C_22 PODs and Horizontal Asymptotes

Tuesday, November 26, 2019   3:56 PM

W   C_22 PODs and Horizontal Asymptotes

Point of Discontinuity (POD)
  —is a "hole" in the graph that happens whenever a factor of the denominator cancels with a factor in the numerator. The NPV that comes from that factor is the x-value of the POD.

For each rational equation, decide whether it has a POD when graphed.

a) $y = \dfrac{x^2 + 7x + 10}{x + 10}$ b) $y = \dfrac{x^2 + 7x + 10}{x^2 + 12x + 20}$

c) $y = \dfrac{x^2 - 3x - 10}{x^2 - 25}$ d) $y = \dfrac{x^2 - 9}{2x + 6}$

Find the (x, y) coordinates of this equation's POD: $y = \dfrac{x^2 - 4x - 5}{x^2 + x - 30}$
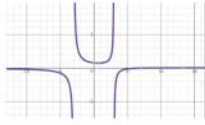
Horizontal Asymptote
  —is a horizontal line that a function's graph approaches as x-values get extremely large

For each rational function below, can you figure out the equation of its horizontal asymptote?

1a) $y = \dfrac{3}{x}$ b) $y = \dfrac{2}{x - 4} + 7$

2a) $y = \dfrac{5}{x^2}$ b) $y = \dfrac{-3}{(x+2)^2} - 6$

For each rational equation, decide whether it has a POD when graphed.

a) $y = \dfrac{x^2 + 7x + 10}{x + 10} = \dfrac{(x+2)(x+5)}{x+10}$ **NO**

b) $y = \dfrac{x^2 + 7x + 10}{x^2 + 12x + 20} = \dfrac{(x+2)(x+5)}{(x+10)(x+2)}$ **Yes**

POD when $x = -2$
$y = \dfrac{-2+5}{-2+10}$ $\left(-2, \tfrac{3}{8}\right)$
$y = \tfrac{3}{8}$

c) $y = \dfrac{x^2 - 3x - 10}{x^2 - 25} = \dfrac{(x+2)(x-5)}{(x+5)(x-5)}$
POD when $x = 5$
$y = \dfrac{5+2}{5+5} = \tfrac{7}{10}$ $\left(5, \tfrac{7}{10}\right)$ **Yes**

d) $y = \dfrac{x^2 - 9}{2x + 6} = \dfrac{(x+3)(x-3)}{2(x+3)}$ **Yes**
POD when $x = -3$
$y = \dfrac{-3-3}{2} = \dfrac{-6}{2}$
$(-3, -3)$

Find the (x, y) coordinates of this equation's POD: $y = \dfrac{x^2 - 4x - 5}{x^2 + x - 30}$

$y = \dfrac{(x+1)(x-5)}{(x-5)(x+6)}$

$y = \dfrac{x+1}{x+6}$, $x \neq 5$

$y = \dfrac{5+1}{5+6} = \dfrac{6}{11}$

$\left(5, \dfrac{6}{11}\right)$

For each rational function below, can you figure out the equation of its horizontal asymptote?

1a) $y = \dfrac{3}{x}$ $y = 0$ b) $y = \dfrac{2}{x - 4} + 7$ $y = 7$

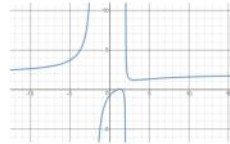2a) $y = \dfrac{5}{x^2}$ $y = 0$ b) $y = \dfrac{-3}{(x+2)^2} - 6$ $y = -6$

Left column:

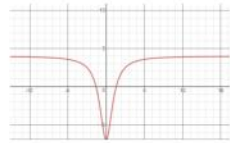3. $y = \dfrac{(x-7)}{x^2 - 9}$

4. $y = \dfrac{(8x-1)}{5x^2 - 9}$
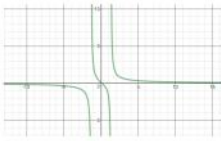
5. $y = \dfrac{(2x^2 - 5x + 3)}{x^2 - 4}$
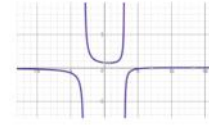
6. $y = \dfrac{(4x^2 - 7)}{x^2 + 1}$

7. Can you tell what the horizontal asymptote equation will be for each function below, without having to graph it?

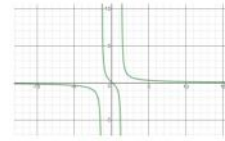a) $y = \dfrac{(3x^2 + 8x - 2)}{(5x^2 - 4x + 7)}$ b) $y = \dfrac{6x + 8}{5x^2 - 9}$

Right column:

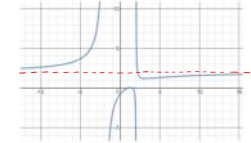3. $y = \dfrac{(x-7)}{x^2 - 9}$

$y = 0$

4. $y = \dfrac{(8x-1)}{5x^2 - 9}$
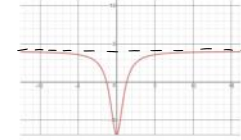
$y = 0$

5. $y = \dfrac{(2x^2 - 5x + 3)}{x^2 - 4}$

$y = 2$

6. $y = \dfrac{(4x^2 - 7)}{x^2 + 1}$

$y = 4$

7. Can you tell what the horizontal asymptote equation will be for each function below, without having to graph it?

a) $y = \dfrac{(3x^2 + 8x - 2)}{(5x^2 - 4x + 7)}$ b) $y = \dfrac{6x + 8}{5x^2 - 9}$

$y = \dfrac{3}{5}$ $y = 0$