C_22 Ch 9 Key Hand-in 2022

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C_23 Ch 9 Hand-in 2022

Chapter 9 Hand-in Assignment – Rational Functions

Name: Key

1. Given the original rational function $y=\frac{1}{x}$ and the transformed function, $y=\frac{-3}{x-7}+4$:

a) List the transformations taking place.

VE by 3
reflect across x-axis
right 7
up 4

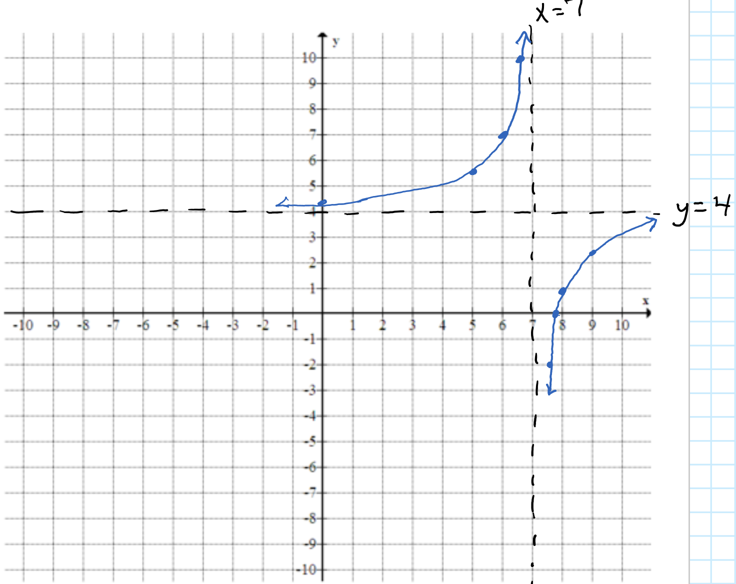
b) Complete the tables below. For the first table, give 6 points found on the graph of the original function $y=\frac{1}{x}$. For the second table, transform each point in the original table to get its image point. Write the mapping notation in the headings of the final table.

$(x,y) \rightarrow (x+7, -3y+4)$

$y=\frac{1}{x}$

x	y
-2	$-\frac{1}{2}$
-1	-1
$-\frac{1}{2}$	-2
$\frac{1}{2}$	2
1	1
2	$\frac{1}{2}$

$x+7$	$-3y+4$
5	$5\frac{1}{2}$
6	7
$6\frac{1}{2}$	10
$7\frac{1}{2}$	-2
8	1
9	$2\frac{1}{2}$



c) Accurately sketch the final transformed function. Include its asymptotes, using dotted lines. Label each asymptote with its equation.

d) Find the coordinates of the final graph's *x*-intercept and *y*-intercept.

x-int

$0 = \frac{-3}{x-7} + 4$

$\cancel{x-7}\left(\frac{3}{\cancel{x-7}}\right) = (4)(x-7)$

$3 = 4x - 28$

$\frac{31}{4} = \frac{4x}{4}$

$x = \frac{31}{4}$

$\boxed{\left(\frac{31}{4}, 0\right) \text{ or } (7.75, 0)}$

y-int

$y = \frac{-3}{0-7} + 4$

$y = \frac{-3}{-7} + 4$

$y = 4\frac{3}{7}$

$y = \frac{31}{7}$

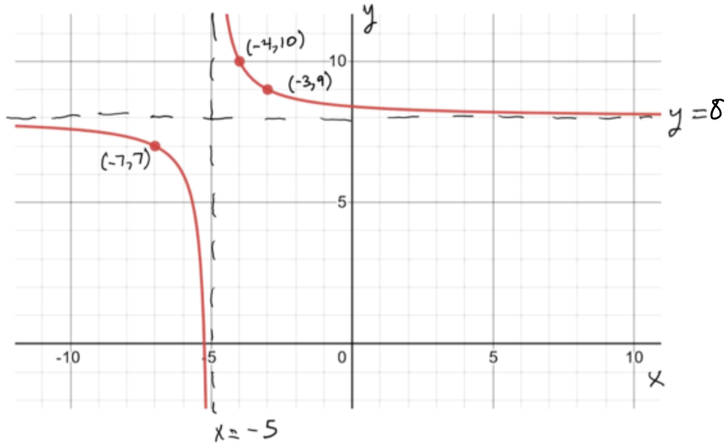
$\boxed{\left(0, \frac{31}{7}\right)}$

2. What is the equation of the rational function shown below?

Using the asymptotes, we get

$$y = \frac{a}{x+5} + 8$$

We use any point on the graph to calculate the "a" value:



Using (-7,7), we get

$$7 = \frac{a}{-7+5} + 8$$

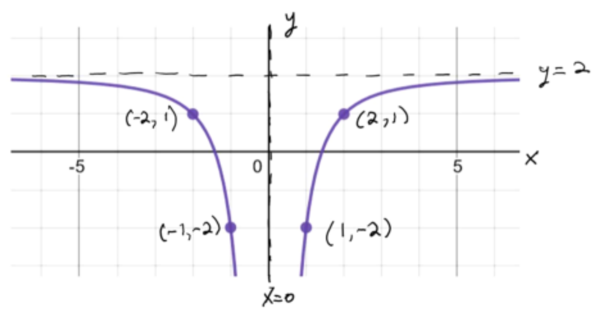
$$-1 = \frac{a}{-2}$$

$$a = 2 \Rightarrow \boxed{y = \frac{2}{x+5} + 8}$$

3. What is the equation of the rational function shown below?

Using the asymptotes, we get:

$$y = \frac{a}{x^2} + 2$$



Let's use (2,1) to find the "a" value:

$$1 = \frac{a}{2^2} + 2$$

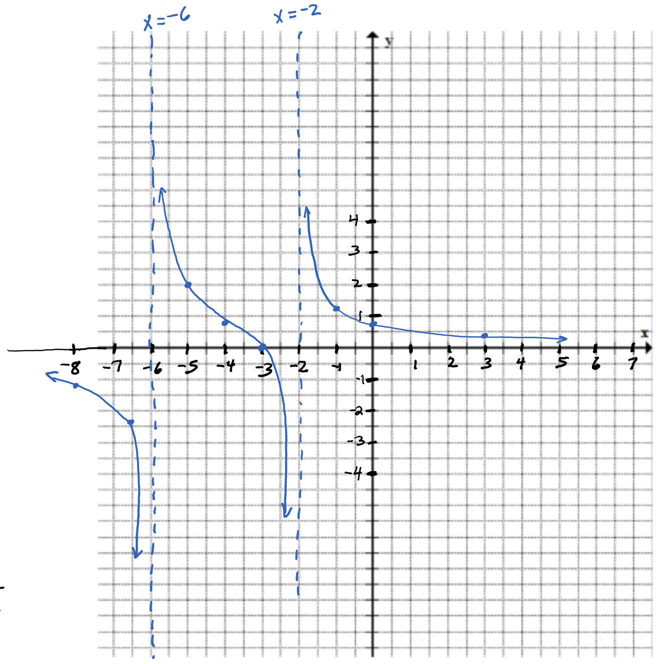
$$-1 = \frac{a}{4}$$

$$a = -4 \Rightarrow \boxed{y = \frac{-4}{x^2} + 2}$$

4. For each of the following functions

- factor and simplify (if possible)
- determine all characteristics of each (show work).
- graph each function, showing asymptotes, intercepts and any points of discontinuity

a) $y = \dfrac{3x+9}{x^2+8x+12}$



x	y
-8	-1.25
-7	-2.4
-5	2
-4	0.75
-3	0
-1	1.2
0	3/4 = 0.75
3	0.4

simplified form of equation:

$y = \dfrac{3(x+3)}{(x+2)(x+6)}$

NPVs (restrictions) $\boxed{x=-2 \quad x=-6}$

vertical asymptote equation(s)

$\boxed{x=-2 \quad x=-6}$

horizontal asymptote equation $\dfrac{\text{degree } 1}{\text{degree } 2}$

$\Rightarrow \boxed{y=0}$

coordinates of points of discontinuity (PODs) if any

$\boxed{\text{none}}$

x-intercept

let $y=0$

$(x^2+8x+12)\ 0 = \left(\dfrac{3x+9}{\cancel{x^2+8x+12}}\right)\cancel{(x^2+8x+12)}$

$0 = 3x+9$

$3x = -9$

$x = -3$

$\boxed{(-3, 0)}$

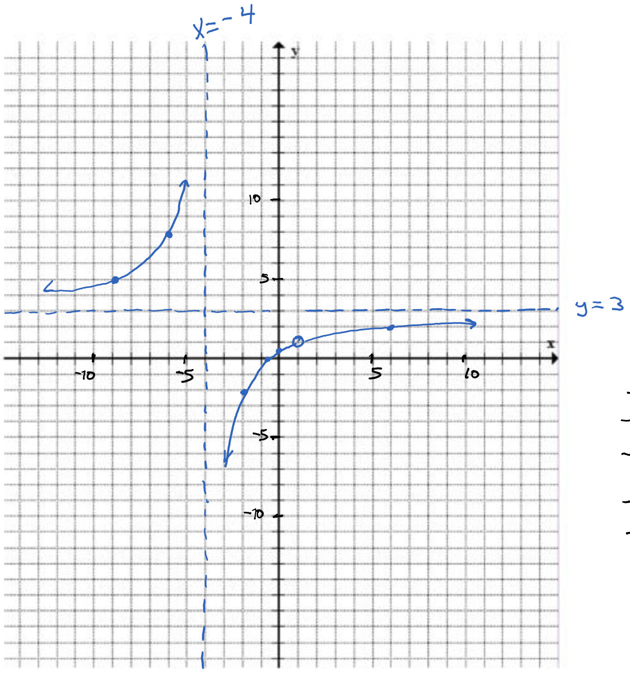
y-intercept

let $x=0$

$y = \dfrac{3(0)+9}{0^2+8(0)+12}$, $y = \dfrac{9}{12} = 3/4$

$\boxed{(0, 3/4)}$

b) $y = \frac{3x^2 - x - 2}{x^2 + 3x - 4} = \frac{(3x+2)\cancel{(x-1)}}{(x+4)\cancel{(x-1)}}$



x	y
-9	5
-6	8
-2	-2
$-\frac{2}{3}$	0
0	1/2
6	2

simplified form of equation:

$y = \frac{3x+2}{x+4}$, where $x \neq 1$

NPVs (restrictions) $\boxed{x = -4 \quad x = 1}$

vertical asymptote equation(s)

$\boxed{x = -4}$

horizontal asymptote equation

$y = \frac{\boxed{3}x^2 - x - 2}{\boxed{1}x^2 + 3x - 4}$ degree = 2, degree = 2

same degree $\Rightarrow$ $y = \boxed{\frac{3}{1}}$ $\boxed{y = 3}$

coordinates of points of discontinuity (PODs) if any

POD when $x = 1$

$y = \frac{3x+2}{x+4}$, let $x = 1$

$y = \frac{3(1)+2}{1+4}$

$y = \frac{5}{5}$, $y = 1$

$\boxed{(1, 1)}$

x-intercept

let $y = 0$

$(x+4)\,0 = \frac{(3x+2)}{\cancel{x+4}}\cancel{(x+4)}$

$0 = 3x + 2$

$3x = -2$

$x = -\frac{2}{3}$

$\boxed{(-\frac{2}{3}, 0)}$

y-intercept

let $x = 0$

$y = \frac{3(0)+2}{0+4}$

$y = \frac{2}{4}$

$y = \frac{1}{2}$

$\boxed{(0, \frac{1}{2})}$

5. Given the following information, determine a possible equation for the rational function. Show final answer in simplified form (multiply everything out).

a) vertical asymptote equations $x=-4$ and $x=2$; and x-intercepts at $x=3$ and $x=-8$.

$$y=\frac{(x-3)(x+8)}{(x+4)(x-2)}$$, simplifies to: $$\boxed{y=\frac{x^2+5x-24}{x^2+2x-8}}$$

b) vertical asymptote equation $x=6$; x-intercept at $x=-1$; a point of discontinuity at $x=4$; horizontal asymptote equation $y=2$.

$$y=\frac{2(x+1)(x-4)}{(x-6)(x-4)}$$, simplifies to: $$\boxed{y=\frac{2x^2-6x-8}{x^2-10x+24}}$$

$$y=\frac{2(x^2-3x-4)}{x^2-10x+24}$$

6. Solve the following rational equations algebraically. Identify all restrictions (NPVs).

a) $2x+9=\frac{5}{x}$

$\boxed{\text{NPV } x=0}$

Multiply by x to eliminate fractions

$$x(2x)+x(9)=\cancel{x}\left(\frac{5}{\cancel{x}}\right)$$

$$2x^2+9x=5$$

$$2x^2+9x-5=0$$

$$2x^2+10x-x-5=0$$

$$2x(x+5)-1(x+5)=0$$

$$(x+5)(2x-1)=0$$

$AC=2(-5)$
$=-10$

Prod $=-10$
Sum $=9$ } 10 , −1

$x=-5$

$2x-1=0$
$2x=1$
$\boxed{x=\tfrac{1}{2}}$

b) $\frac{8}{x^2-16}+1=\frac{1}{x-4}$

Factor first:

$$\frac{8}{(x+4)(x-4)}+1=\frac{1}{x-4}$$

$\boxed{\text{NPVs: } x=-4,\ x=4}$

Multiply by $(x+4)(x-4)$

$$\cancel{(x+4)}\cancel{(x-4)}\left(\frac{8}{\cancel{(x+4)}\cancel{(x-4)}}\right)+(x+4)(x-4)(1)=(x+4)\cancel{(x-4)}\left(\frac{1}{\cancel{x-4}}\right)$$

$$8+x^2+\cancel{4x}-\cancel{4x}-16=x+4$$

$$8+x^2-16-x-4=0$$

$$x^2-x-12=0$$

$$(x+3)(x-4)=0$$

$\boxed{x=-3}$

$\cancel{x=4}$ reject, it's an NPV

c) $5-\frac{4x}{3}=\frac{1+2x-x^2}{3x+12}$

$$5-\frac{4x}{3}=\frac{1+2x-x^2}{3(x+4)}$$

Use quadratic formula,

$$5-\frac{4x}{3}=\frac{1+2x-x^2}{3(x+4)}$$

$\boxed{\text{NPV: } x=-4}$

Multiply by $3(x+4)$

$$(3)(x+4)(5)-(\cancel{3})(x+4)\left(\frac{4x}{\cancel{3}}\right)=(\cancel{3})(\cancel{x+4})\left(\frac{1+2x-x^2}{\cancel{3(x+4)}}\right)$$

$$15(x+4)-4x(x+4)=1+2x-x^2$$

$$15x+60-4x^2-16x=1+2x-x^2$$

$$-4x^2-x+60+x^2-2x-1=0$$

$$\frac{-3x^2-3x+59}{-1}=\frac{0}{-1}$$

$$3x^2+3x-59=0 \leftarrow \text{doesn't factor}$$

Use quadratic formula,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$3x^2+3x-59=0$$

$a=3$
$b=3$
$c=-59$

$$x=\frac{-3\pm\sqrt{3^2-(4)(3)(-59)}}{2(3)}$$

$$x=\frac{-3\pm\sqrt{9+708}}{6}$$

$$\underbrace{\boxed{x=\frac{-3\pm\sqrt{717}}{6}}}_{\text{exact}},\quad \underbrace{\boxed{x\doteq 3.96,\ x\doteq -4.96}}_{\text{approximate}}$$

7. Tom has succeeded in 10 of his 31 attempts, when putting at the golf course. If he succeeds in half of his attempts from now on, how many more attempts will he need in order to get his average up to 45%?

Create a rational equation describing this situation. Solve the equation algebraically.

Let $x=$ number of additional attempts

$$\text{Tom's success rate}=\frac{10+0.5x}{31+x}$$

$$\frac{10+0.5x}{31+x}=0.45$$

$\boxed{\begin{array}{c}\text{NPV}\\ x=-31\end{array}}$

Multiply by $31+x$ to eliminate fraction

$$(\cancel{31+x})\left(\frac{10+0.5x}{\cancel{31+x}}\right)=(0.45)(31+x)$$

$$10+0.5x=13.95+0.45x$$

$$\frac{0.05x}{0.05}=\frac{3.95}{0.05}$$

$\boxed{x=79 \text{ more attempts}}$

$$\text{Check: } \frac{10+0.5(79)}{31+79}=\frac{49.5}{110}=0.45 \checkmark$$