$ D. $\log\frac{ac^3}{b^3}$

$\log\left(\frac{a}{b}\right) - \log c^3$

$= \log\left(\frac{\frac{a}{b}}{c^3}\right)$

$= \log\left(\frac{a}{b} \cdot \frac{1}{c^3}\right) = \log\left(\frac{a}{bc^3}\right)$

9. An investment earns 2.25% per year compounded daily. How many years would be required for an investment to triple in value? (Assume all years have 365 days.)

A. 4.88 years B. 5.41 years C. 48.83 years D. 49.37 years

$= 0.0225$

$\text{rate} = 1 + \frac{0.0225}{365}$

$= 1.000061644$

If original amount is \$1, how long until we have \$3 ?

$3 = 1(1.000061644)^n$ ← n = # of compounding periods = days

$\log 3 = \log(1.000061644)^n$

$\frac{\log 3}{\log 1.000061644} = n$

$n = \frac{\log 3}{\log 1.000061644}$

$n = 17822.43446$ days,

dividing by 365, $n \doteq 48.83$ years

10. What is the value of $\log_3 30$?

A. 3.1 B. 3.4 C. 3.6 D. 3.9

$\log_3 30 = \frac{\log 30}{\log 3}$

$\doteq 3.095$

11. Determine the Richter scale reading for an earthquake that is 5 times more intense than another earthquake that measures 4.0 on the Richter scale.

A. 9 B. 20 C. $4 + \log 5$ D. $5 + \log 4$

$I = I_0(10)^{R-r}$

$5I_0 = I_0(10)^{R-4}$

$\log 5 = \log 10^{R-4}$

$\log 5 = (R-4)\log 10$ ← $\log 10 = 1$

$\log 5 = R - 4$

$R = \log(5) + 4$

12. Determine the domain of the function $y = \log(4 - x^2)$

A. $-2 < x < 2$ B. $-2 \le x \le 2$

C. $x < -2,\ x > 2$ D. $x \le -2,\ x \ge 2$

argument > 0

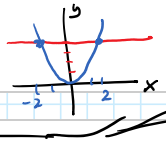
$4 - x^2 > 0$

$4 > x^2$

or $x^2 < 4$

$\Rightarrow -2 < x < 2$

Visualize graph of $y = x^2$. When is it smaller than 4?



13. Write $3^y = 73$ in logarithmic form.

A. $73 = \log_3 y$ B. $73 = \log_y 3$ C. $3 = \log_y 73$ D. $3 = \log_{73} y$ E. $y = \log_3 73$

14. Find the value of x if $\log_x 18 = 2$.

A. $2\sqrt{3}$ B. $3\sqrt{2}$ C. 9 D. $18\sqrt{2}$ E. 324

$\log_x 18 = 2$

$x^2 = 18$

$x = \sqrt{18}$

$x = \sqrt{3 \cdot 3 \cdot 2} \Longrightarrow x = 3\sqrt{2}$

15. Which of the following is the inverse relation of $y = \log 3x$?

A. $y = 3^x$ B. $y = 3x$ C. $y = \dfrac{1}{\log 3x}$ D. $y = \dfrac{10^x}{3}$ E. $y = \log_{3x} 10$

inverse of $y = \log 3x$ is

$x = \log 3y$

change form: $10^x = 3y$

$y = \frac{10^x}{3}$

16. Write as a single logarithm: $3\log 2 - \dfrac{1}{2}\log 16$

A. $\log 1$ B. $\log 2$ C. $\log \dfrac{3}{4}$ D. $\log \dfrac{9}{4}$ E. $\frac{3}{2}\log 32$

$3\log 2 - \frac{1}{2}\log 16$

$= \log 2^3 - \log 16^{1/2}$

$= \log 8 - \log \sqrt{16}$

$= \log 8 - \log 4$

$= \log\left(\frac{8}{4}\right)$

$= \log 2$

17. Simplify: $\log_a(bc) - \log_a\left(\frac{c}{b}\right)$

A. $(\log_a b)^2$ B. $2\log_a(b)$ C. $\log_a(2b)$ D. $a\log b^2$ E. 0

$$\log_a\left(\frac{bc}{\frac{c}{b}}\right) = \log_a\left(\frac{bc}{1}\cdot\frac{b}{c}\right)$$
$$= \log_a\left(\frac{b^2 c}{c}\right)$$
$$= \log_a b^2$$
$$= 2\log_a b$$

18. Which of the following is a simplified value of $\frac{\log x^2 + \log x^5}{\log x^5 - \log x^3}$?

A. $\frac{7}{2}$ B. $\frac{10}{2}$ C. $\frac{\log 7}{\log 2}$ D. $\frac{\log 10}{\log 2}$ E. $\frac{\log 7x}{\log 2x}$

$$\frac{\log x^2 + \log x^5}{\log x^5 - \log x^3} = \frac{\log(x^2 \cdot x^5)}{\log\left(\frac{x^5}{x^3}\right)}$$
$$= \frac{\log(x^7)}{\log(x^2)}$$
$$= \frac{7\log x}{2\log x} = \frac{7}{2}$$

19. Which of the following is equal to $\log x$ if $x = \frac{ab}{\sqrt[4]{c}}$?

A. $\log a + \log b - \log 4c$ B. $\log a + \log b - 4\log c$ C. $\log a + \log b - \frac{1}{4}\log c$

D. $\frac{\log a + \frac{1}{4}\log b}{\log c}$ E. $\frac{\log a + 4\log b}{\log c}$

$$x = \frac{ab}{\sqrt[4]{c}}$$
$$\log x = \log\left(\frac{ab}{\sqrt[4]{c}}\right)$$
$$= \log ab - \log \sqrt[4]{c}$$
$$= \log a + \log b - \log c^{1/4}$$
$$= \log a + \log b - \tfrac{1}{4}\log c$$

20. Find the exact value of t given that $7^{2t} = 3^5$.

A. $5\log 3 - 2\log 7$ B. $\frac{3^5}{49}$ C. $\frac{5\log 3}{2\log 7}$

D. $3^5 - 49$ E. $3\log 5 - 7\log 2$

$7^{2t} = 3^5$

$\log 7^{2t} = \log 3^5$

$\frac{2t\log 7}{2\log 7} = \frac{5\log 3}{2\log 7}$

$t = \frac{5\log 3}{2\log 7}$

21. A bacteria population doubles every 9 days. Determine an expression for the number of bacteria N after t days, given an initial amount of 500 bacteria.

this is the base

A. ~~$N = 500(9)^{t/9}$~~ B. $N = 500(2)^{t/9}$ C. ~~$N = 500(9)^{t/2}$~~

D. $N = 500(2)^{9t}$ E. ~~$N = 500(9)^{2t}$~~

22. Determine the initial investment needed, P, if an interest rate of 12% per year compounded quarterly yields \$1 000 in 5 years.

A. $P = \frac{1000}{1.03^5}$ B. $P = \frac{1000}{1.04^5}$

C. $P = \frac{1000}{1.03^{20}}$ D. $P = \frac{1000}{1.04^{20}}$

(# of compounding periods in 5 years)

20

$1000 = P\left(1 + \frac{0.12}{4}\right)^{20}$

$\frac{1000}{(1+0.03)^{20}} = P$

$P = \frac{1000}{(1.03)^{20}}$

23. The half-life of a radioactive substance is 14 years. Determine an expression for the mass of the substance remaining from 50 grams, after t years have gone by

A. $0.5(50)^{14/t}$ B. $50\left(\frac{1}{2}\right)^{14/t}$

C. $0.5(50)^{t/14}$ D. $50\left(\frac{1}{2}\right)^{t/14}$

$A = 50\left(\frac{1}{2}\right)^{t/14}$

24. A strain of bacteria triples every 5 days. Determine an expression for the initial number of bacteria if after t days, 200 bacteria are present.

A. $A = \frac{200}{3^{t/5}}$ B. $A = \frac{200}{3^{5/t}}$

C. $A = 200(3)^{t/5}$ D. $A = 200(3)^{5/t}$

$200 = A(3)^{t/5}$

$\frac{200}{(3)^{t/5}} = A$

Written Questions

1. Solve algebraically: $\log_{15}(3-x) + \log_{15}(1-x) = 1$

$\log_{15}[(3-x)(1-x)] = 1$

$\log_{15}(3-3x-x+x^2) = 1$

$\log_{15}(3-4x+x^2) = 1$

$15^1 = x^2-4x+3$

$0 = x^2-4x+3-15$

$0 = x^2-4x-12$

$0 = (x+2)(x-6)$

$x+2=0$

$x=-2$

$x-6=0$

$x=6$

reject, because it makes the arguments negative

7

base = 2

2. A food sample contains 300 bacteria. The doubling time for bacteria left at room temperature is 20 minutes. How many minutes will it take to reach an unsafe level of 100 000 bacteria? Solve algebraically using logarithms. Give answer as a decimal accurate to 2 decimal places.

$\frac{100\,000}{300} = \frac{300(2)^{t/20}}{300}$

$\log\left(\frac{1000}{3}\right) = \log 2^{t/20}$

$20 \times \left[\log\left(\frac{1000}{3}\right)\right] = \left[\frac{t}{20}\log 2\right] \times 20$

$\frac{20\log\left(\frac{1000}{3}\right)}{\log 2} = \frac{t\log 2}{\log 2}$

$\frac{20\log\left(\frac{1000}{3}\right)}{\log 2} = t$

$t = 167.62$ minutes

3. Given the function $y = \log_3(x+2) - 2$.

a) Graph the function. Clearly show at least 4 points on the curve and state the equation of the asymptote.

b) Calculate the x-intercept of this function.

a) $y = \log_3(x+2) - 2$ is a transformation of $y = \log_3 x$, and this is the inverse of $y = 3^x$

2 left
2 down

$(x, y) \to (x-2, y-2)$

$y = 3^x$

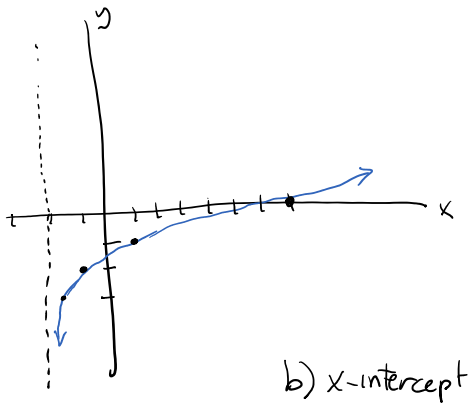
x	y
-2	$3^{-2} = \frac{1}{9}$
-1	$3^{-1} = \frac{1}{3}$
0	$3^0 = 1$
1	$3^1 = 3$
2	$3^2 = 9$

$y = \log_3 x$

$\frac{1}{9}$	-2
$\frac{1}{3}$	-1
1	0
3	1
9	2

$y = \log_3(x+2) - 2$

$x-2$	$y-2$
$\frac{1}{9} - 2 = -\frac{17}{9}$	$-2-2 = -4$
$\frac{1}{3} - 2 = -\frac{5}{3}$	$-1-2 = -3$
$1-2 = -1$	$0-2 = -2$
$3-2 = 1$	$1-2 = -1$
$9-2 = 7$	$2-2 = 0$



vertical asymptote at $x = -2$

b) x-intercept is $(7, 0)$, which is on our table.

We can find it using the equation, too:

$y = \log_3(x+2) - 2$

Let $y = 0$

$0 = \log_3(x+2) - 2$

$+2 \quad +2$

$$2 = \log_3(x+2)$$

change form: $3^2 = x+2$

$$9 = x+2$$
$$-2 \quad -2$$
$$7 = x$$

$(7, 0)$