C_25 Other Sequences and Series Sheet

(Solutions at right)

Sequences and Series

1. A geometric sequence has $t_3 = 54$ and $t_5 = 4374$. Find the tenth term.

2. Use the appropriate formulas to find the exact sum of the geometric series: $1 + 6 + \ldots + 46656$

3. Carter has 262,143 boxes of cheese puffs. His daily consumption follows a geometric series. On the third day he consumes 48 boxes and on the fourth day he consumes 192 boxes. (Okay, these boxes must be microscopic in size!!!) Find the number of days his supply will last.

4. The first year that a cow was milked she produced 7570 L of milk. Each year she produced 9% less than the previous year. Find the total amount of milk the cow produced in 12 years.

Sequences and Series

54, —, —, —, 4374

1. A geometric sequence has $t_3 = 54$ and $t_5 = 4374$. Find the tenth term.

$\frac{54}{54}r^4 = \frac{4374}{54}$

$r^4 = 81$

$r = \pm 3$

For $r=3$: $t_3 = ar^2$, $54 = a(3)^2$, $\frac{54}{9} = a$, $a = 6$; $t_{10} = ar^9 = 6(3)^9 = 118098$

For $r=-3$: $t_3 = ar^2$, $54 = a(-3)^2$, $\frac{54}{9} = a$, $a = 6$; $t_{10} = ar^9 = 6(-3)^9 = -118098$

2. Use the appropriate formulas to find the exact sum of the geometric series: $1 + 6 + \ldots + 46656$

$a = 1$, $r = 6$, $n = ?$

$t_n = ar^{n-1}$

$46656 = 1(6)^{n-1}$

$6^6 = 6^{n-1}$

$\Rightarrow 6 = n-1$

$n = 7$

$S_n = \frac{a(1-r^n)}{1-r}$

$= \frac{1(1-6^7)}{1-6}$

$= \frac{-279935}{-5} = 55987$

3. Carter has 262,143 boxes of cheese puffs. His daily consumption follows a geometric series. On the third day he consumes 48 boxes and on the fourth day he consumes 192 boxes. (Okay, these boxes must be microscopic in size!!!) Find the number of days his supply will last.

—, —, 48, 192, ...

$r = \frac{192}{48}$

$r = 4$

Dividing by 4, we find $t_2 = 48/4 = 12$, $t_1 = 12/4 = 3$

So, $a = 3$

$S_n = \frac{a(1-r^n)}{1-r}$

$262\,143 = \frac{3(1-4^n)}{1-4}$

$262\,143 = \frac{3(1-4^n)}{-3}$

$262\,143 = -1(1-4^n)$

$262\,143 = -1 + 4^n$

$262\,144 = 4^n$

$4^9 = 4^n$

$\Rightarrow n = 9$ days

4. The first year that a cow was milked she produced 7570 L of milk. Each year she produced 9% less than the previous year. Find the total amount of milk the cow produced in 12 years.

7570, 7570(0.91), ...

$a = 7570$

$r = 0.91$

$n = 12$

$S_n = \frac{a(1-r^n)}{1-r}$

$S_{12} = \frac{7570(1-0.91^{12})}{(1-0.91)} = 56\,987.33956$

$\doteq 56\,987$ L