Ch G Hand-in full key 2022

Sunday, December 5, 2021 4:38 PM

C_22 Geometric Sequences and Series

Scroll down for solutions

Hand-In Assignment - Geometric Sequences & Series

Name: ____.

Formulas:

$t_n = ar^{n-1}$ $\qquad S_n = \dfrac{a(1-r^n)}{1-r}$ $\qquad S_n = \dfrac{a-lr}{1-r}$ $\qquad S = \dfrac{a}{1-r}$

Where appropriate, show the process - not just the final answer!

1. Show the using of a formula to find t_7 for the geometric sequence that begins:
   0.01, 0.08, 0.64, 5.12 . . .

2. Show the using of a formula to find the **sum** of the first 8 terms, S_8, given the sequence that begins: $50 + 37.5 + 28.125 + 21.09375 + \ldots$

3. Show the using of a formula to find the first term of a geometric series, given that $S_6 = 1092$ and $r = -3$.

4. Use an algebraic method to find t_{10} for the geometric sequence with $t_3 = 10,\ t_6 = 80$

5. Consider the geometric series: $3/4 + 9/4 + \ldots + 729/4$.
a) Determine the sum of this series.

b) Determine the number of terms in the series.

c) Express the series in sigma notation.

6. Write each series below using sigma notation. Do not find the sum.
a) $1/3 + 1/6 + 1/12 + 1/24 + 1/48$

b) $3 - 6 + 12 - 24 + 48 - 96$

7.. Determine the sum of the series. $\sum_{2}^{14} -3(-2)^k$

8. Determine t_5, given that an infinite geometric series has $S = 18$ and $r = \frac{2}{3}$

9. Where possible, find the sum of the following infinite series. If it is not possible, state "No finite sum."

a) 81 + 27 + 9 + 3 + . . .

b) $20 - 30 + 45 - 67.5 + ...$

10. Determine the infinite sum: $S = 0.3 + \frac{2}{100} + \frac{2}{1000} + \frac{2}{10000} + ...$

Give answer as a fraction.
(Hint – think carefully about what you use as the first term in the formula!)

11. A ball is dropped from a height of 4.0 meters to a floor. After each bounce, the ball rises to 60% of its previous height.

a) Determine the total vertical distance traveled when the ball hits the ground the 8^{th} time. (Correct to 2 decimal places.)

b) Determine the total vertical distance that the ball travels before coming to rest. (Correct to 2 decimal places.)

12. A geometric sequence consists of these terms: $2x+5,\ 3x,\ m$. The common ratio is $r=4$. What is the value of m ?

A. -4 B. -12 C. -16 D. -48

13. Determine the first term in the expansion of $\sum_{k=2}^{6} 4(5)^k$.

A. 4 B. 20 C. 100 D. 500

14. The number of terms in the series defined by $\sum_{n=8}^{w} (2n-10)$ is

A. w B. w – 7 C. w – 8 D. w – 9

Hand-In Assignment - Geometric Sequences & Series

Name: Key

Formulas:

$t_n = ar^{n-1}$ $S_n = \frac{a(1-r^n)}{1-r}$ $S_n = \frac{a-lr}{1-r}$ $S = \frac{a}{1-r}$

Where appropriate, show the process - not just the final answer!

1. Show the using of a formula to find t_7 for the geometric sequence that begins:

0.01, 0.08, 0.64, 5.12 . . .

$t_n = ar^{n-1}$

$t_7 = 0.01(r)^6$ $\left\{ r = \frac{0.08}{0.01},\ r = 8 \right.$

$t_7 = 0.01(8)^6$

$t_7 = \boxed{2621.44}$

2. Show the using of a formula to find the **sum** of the first 8 terms, S_8, given the sequence that begins: 50 + 37.5 + 28.125 + 21.09375 + . . .

$S_n = \frac{a(1-r^n)}{1-r}$ $\left\{ r = \frac{37.5}{50},\ r = 0.75 \right.$

$S_8 = \frac{50(1-0.75^8)}{1-0.75}$

$S_8 \doteq \boxed{179.98}$

3. Show the using of a formula to find the first term of a geometric series, given that $S_6 = 1092$ and $r = -3$.

$S_n = \frac{a(1-r^n)}{1-r}$

$1092 = \frac{a(1-(-3)^6)}{1-(-3)}$

$1092 = \frac{a(1-729)}{4}$

$1092 = \frac{a(-728)}{4}$

$\frac{1092}{-182} = \frac{a(-182)}{-182}$

$a = \frac{1092}{-182}$

$\boxed{a = -6}$

4. Use an algebraic method to find t_{10} for the geometric sequence with $t_3 = 10,\ t_6 = 80$

$t_n = ar^{n-1}$

$t_3 = ar^2 = 10$

$t_6 = ar^5 = 80$

We know $ar^5 = 80$.

Because $ar^2 = 10$, we can divide on one side by ar^2, and the other side by 10

$\frac{\cancel{a}r^5}{\cancel{a}r^2} = \frac{80}{10}$

$r^3 = 8$

$\Rightarrow r = 2$

Now we know $r = 2$, we can find the value of a:

$ar^2 = 10$

$a(2^2) = 10$

$a = \frac{10}{4} = \frac{5}{2}$

Finally, $t_{10} = ar^9$

$= \left(\frac{5}{2}\right)(2)^9$

$= \boxed{1280}$

$\times r \quad \times r \quad \times r$

$\underset{t_3}{10} \quad \_ \quad \_ \quad \underset{t_6}{80}$

$t_{10} = ?$

$\frac{\cancel{10}r^3}{\cancel{10}} = \frac{80}{10}$

$r^3 = 8$

$r = 2$

$t_{10} = ar^9$ $a = 2.5$

$\boxed{t_{10} = (2.5)(2)^9}$

5. Consider the geometric series: 3/4 + 9/4 + . . . + 729/4.

a) Determine the sum of this series.

$S_n = \frac{a - lr}{1-r}$ $\left\{ r = \frac{9/4}{3/4},\ r = \frac{9}{4}\cdot\frac{4}{3},\ r = 3 \right.$

$= \frac{\frac{3}{4} - \left(\frac{729}{4}\right)(3)}{1-3}$

$= \frac{\frac{3}{4} - \frac{2187}{4}}{-2}$

$= \frac{-2184}{4} \cdot \frac{1}{-2}$

$= \boxed{273}$

b) Determine the number of terms in the series.

The nth term is 729

$\rightarrow \frac{729}{\cancel{4}} \cdot \frac{\cancel{4}}{3} = 3^{n-1}$

$1-3$ $= \boxed{273}$

b) Determine the number of terms in the series.

The n^{th} term is $\frac{729}{4}$

$t_n = ar^{n-1}$

$\frac{729}{4} = \left(\frac{3}{4}\right)(3)^{n-1}$

$\frac{729}{4} \cdot \frac{4}{3} = 3^{n-1}$

$243 = 3^{n-1}$

$3^5 = 3^{n-1}$

$\Rightarrow 5 = n-1,$ $\boxed{n=6}$

c) Express the series in sigma notation.

For sigma notation, we put ar^{n-1} into the sigma, as the expression.

$ar^{n-1} = \frac{3}{4}(3)^{n-1}$

We know we have 6 terms, so we get:

$$\sum_{n=1}^{6} \frac{3}{4}(3)^{n-1}$$

6. Write each series below using sigma notation. Do not find the sum.

a) 1/3 + 1/6 + 1/12 + 1/24 +1/48

$r = \frac{\frac{1}{6}}{\frac{1}{3}} = \frac{1}{6} \cdot \frac{3}{1} = \frac{3}{6} = \frac{1}{2}$

$ar^{n-1} = \frac{1}{3}\left(\frac{1}{2}\right)^{n-1}$

We count, and see there are 5 terms.

$$\sum_{n=1}^{5} \frac{1}{3}\left(\frac{1}{2}\right)^{n-1}$$

b) $3-6+12-24+48-96$

$r = \frac{-6}{3} = -2$

$ar^{n-1} = 3(-2)^{n-1}$

6 terms

$$\sum_{n=1}^{6} 3(-2)^{n-1}$$

expand a few terms, to figure out what "a" and "r" values are.

7.. Determine the sum of the series. $\sum_{2}^{14} -3(-2)^k$ $= -3(-2)^2 + -3(-2)^3 + \dots + -3(-2)^{14}$

$= -3(4) + -3(-8) + \dots + -3(16384)$

$= -12 + 24 + \dots + -49152$

$S_n = \frac{a(1-r^n)}{1-r}$

$a = -12$

$r = \frac{24}{-12} = -2$

$n = \#\text{of terms}: (14-2)+1 = 13 \text{ terms}$

$S_{13} = \frac{-12(1-(-2)^{13})}{1-(-2)}$

$= \boxed{-32772}$

8. Determine t_5, given that an infinite geometric series has $S = 18$ and $r = \frac{2}{3}$

$S = \frac{a}{1-r}$

$18 = \frac{a}{1-\frac{2}{3}}$

$\frac{1}{3}(18) = \left(\frac{a}{\frac{1}{3}}\right) \cdot \frac{1}{3}$

$6 = a$

$t_n = ar^{n-1}$

$t_5 = 6\left(\frac{2}{3}\right)^4$

$= \frac{6}{1}\left(\frac{16}{81}\right) = \frac{96}{81} = \boxed{\frac{32}{27}}$

9. Where possible, find the sum of the following infinite series. If it is not possible, state "No finite sum."

Check: $-1 < r < 1$ is needed, to have a sum

a) 81 + 27 + 9 + 3 + . . .

$r = \frac{27}{81} = \frac{1}{3}$ $\quad -1 < \frac{1}{3} < 1$, so we CAN sum.

$S = \frac{a}{1-r}$

$= \frac{81}{1-\frac{1}{3}} = \boxed{121.5}$

b) $20 - 30 + 45 - 67.5 + \dots$

$r = \frac{-30}{20} = \frac{-3}{2}$ This is not between -1 and 1,

so: $\boxed{\text{No finite sum.}}$

10. Determine the infinite sum: $S = 0.3 + \frac{2}{100} + \frac{2}{1000} + \frac{2}{10000} + \dots$

Give answer as a fraction.
(Hint – think carefully about what you use as the first term in the formula!)

What is r?

$\frac{\frac{2}{100}}{0.3} = 0.0\overline{6}$

$\frac{\frac{2}{1000}}{\frac{2}{100}} = \frac{1}{10}$

$\frac{\frac{2}{10\,000}}{\frac{2}{1000}} = \frac{1}{10}$

That first term, 0.3, doesn't give the same r-value. We'll set 0.3 aside for now, and add up the rest of the series.

Let's add up this part:

$\frac{2}{100} + \frac{2}{1000} + \dots$

$S = \frac{a}{1-r}$

$= \frac{\frac{2}{100}}{1-\frac{1}{10}}$

$= \frac{\frac{2}{100}}{\frac{9}{10}}$

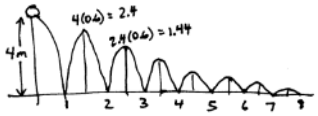
$= \frac{2}{100} \cdot \frac{10}{9}$

$= \frac{20}{900} = \frac{2}{90}$

Total sum $= 0.3 + \frac{2}{90}$

$= \frac{3}{10}\cdot\frac{9}{9} + \frac{2}{90}$

$= \frac{27}{90} + \frac{2}{90} = \boxed{\frac{29}{90}}$



11. A ball is dropped from a height of 4.0 meters to a floor. After each bounce, the ball rises to 60% of its previous height.

a) Determine the total vertical distance traveled when the ball hits the ground the 8th time. (Correct to 2 decimal places.)

Vertical distance = (initial drop) + (up distances) + (downs)

"up distances" $= \underbrace{2.4 + 1.44 + \cdots}_{7 \text{ terms}}$

"down distances" $= \underbrace{2.4 + 1.44 + \cdots}_{7 \text{ terms}}$

$a = 2.4$
$r = 0.6$
$n = 7$

$$S_7 = \frac{2.4(1 - 0.6^7)}{1 - 0.6} = 5.8320384$$

$$\text{Total} = 4 + 2(5.8320384) \doteq \boxed{15.66 \text{ m}}$$

b) Determine the total vertical distance that the ball travels before coming to rest. (Correct to 2 decimal places.)

"come to a rest" means we do the sum of the infinite series: $\underbrace{2.4 + 1.44 + \cdots}_{\text{infinitely many terms,}}$

$a = 2.4$
$r = 0.6$

$-1 < 0.6 < 1$, so we CAN find the sum.

$$S = \frac{2.4}{1 - 0.6} = 6$$

$$\text{Total} = \left(\begin{matrix}\text{initial}\\ \text{drop}\end{matrix}\right) + (\text{up's}) + (\text{down's})$$

$$= 4 + 6 + 6 = \boxed{16 \text{ m}}$$

12. A geometric sequence consists of these terms: $2x+5$, $3x$, m. The common ratio is $r = 4$. What is the value of m ?

A. -4 B. -12 C. -16 D. -48

$$(2x+5)\left(\frac{3x}{2x+5}\right) = 4(2x+5)$$

$3x = 8x + 20$
$-5x = 20$
$x = -4$

First term: $2(-4)+5 = -3$
2^{nd} term: $3(-4) = -12$

Third term is "r" times the 2^{nd} term: $(4)(-12)$
$= -48$

13. Determine the first term in the expansion of $\sum_{k=2}^{6} 4(5)^k$.

A. 4 B. 20 C. 100 D. 500

Substitute k=2 into the expression: $4(5)^2 = 4(25)$
$= 100$

14. The number of terms in the series defined by $\sum_{n=8}^{w} (2n-10)$ is

A. w B. w – 7 C. w – 8 D. w – 9

number of terms = (top number) – (bottom number) + 1
$= w - 8 + 1$
$= w - 7$