C_25 More Sequences and Series Practice

Sequences and Series – more practice

1. Is the following sequence geometric?

a) 10, 15, 22.5, 37.5,... b) 7, 14, 21, 28,...

2. Find the common ratio, r, of each geometric sequence

a) $-1, -5, -25, -125,...$ b) $-200, 100, -50, -25,...$

3. Find the next three terms of the following sequence

a) 386561, 55223, 7889, ____, ____, ____ b) $-\frac{1}{5}, -\frac{1}{15}, -\frac{1}{45}$, ____, ____, ____

4. Find a formula for the nth term of each geometric sequence.

a) $a = 4, t_{13} = 16384$ b) $t_3 = 5, t_6 = 135$

5. The seventh term of a geometric sequence is 1215 and the fourth term is 45. Find the common ratio, then find the value of the ninth term.

6. A population of rabbits is growing at a rate of 8% a year. If there are 160 rabbits in the initial population, create a general term equation, t_n, describing this sequence. Use it to find the number of rabbits after 6 years.

7. Find the sum of the following geometric series. If necessary, round to 2 decimal places.

a) $729 - 243 + 81 - 27 + ...$ (10 terms)

b) $7 + 14 + 28 + 56 + ... + 7168$

c) $\sum_{n=4}^{10} 5(2)^n$

8. Find the common ratio of a geometric series with a first term of 38 and a sum to infinity of 76.

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9. Find the general term, t_n, for the described sequences:

a) geometric, beginning: $-2, 1, -\frac{1}{2}, \frac{1}{4}, \ldots$

b) geometric, with $t_3 = 75$ and $r = 5$

c) geometric, with $t_4 = 5$ and $r = \frac{1}{4}$

10. Find the 25th term of the following geometric sequence: $2, 2\sqrt{3}, 6, \ldots$

11. List the first five terms of the geometric sequence with $t_3 = 8$ and $r = -\frac{1}{2}$.

12. Find the requested sum for each geometric sequence.

a) Find S_{12} correct to 2 decimal places, for $a = 5$, $r = \frac{2}{3}$

b) Find S_9 for $a = -3$ and $r = 2$

c) Find the sum of the first 11 terms of the geometric series that begins $7 - 14 + 28 - \ldots$

13. Determine the sum, if possible:

a) $\sum_{i=1}^{\infty} -4\left(\frac{4}{5}\right)^i$

b) $\sum_{i=1}^{6} 2(3)^i$

c) $\sum_{i=1}^{\infty} 5\left(\frac{4}{3}\right)^i$

d) $\sum_{i=1}^{\infty} 5\left(\frac{2}{3}\right)^i$

14. A helium balloon rises 80 meters the first minute after it is released. Each minute after that it rises 15% less than the previous minute. How high does the balloon rise in total?

9. Find the general term, t_n, for the described sequences:

a) geometric, beginning: $-2, 1, -\frac{1}{2}, \frac{1}{4}, \ldots$ $t_n = -2\left(-\frac{1}{2}\right)^{n-1}$

b) geometric, with $t_3 = 75$ and $r = 5$ $t_3 = 75 = a(5)^2$, $75 = 25a$, $a = 3$, $t_n = 3(5)^{n-1}$

c) geometric, with $t_4 = 5$ and $r = \frac{1}{4}$ $t_4 = 5 = a\left(\frac{1}{4}\right)^3$, $5 = a\left(\frac{1}{64}\right)$, $a = 320$, $t_n = 320\left(\frac{1}{4}\right)^{n-1}$

10. Find the 25th term of the following geometric sequence: $2, 2\sqrt{3}, 6, \ldots$ $r = \frac{2\sqrt{3}}{2} = \sqrt{3}$

$t_{25} = 2(\sqrt{3})^{24} = 2(3^{12}) = 1062882$

11. List the first five terms of the geometric sequence with $t_3 = 8$ and $r = -\frac{1}{2}$.

$8 = a\left(-\frac{1}{2}\right)^2$ $8 = a\left(\frac{1}{4}\right)$ $a = 32$ 32, −16, 8, −4, 2

12. Find the requested sum for each geometric sequence.

a) Find S_{12} correct to 2 decimal places, for $a = 5$, $r = \frac{2}{3}$ $S_{12} = \frac{5\left(1 - \left(\frac{2}{3}\right)^{12}\right)}{1 - \frac{2}{3}} \approx 14.88$

b) Find S_9 for $a = -3$ and $r = 2$ $S_9 = \frac{-3(1 - (2)^9)}{1 - 2} = -1533$

c) Find the sum of the first 11 terms of the geometric series that begins $7 - 14 + 28 - \ldots$

$S_{11} = \frac{7(1 - (-2)^{11})}{1 - (-2)} = 4781$

13. Determine the sum, if possible:

a) $\sum_{i=1}^{\infty} -4\left(\frac{4}{5}\right)^i = -4\left(\frac{4}{5}\right) + -4\left(\frac{4}{5}\right)^2 + \ldots$ $= -\frac{16}{5} - \frac{64}{25} \ldots$ $r = \frac{-64/25}{-16/5} = 4/5$

$S = \frac{-16/5}{1 - 4/5} = \frac{-16/5}{1/5} = -16$

b) $\sum_{i=1}^{6} 2(3)^i = 2(3) + 2(3)^2 + \ldots = 6 + 18 + \ldots$ $S_6 = \frac{6(1 - 3^6)}{1 - 3} = 2184$

c) $\sum_{i=1}^{\infty} 5\left(\frac{4}{3}\right)^i = 5\left(\frac{4}{3}\right) + 5\left(\frac{4}{3}\right)^2 + \ldots$ $= \frac{20}{3} + \frac{80}{9} + \ldots$ $r = \frac{80/9}{20/3} = \frac{4}{3}$ (no sum, $r > 1$)

d) $\sum_{i=1}^{\infty} 5\left(\frac{2}{3}\right)^i = 5\left(\frac{2}{3}\right) + 5\left(\frac{2}{3}\right)^2 + \ldots$ $= 10/3 + \frac{20}{9} + \ldots$ $r = \frac{20/9}{10/3} = 2/3$ $S = \frac{10/3}{1 - 2/3} = \frac{10/3}{1/3} = 10$

14. A helium balloon rises 80 meters the first minute after it is released. Each minute after that it rises 15% less than the previous minute. How high does the balloon rise in total?

80, 68, 57.8 ...

$S = \frac{80}{1 - 0.85} = 533.33 \text{ m}$