C_25 Unit 4 Practice Test Rationals and Series

(Solutions at right)

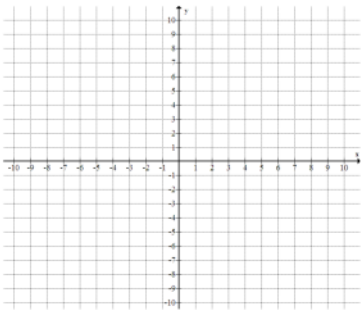
Unit 4 Practice Test

1. Given the original rational function $y = \frac{1}{x}$ and the transformed function, $y = \frac{5}{x+4} - 2$:

a) List the transformations taking place.

b) Complete the tables below. For the first table, give 6 points found on the graph of the original function $y = \frac{1}{x}$. In the second and third tables, show the transformed points after stretches/reflections, and finally after translations. Write the mapping notation in the headings of each table.

x	y



c) Accurately sketch the final transformed function. Include its asymptotes, using dotted lines. Label each asymptote with its equation.

d) Find the coordinates of the final graph's x-intercept and y-intercept.

Unit 4 Practice Test

1. Given the original rational function $y = \frac{1}{x}$ and the transformed function, $y = \frac{5}{x+4} - 2$:

a) List the transformations taking place.

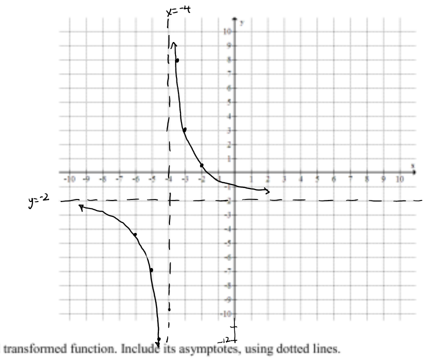
VS 5
4 left
2 down

b) Complete the tables below. For the first table, give 6 points found on the graph of the original function $y = \frac{1}{x}$. In the second and third tables, show the transformed points after stretches/reflections, and finally after translations. Write the mapping notation in the headings of each table.

x	y
-2	$-\frac{1}{2}$
-1	-1
$-\frac{1}{2}$	-2
$\frac{1}{2}$	2
1	1
2	$\frac{1}{2}$

x	$5y$
-2	$-\frac{5}{2} = -2.5$
-1	-5
$-\frac{1}{2}$	-10
$\frac{1}{2}$	10
1	5
2	$\frac{5}{2} = 2.5$

$x-4$	$5y-2$
-6	-4.5
-5	-7
$-4\frac{1}{2}$	-12
$-3\frac{1}{2}$	8
-3	3
-2	0.5



c) Accurately sketch the final transformed function. Include its asymptotes, using dotted lines. Label each asymptote with its equation.

d) Find the coordinates of the final graph's x-intercept and y-intercept.

x-int, let $y=0$

$0 = \frac{5}{x+4} - 2$

$2 = \frac{5}{x+4}$

$2(x+4) = 5$

$2x + 8 = 5$

$2x = -3$

$x = -\frac{3}{2}$

$\left(-\frac{3}{2}, 0\right)$

y-int, let $x=0$

$y = \frac{5}{0+4} - 2$

$y = \frac{5}{4} - 2\left(\frac{4}{4}\right)$

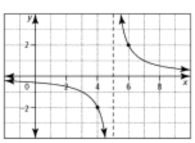
$y = \frac{5}{4} - \frac{8}{4}$

$y = -\frac{3}{4}$

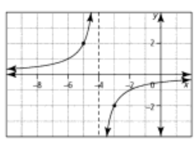
$\left(0, -\frac{3}{4}\right)$

2. Write the equation of each function in the form $y = \frac{a}{x-h} + k$

a)



b)



3. Given the original function $y = \frac{1}{x^2}$ and the transformed function $y = \frac{1}{x^2+6x+9}$. What is the transformation that takes place?

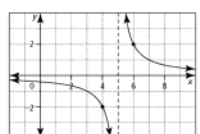
4. Does the graph of the rational function $y = \frac{(x+a)(x+b)}{(x+b)}$ have a vertical asymptote or a point of discontinuity?

5. Complete the table:

	$y = \frac{(x+3)(x-2)}{(x+5)(x+3)}$
Non-permissible value(s)	
Equation of vertical asymptote	
Coordinates of POD	
Equation of horizontal asymptote	

2. Write the equation of each function in the form $y = \frac{a}{x-h} + k$

a)

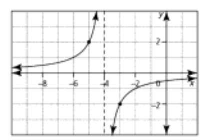


$y = \frac{a}{x-5}$, contains $(6,2)$

$2 = \frac{a}{6-5}$ $2 = \frac{a}{1}$, $a=2$

$\boxed{y = \frac{2}{x-5}}$

b)



$y = \frac{a}{x+4}$, contains $(-3,-2)$

$-2 = \frac{a}{-3+4}$, $-2 = \frac{a}{1}$, $a=-2$

$\boxed{y = \frac{-2}{x+4}}$

3. Given the original function $y = \frac{1}{x^2}$ and the transformed function $y = \frac{1}{x^2+6x+9}$. What is the transformation that takes place?

$y = \frac{1}{x^2+6x+9} = \frac{1}{(x+3)(x+3)}$

$= \frac{1}{(x+3)^2}$

Comparing $y = \frac{1}{x^2}$ and $y = \frac{1}{(x+3)^2}$, we've moved $\boxed{\text{3 left}}$

4. Does the graph of the rational function $y = \frac{(x+a)\cancel{(x+b)}}{\cancel{(x+b)}}$ have a vertical asymptote or a point of discontinuity?

point of discontinuity

5. Complete the table:

	$y = \frac{\cancel{(x+3)}(x-2)}{(x+5)\cancel{(x+3)}}$ OR $y = \frac{x^2+x-6}{x^2+8x+15}$
Non-permissible value(s)	$x \neq -5$, $x \neq -3$
Equation of vertical asymptote	$x = -5$
Coordinates of POD	$\left(-3, -\frac{5}{2}\right)$
Equation of horizontal asymptote	degree top = 2 degree bottom = 2 $\Rightarrow y = \frac{1}{1}$, $\boxed{y=1}$

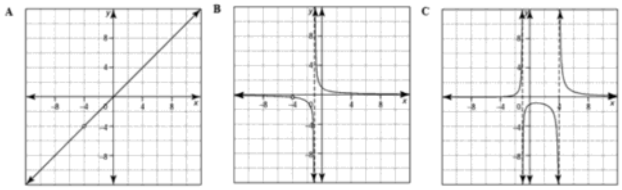
Use $x=-3$ in simple equation:

$y = \frac{x-2}{x+5} = \frac{-3-2}{-3+5}$

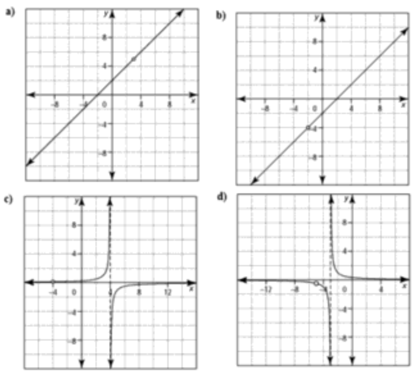
$= \frac{-5}{2}$

6. Without using technology, match the equation of each rational function with the appropriate graph.

a) $y=\frac{x+4}{x^2-3x-4}$ b) $y=\frac{x+4}{x^2+5x+4}$ c) $y=\frac{x^2+4x}{x+4}$



7. Write the equation for each graphed rational function.

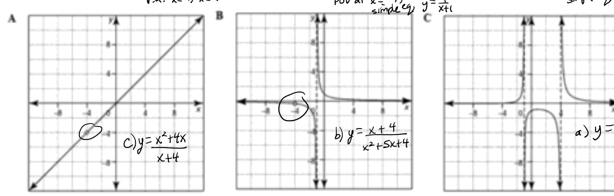


6. Without using technology, match the equation of each rational function with the appropriate graph.

a) $y=\frac{x+4}{x^2-3x-4}=\frac{x+4}{(x+1)(x-4)}$ v.a. x=-1, x=4

b) $y=\frac{x+4}{x^2+5x+4}=\frac{\cancel{x+4}\ 1}{\cancel{(x+4)}(x+1)}$ POD at x=-4, simple eq. $y=\frac{1}{x+1}$

c) $y=\frac{x^2+4x}{x+4}=\frac{x\cancel{(x+4)}}{\cancel{x+4}}$ ⇒ POD at x=-4, simple equation, y=x



c) $y=\frac{x^2+4x}{x+4}$

b) $y=\frac{x+4}{x^2+5x+4}$

a) $y=\frac{x+4}{x^2-3x-4}$

7. Write the equation for each graphed rational function.

a) POD x=3, so we need x-3 in top + bottom

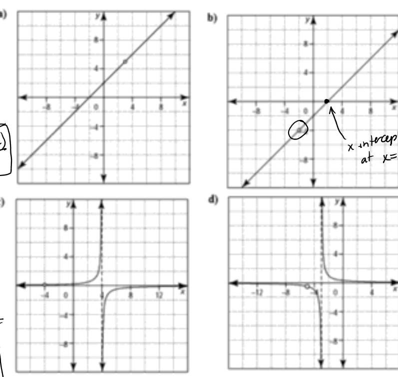
x-int. x=-2, so we need x+2 in top

$\boxed{y=\frac{(x-3)(x+2)}{x-3}}$

b) POD x=-2

$\boxed{y=\frac{(x+2)(x-2)}{(x+2)}}$

x intercept at x=2, so we need x-2 in top



c) v.a. x=4, so we need x-4 in bottom

POD x=-4, so x+4 in top + bottom

it's reflected, so, need vs of -1.

$\boxed{y=\frac{-1(x+4)}{(x+4)(x-4)}}$

d) V.a. x=-3, so we need x+3 in denominator

POD x=-5, so x+5 in top + bottom

$\boxed{y=\frac{x+5}{(x+5)(x+3)}}$

8. Write the equation of a possible rational function that has a vertical asymptote at $x=2$, a point of discontinuity at $x=-2.5$, and passes through the point $(6,-3)$

9. Solve algebraically. List all restrictions/NPVs.

a) $\frac{18}{x^2-9}+1=\frac{x}{x+3}$

b) $\frac{3}{5x-2}+x=5$

10. Solve graphically.

a) $3x=\frac{6x}{2x-5}$

b) $\frac{17-3x+x^2}{x-1}=2x-5$

8. Write the equation of a possible rational function that has a vertical asymptote at $x=2$, a point of discontinuity at $x=-2.5$, and passes through the point $(6,-3)$

$y=\frac{a(x+2.5)}{(x-2)(x+2.5)}$

Substitute (6,-3):

$-3=\frac{a(6+2.5)}{(6-2)(6+2.5)}$

$-3=\frac{a\cancel{(8.5)}}{(4)\cancel{(8.5)}}$

$-3=\frac{a}{4}$, a=-12

$y=\frac{-12(x+2.5)}{(x-2)(x+2.5)}$

9. Solve algebraically. List all restrictions/NPVs.

a) $\frac{18}{x^2-9}+1=\frac{x}{x+3}$

$\boxed{\text{NPVs: } x\neq -3,\ x\neq 3}$

$(x+3)(x-3)\left[\frac{18}{(x+3)(x-3)}+1=\frac{x}{x+3}\right]$

$\cancel{(x+3)}\cancel{(x-3)}\left(\frac{18}{\cancel{(x+3)}\cancel{(x-3)}}\right)+(x+3)(x-3)(1)=\cancel{(x+3)}(x-3)\left(\frac{x}{\cancel{x+3}}\right)$

$18+x^2\cancel{-3x}\cancel{+3x}-9=x(x-3)$

$\cancel{x^2}+9=\cancel{x^2}-3x$

$\frac{9}{-3}=\frac{-3x}{-3}$

x=-3, but that's an NPV

⇒ $\boxed{\text{no solution}}$

b) $\frac{3}{5x-2}+x=5$

NPV $5x-2\neq 0$, $5x\neq 2$, $x\neq\frac{2}{5}$

$(5x-2)\left[\frac{3}{5x-2}+x=5\right]$

$\cancel{(5x-2)}\left(\frac{3}{\cancel{5x-2}}\right)+(5x-2)(x)=(5x-2)(5)$

$3+5x^2-2x=25x-10$

$5x^2-2x+3-25x+10=0$

$5x^2-27x+13=0$

Use quadratic formula to solve

$x=\frac{27\pm\sqrt{(-27)^2-(4)(5)(13)}}{2(5)}$

$x=\frac{27\pm\sqrt{729-260}}{10}$

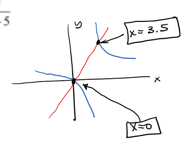
$x=\frac{27\pm\sqrt{469}}{10}$ $\doteq\boxed{4.87}$, $\doteq\boxed{0.53}$

10. Solve graphically.

a) $3x=\frac{6x}{2x-5}$

$Y_1=3x$

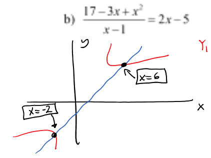
$Y_2=\frac{6x}{2x-5}$



b) $\frac{17-3x+x^2}{x-1}=2x-5$

$Y_1=\frac{17-3x+x^2}{x-1}$

$Y_2=2x-5$



11. Is each sequence geometric? If it is, what is the common ratio, r?

a) 1, 2, 4, 8, 16… b) 0.6, 0.06, 0.006 c) 2, 4, 6, 10, 16…

12. State the common ratio, then list the next 3 terms of each geometric sequence.

a) 1, 3, 9, 27, … b) 5, -15, 45, -135, … c) $6, 2, \frac{2}{3}, \frac{2}{9}, \ldots$

13. For each geometric sequence, write a formula for t_n and use it to find the indicated term.

a) 0.01, 0.08, 0.64, 5.12, . . Give t_n formula. Use it to find t_6.

b) 12 000, 8400, 5880, 4116, . . . Give t_n formula. Use it to find t_8.

14. Use the S_n formula to find the **sum** of the first 10 terms of each geometric series.

a) 20 + 60 + 180 + 540 + . . .

b) 50 + 37.5 + 28.125 + 21.09375 + . . .

c) 2.5 – 6.25 + 15.625 – 39.0625 + . . .

15. A doctor prescribes 200 mg of medication on the first day of treatment. The dosage is halved on each successive day. The medication lasts for seven days. To the nearest milligram, what is the total amount of medication administered?

16. A student band has two choices of payment to receive for 50 hours of performance.
Choice 1: $0.10 for the first hour, $0.12 for the second hour, $0.144 for the third hour, etc.
Choice 2: $100 for the first 10 hours, $200 for the second 10 hours, $400 for the third 10 hours, etc.
Which set of payments should the students choose to receive?

17. If the infinite geometric series has a finite sum, find the sum.

a) 81 + 27 + 9 + 3 + . . .

b) 140 + 35 + 8.75 + 2.1875 + . . .

c) 20 + 30 + 45 + 67.5 + . . .

d) 50 – 25 + 12.5 – 6.25 + . . .

18. Write each series using sigma notation.

a) 40 + 20 + 10 + 5 + 2.5 b) 3 + 9 + 27 + 81 + 243 + 729

19. A ball is dropped from a height of 2 meters to a floor. On each bounce, the ball rises to 50% of the height of the previous bounce. Calculate the total vertical distance (up and down) the ball travels before coming to rest.

20. Write each series in sigma notation.

a) 5 + 1 + 1/5 + 1/25 + 1/25

b) 2 + 6 + 18 + . . .

c) 3 + 6 + 12 + . . . + 768

11. Is each sequence geometric? If it is, what is the common ratio, r?

a) 1, 2, 4, 8, 16… b) 0.6, 0.06, 0.006 c) 2, 4, 6, 10, 16…

yes, $r=2$ yes, $r=0.1$ not geometric

12. State the common ratio, then list the next 3 terms of each geometric sequence.

a) 1, 3, 9, 27, … b) 5, -15, 45, -135, … c) $6, 2, \frac{2}{3}, \frac{2}{9}, \ldots$

$r=3$ $r=-3$ $r=\frac{1}{3}$

81, 243, 729 405, -1215, 3645 $\frac{2}{27}, \frac{2}{81}, \frac{2}{243}$

13. For each geometric sequence, write a formula for t_n and use it to find the indicated term.

a) 0.01, 0.08, 0.64, 5.12, . . Give t_n formula. Use it to find t_6. $t_n = 0.01\,(8)^{n-1}$

$r = \frac{0.08}{0.01} = 8$ $t_6 = 0.01\,(8)^5 = 327.68$

b) 12 000, 8400, 5880, 4116, . . . Give t_n formula. Use it to find t_8.

$r = \frac{8400}{12\,000} = 0.7$ $t_n = 12\,000\,(0.7)^{n-1}$ $t_8 = 12\,000\,(0.7)^7 \doteq 988.2516$

14. Use the S_n formula to find the **sum** of the first 10 terms of each geometric series.

$S_n = \frac{a(1-r^n)}{1-r}$

a) 20 + 60 + 180 + 540 + . . . $S_{10} = \frac{20\,(1-3^{10})}{1-3} = 590\,480$

b) 50 + 37.5 + 28.125 + 21.09375 + . . . $S_{10} = \frac{50\,(1-0.75^{10})}{-0.75} \doteq 188.74$

c) 2.5 – 6.25 + 15.625 – 39.0625 + . . . $S_{10} = \frac{2.5\,(1-(-2.5)^{10})}{1-(-2.5)} \doteq -6811.25$

15. A doctor prescribes 200 mg of medication on the first day of treatment. The dosage is halved on each successive day. The medication lasts for seven days. To the nearest milligram, what is the total amount of medication administered?

$S_7 = \frac{200\,(1-0.5^7)}{1-0.5} \doteq 397$ mg

16. A student band has two choices of payment to receive for 50 hours of performance.
Choice 1: $0.10 for the first hour, $0.12 for the second hour, $0.144 for the third hour, etc.
Choice 2: $100 for the first 10 hours, $200 for the second 10 hours, $400 for the third 10 hours, etc.
Which set of payments should the students choose to receive?

$S_{50} = \frac{0.10\,(1-1.2^{50})}{1-1.2} \doteq \$4549.72$ $S_5 = \frac{100\,(1-2^5)}{1-2} = \$3100$ choice 1

17. If the infinite geometric series has a finite sum, find the sum.

a) 81 + 27 + 9 + 3 + . . . $r=\frac{1}{3}$ $S = \frac{81}{(1-\frac{1}{3})} = 121.5$

b) 140 + 35 + 8.75 + 2.1875 + . . . $r = 0.25$ $S = \frac{140}{(1-0.25)} \doteq 186.67$

c) 20 + 30 + 45 + 67.5 + . . . $r = 1.5$ no finite sum

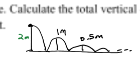
d) 50 – 25 + 12.5 – 6.25 + . . . $r = -\frac{1}{2}$ $S = \frac{50}{1-(-\frac{1}{2})} = 33.\overline{3}$

18. Write each series using sigma notation.

a) 40 + 20 + 10 + 5 + 2.5 b) 3 + 9 + 27 + 81 + 243 + 729

$r = \frac{20}{40} = \frac{1}{2}$ $\sum_{n=1}^{5} 40\left(\frac{1}{2}\right)^{n-1}$ $r = \frac{9}{3} = 3$ $\sum_{n=1}^{6} 3(3)^{n-1}$

19. A ball is dropped from a height of 2 meters to a floor. On each bounce, the ball rises to 50% of the height of the previous bounce. Calculate the total vertical distance (up and down) the ball travels before coming to rest.



Sum = drop + ups + downs
= 2 + 2 + 2
= $\boxed{6\text{ m}}$ total vertical distance

up: 1 + 0.5 + 0.25 . . .
down: 1 + 0.5 + 0.25 . . .

$S_{up} = \frac{1}{1-0.5} = 2$ $S_{down} = \frac{1}{1-0.5} = 2$

20. Write each series in sigma notation.

a) 5 + 1 + 1/5 + 1/25 + 1/25 $\sum_{n=1}^{5} 5\left(\frac{1}{5}\right)^{n-1}$

b) 2 + 6 + 18 + . . . $r = 3$, infinite sum $\sum_{n=1}^{\infty} 2(3)^{n-1}$ (This will have no finite sum, because $r=3$, and $-1<r<1$ is not met.)

c) 3 + 6 + 12 + . . . + 768

How many terms?
$t_n = 768$, $t_n = ar^{n-1}$
$768 = 3(2)^{n-1}$
$256 = 2^{n-1}$
$2^8 = 2^{n-1}$
$\Rightarrow 8 = n-1$
$n=9$

Now we know there are 9 terms:

$\sum_{n=1}^{9} 3(2)^{n-1}$