C_25 corrected More Sequences and Series Practice

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C_25 More Sequences and Series Practice

(Solutions at right)

Sequences and Series – more practice

1. Is the following sequence geometric?

a) 10, 15, 22.5, 37.5,...

b) 7, 14, 21, 28,...

2. Find the common ratio, r, of each geometric sequence

a) $-1, -5, -25, -125, \ldots$

b) $-200, 100, -50, -25, \ldots$

3. Find the next three terms of the following sequence

a) 386561, 55223, 7889, ____, ____, ____

b) $-\frac{1}{5}, -\frac{1}{15}, -\frac{1}{45}$, ____, ____, ____

4. Find a formula for the nth term of each geometric sequence.

a) $a = 4$, $t_{13} = 16384$

b) $t_3 = 5$, $t_6 = 135$

5. The seventh term of a geometric sequence is 1215 and the fourth term is 45. Find the common ratio, then find the value of the ninth term.

6. A population of rabbits is growing at a rate of 8% a year. If there are 160 rabbits in the initial population, create a general term equation, t_n, describing this sequence. Use it to find the number of rabbits after 6 years.

7. Find the sum of the following geometric series. If necessary, round to 2 decimal places.

a) $729 - 243 + 81 - 27 + \ldots$ (10 terms)

b) $7 + 14 + 28 + 56 + \ldots + 7168$

c) $\sum_{n=4}^{10} 5(2)^n$

8. Find the common ratio of a geometric series with a first term of 38 and a sum to infinity of 76.

Sequences and Series – more practice

1. Is the following sequence geometric?

a) 10, 15, 22.5, 37.5,...

$r = 1.5$

$\frac{15}{10} = 1.5 \quad \frac{22.5}{15} = 1.5$ } yes

b) 7, 14, 21, 28,... no. To get the next term, we're adding the same number, not multiplying by the same number.

2. Find the common ratio, r, of each geometric sequence

a) $-1, -5, -25, -125, \ldots$

$r = \frac{-5}{-1} = 5$ $\quad \boxed{r = 5}$

b) $-200, 100, -50, -25, \ldots$ $\quad r = \frac{100}{-200} = -\frac{1}{2}$

$\boxed{r = -\frac{1}{2}}$

3. Find the next three terms of the following sequence

a) 386561, 55223, 7889, 1127, 161, 23

$r = \frac{55223}{386561} = \frac{1}{7}$

b) $-\frac{1}{5}, -\frac{1}{15}, -\frac{1}{45}, \frac{-1}{135}, \frac{-1}{405}, \frac{-1}{1215}$

$r = \frac{-1/15}{-1/5} = -\frac{1}{15} \cdot \frac{5}{-1} = \frac{-5}{-15} = \frac{1}{3}$

4. Find a formula for the nth term of each geometric sequence.

a) $a = 4$, $t_{13} = 16384$

$t_{13} = ar^{12}$

$16384 = 4(r^{12})$

$4096 = r^{12}$

$2^{12} = r^{12}$ $\quad r = 2$

$\boxed{t_n = 4(2)^{n-1}}$

b) $t_3 = 5$, $t_6 = 135$

$\frac{135}{5} = \frac{ar^5}{ar^2}$ $\quad r^3 = 27$ $\quad r = 3$

$ar^2 = 5$

$a = \frac{5}{9}$

$\boxed{t_n = \frac{5}{9}(3)^{n-1}}$

5. The seventh term of a geometric sequence is 1215 and the fourth term is 45. Find the common ratio, then find the value of the ninth term.

$t_7 = 1215 = ar^6$

$t_4 = 45 = ar^3$

$27 = r^3$, $r = 3$

$ar^3 = 45$

$a = \frac{45}{r^3}$

$a = \frac{45}{27} = \frac{5}{3}$

$t_9 = \frac{5}{3}(3)^8$

$\boxed{t_9 = 10\,935}$

6. A population of rabbits is growing at a rate of 8% a year. If there are 160 rabbits in the initial population, create a general term equation, t_n, describing this sequence. Use it to find the number of rabbits after 6 years.

160, 172.8, ...
1 yr, 2 yrs

$t_n = 160(1.08)^{n-1}$

$t_7 = 160(1.08)^6$

$\doteq \boxed{253 \text{ rabbits}}$

7. Find the sum of the following geometric series. If necessary, round to 2 decimal places.

a) $729 - 243 + 81 - 27 + \ldots$ (10 terms) $\quad r = \frac{-243}{729} = -\frac{1}{3}$

$S_{10} = \frac{729(1 - (-1/3)^{10})}{1 - (-1/3)} \doteq \boxed{546.74}$

b) $7 + 14 + 28 + 56 + \ldots + 7168$

$7168 = 7(2)^{n-1}$

$1024 = 2^{n-1}$

$2^{10} = 2^{n-1}$ $\Rightarrow n = 11$

$S_{11} = \frac{7(1 - 2^{11})}{1-2} = 14329$

c) $\sum_{n=4}^{10} 5(2)^n$

# of terms: $10 - 4 + 1 = 7$ terms

$= 5(2)^4 + 5(2)^5 + \ldots$

$= 80 + 160 + \ldots$ $\quad r = \frac{160}{80} = 2$

$S_7 = \frac{80(1 - 2^7)}{1-2} = \boxed{10160}$

8. Find the common ratio of a geometric series with a first term of 38 and a sum to infinity of 76.

$S = \frac{a}{1-r}$

$(1-r)(76) = \left(\frac{38}{\cancel{1-r}}\right)\cancel{(1-r)}$

$76(1-r) = 38$

$76 - 76r = 38$

$38 = 76r$

$\frac{38}{76} = r$, $\quad \boxed{r = \frac{1}{2}}$

9. Find the general term, t_n, for the described sequences:

a) geometric, beginning: $-2, 1, -\frac{1}{2}, \frac{1}{4}, \ldots$

b) geometric, with $t_3 = 75$ and $r = 5$

c) geometric, with $t_4 = 5$ and $r = \frac{1}{4}$

10. Find the 25th term of the following geometric sequence: $2, 2\sqrt{3}, 6, \ldots$

11. List the first five terms of the geometric sequence with $t_3 = 8$ and $r = -\frac{1}{2}$.

12. Find the requested sum for each geometric sequence.

a) Find S_{12} correct to 2 decimal places, for $a = 5$, $r = \frac{2}{3}$

b) Find S_9 for $a = -3$ and $r = 2$

c) Find the sum of the first 11 terms of the geometric series that begins $7 - 14 + 28 - \ldots$

13. Determine the sum, if possible:

a) $\sum_{i=1}^{\infty} -4\left(\frac{4}{5}\right)^i$

b) $\sum_{i=1}^{6} 2(3)^i$

c) $\sum_{i=1}^{\infty} 5\left(\frac{4}{3}\right)^i$

d) $\sum_{i=1}^{\infty} 5\left(\frac{2}{3}\right)^i$

14. A helium balloon rises 80 meters the first minute after it is released. Each minute after that it rises 15% less than the previous minute. How high does the balloon rise in total?

9. Find the general term, t_n, for the described sequences:

a) geometric, beginning: $-2, 1, -\frac{1}{2}, \frac{1}{4}, \ldots$ $\boxed{t_n = -2\left(-\frac{1}{2}\right)^{n-1}}$

b) geometric, with $t_3 = 75$ and $r = 5$

$t_3 = 75 = a(5)^2$

$75 = 25a,\ a = 3$

$\boxed{t_n = 3(5)^{n-1}}$

c) geometric, with $t_4 = 5$ and $r = \frac{1}{4}$

$t_4 = 5 = a\left(\frac{1}{4}\right)^3$ $\quad 5 = a\left(\frac{1}{64}\right)$ $\quad a = 320$

$\boxed{t_n = 320\left(\frac{1}{4}\right)^{n-1}}$

10. Find the 25th term of the following geometric sequence: $2, 2\sqrt{3}, 6, \ldots$ $\quad r = \frac{\cancel{2}\sqrt{3}}{\cancel{2}} = \sqrt{3}$

$t_{25} = 2(\sqrt{3})^{24} = 2(3^{12}) = 1062882$

11. List the first five terms of the geometric sequence with $t_3 = 8$ and $r = -\frac{1}{2}$.

$8 = a\left(-\frac{1}{2}\right)^2$ $\quad 8 = a\left(\frac{1}{4}\right)$ $\quad a = 32$ $\quad \boxed{32, -16, 8, -4, 2}$

12. Find the requested sum for each geometric sequence.

a) Find S_{12} correct to 2 decimal places, for $a = 5$, $r = \frac{2}{3}$ $\quad S_{12} = \frac{5\left(1 - \left(\frac{2}{3}\right)^{12}\right)}{1 - \frac{2}{3}} \doteq 14.88$

b) Find S_9 for $a = -3$ and $r = 2$ $\quad S_9 = \frac{-3\left(1 - (2)^9\right)}{1-2} = -1533$

c) Find the sum of the first 11 terms of the geometric series that begins $7 - 14 + 28 - \ldots$

$S_{11} = \frac{7\left(1 - (-2)^{11}\right)}{1 - (-2)} = 4781$

13. Determine the sum, if possible:

a) $\sum_{i=1}^{\infty} -4\left(\frac{4}{5}\right)^i = -4\left(\frac{4}{3}\right) + -4\left(\frac{4}{3}\right)^2 + \ldots$

$= -\frac{16}{5} - \frac{64}{25} \ldots$ $\quad r = \frac{-\frac{64}{25}}{-\frac{16}{5}}$ $\quad r = \frac{4}{5}$

a) $S = \frac{-\frac{16}{5}}{\left(1 - \frac{4}{5}\right)} = \frac{-\frac{16}{5}}{\frac{4}{5}} = -16$

b) $\sum_{i=1}^{6} 2(3)^i = 2(3) + 2(3)^2 + \ldots$

$= 6 + 18 + \ldots$

$S_6 = \frac{6\left(1 - 3^6\right)}{1-3} = 2184$

c) $\sum_{i=1}^{\infty} 5\left(\frac{4}{3}\right)^i = 5\left(\frac{4}{3}\right) + 5\left(\frac{4}{3}\right)^2 + \ldots$

$= \frac{20}{3} + \frac{80}{9} + \ldots$ $\quad r = \frac{\frac{80}{9}}{\frac{20}{3}} = \frac{4}{3}$

no sum, $r > 1$

d) $\sum_{i=1}^{\infty} 5\left(\frac{2}{3}\right)^i = 5\left(\frac{2}{3}\right) + 5\left(\frac{2}{3}\right)^2$

$= \frac{10}{3} + \frac{20}{9} + \ldots$

$r = \frac{20}{9} / \frac{10}{3} = \frac{2}{3}$ $\longrightarrow$ $S = \frac{\frac{10}{3}}{1 - \frac{2}{3}} = \frac{\frac{10}{3}}{\frac{4}{3}} = 10$

14. A helium balloon rises 80 meters the first minute after it is released. Each minute after that it rises 15% less than the previous minute. How high does the balloon rise in total?

$80, 68, 57.8 \ldots$

$S = \frac{80}{1 - 0.85} \doteq 533.33\text{ m}$