Cosine Law Worked Questions

Tuesday, June 7, 2022 9:32 PM

Scroll down for solutions - the resolution on them may not be great, but this is what I've got.

Exercises

1. Use the Law of Cosines to solve ΔABC in each case. Round the answers to one decimal place.

a) $\angle A = 43°, b = 7, c = 5$

b) $\angle B = 120°, a = 11, c = 15$

c) $\angle C = 85°, b = 16, a = 23$

d) $\angle A = 72°, b = 4.3, c = 2.9$

e) $\angle B = 130°, a = 32, c = 27$

f) $a = 14, b = 6, c = 10$

g) $a = 23, b = 31, c = 52$

h) $a = 8.3, b = 9.7, c = 12.5$

i) $a = 7, b = 9, c = 14$

j) $a = 2, b = 6, c = 7$

2. A triangular course is laid out with buoys in a lake for the triathlon competition. If the legs of the course are 850m, 675m and 420m respectively, find the largest and smallest angles made by the legs of the course rounded to the nearest degree.

3. To the left of skier is a tree and to the right is a cliff. The angle between the tree, the skier and the cliff is 74°. The tree is 34m from the skier and the cliff is 42m from her. Find the distance between the tree and the cliff rounded to the nearest meter.

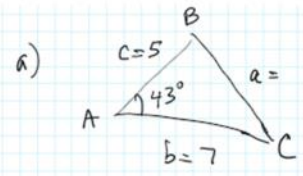
4. A radar tracking station locates a fishing trawler at a distance of 5.4km, and a passenger ferry at a distance of 7.2km. At the station, the angle between the two boats is 118°. How far apart are they? Round your answer to the nearest kilometer.

5. Two ships leave a port, sailing 16km/h and 29km/h. The angle between their directions of travel from the port is 42°. How far apart to the nearest kilometer are the ships after 2 hours?

Law of Cosines worked questions

Law of Cosines Examples

a)



$a^2 = 5^2 + 7^2 - 2(5)(7)\cos 43^\circ$

$a = 4.8$ 4.775483314

$\frac{\sin B}{7} = \frac{\sin 43^\circ}{4.8}$

$B = \sin^{-1}\left(\frac{\sin 43^\circ \cdot 7}{4.775483314}\right)$

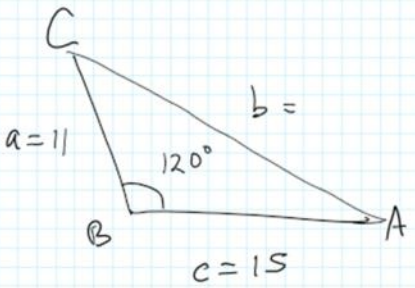
$B \doteq 88.6^\circ$

$\Rightarrow C \doteq 48.4^\circ$

But if we use 4.8, we get $B \doteq 84.0^\circ$, $C \doteq 53^\circ$

Rounding can make a difference.

b) $\angle B = 120^\circ$, $a = 11$, $c = 15$



$b^2 = 11^2 + 15^2 - 2(11)(15)\cos 120^\circ$

$b^2 \doteq 511$

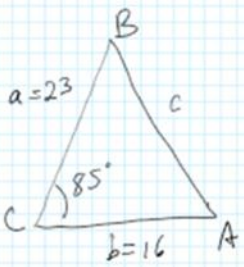
$b \doteq 22.6$

$\frac{\sin 120^\circ}{22.6} = \frac{\sin A}{11}$

$A = \sin^{-1}\left(\frac{11 \sin 120^\circ}{22.6}\right) \Rightarrow A \doteq 24.9^\circ$

$\Rightarrow C \doteq 35.1^\circ$

c) $\angle C = 85°$, $b = 16$, $a = 23$



$c^2 = 23^2 + 16^2 - 2(23)(16)\cos 85°$

$c^2 = 720.8533733$

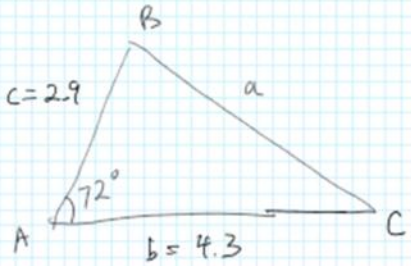
$c \doteq 26.8$ $\quad 26.84871269 = c$

$\frac{\sin 85°}{26.8} = \frac{\sin A}{23}$

$A = \sin^{-1}\left(\frac{23\sin 85°}{26.8}\right) \Rightarrow A = 58.8°$ or, if not rounded: $58.6°$

$\Rightarrow B = 36.2°$ $\quad 36.4°$

d) $\angle A = 72°$, $b = 4.3$, $c = 2.9$



$a^2 = 4.3^2 + 2.9^2 - 2(4.3)(2.9)\cos 72°$

$a^2 = 19.19311616$

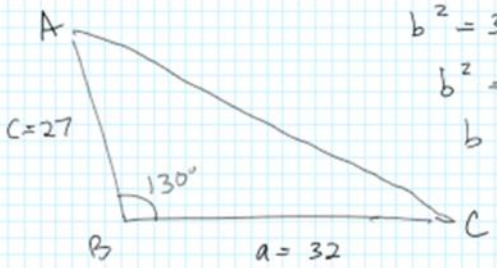
$a = 4.380994882 \doteq 4.4$

$\frac{\sin 72°}{4.4} = \frac{\sin B}{4.3}$

$B = \sin^{-1}\left(\frac{4.3\sin 72°}{4.4}\right)$, $B \doteq 69.0°$ or $68.3°$ if rounded

$\Rightarrow C = 39°$ $\quad 39.7°$

e) $\angle B = 130°$, $a = 32$, $c = 27$



$b^2 = 32^2 + 27^2 - 2(32)(27)\cos 130°$

$b^2 = 2863.73699$

$b = 53.51389529 \doteq \boxed{53.5}$

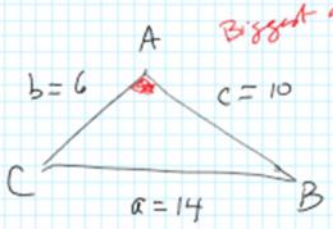
$$\frac{\sin 130°}{53.5} = \frac{\sin A}{32}$$

$$A = \sin^{-1}\left(\frac{32\sin 130°}{53.5}\right) \Rightarrow \boxed{A \doteq 27.3°}$$

rounded ↓ 27.3°

$\boxed{C \doteq 22.7°}$

f) $a = 14$, $b = 6$, $c = 10$

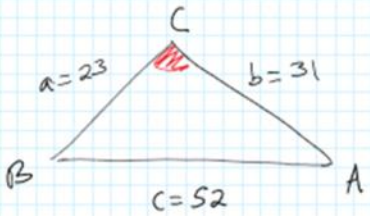


$$\cos A = \frac{6^2 + 10^2 - 14^2}{2(6)(10)}, \quad \boxed{A = 120°}$$

$$\frac{\sin 120°}{14} = \frac{\sin C}{10}, \quad \boxed{C \doteq 38.2°}$$

$\Rightarrow \boxed{B = 21.8°}$

g) a = 23, b = 31, c = 52

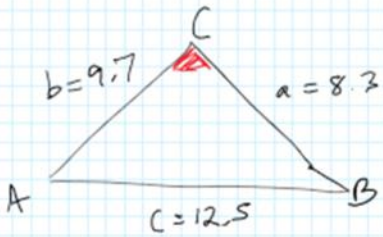


$$\cos C = \frac{23^2 + 31^2 - 52^2}{2(23)(31)}, \quad C \doteq 148.4^\circ$$

$$\frac{\sin 148.4^\circ}{52} = \frac{\sin B}{31}, \quad B \doteq 18.2^\circ$$

$$\Rightarrow A = 13.4^\circ$$

h) a = 8.3, b = 9.7, c = 12.5

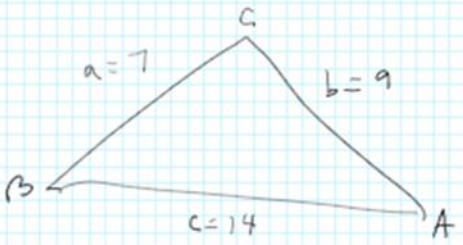


$$\cos C = \frac{8.3^2 + 9.7^2 - 12.5^2}{2(8.3)(9.7)}, \quad C \doteq 87.6^\circ$$

$$\frac{\sin 87.6^\circ}{12.5} = \frac{\sin B}{9.7}, \quad B \doteq 50.8^\circ$$

$$\Rightarrow A = 41.6^\circ$$

i) a = 7, b = 9, c = 14

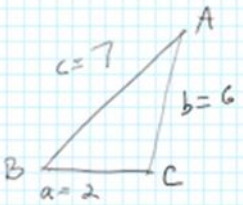


$\cos C = \frac{7^2+9^2-14^2}{2(7)(9)}$, $C \doteq 126°$ (121.588135)

$\frac{\sin 126°}{14} = \frac{\sin B}{9}$, $B \doteq 33.2°$

$\Rightarrow A \doteq 25.2°$

j) a = 2, b = 6, c = 7



Alternative Form

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

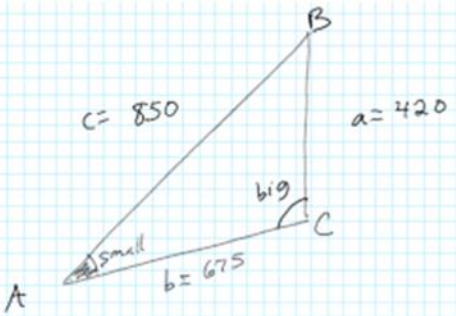
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$\cos C = \frac{2^2+6^2-7^2}{2(2)(6)}$, $C \doteq 112.0°$ (112.02432 8)

$\frac{\sin 112°}{7} = \frac{\sin B}{6}$, $B \doteq 52.6°$

$\Rightarrow A \doteq 15.4°$

2. A triangular course is laid out with buoys in a lake for the triathlon competition. If the legs of the course are 850m, 675m and 420m respectively, find the largest and smallest angles made by the legs of the course rounded to the nearest degree.



$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos C = \frac{420^2 + 675^2 - 850^2}{2(420)(675)}$$

$$C = \cos^{-1}\left(\quad\right)$$

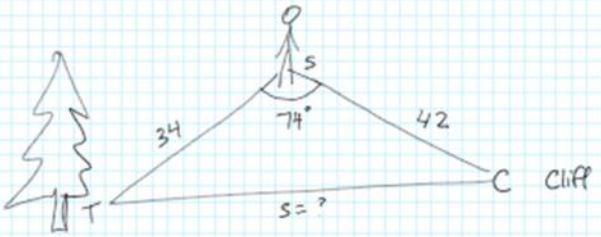
Biggest angle: $C \doteq 99°$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$A = \cos^{-1}\left(\frac{675^2 + 850^2 - 420^2}{2(675)(850)}\right)$$

Small angle: $A \doteq 29°$

3. To the left of skier is a tree and to the right is a cliff. The angle between the tree, the skier and the cliff is 74°. The tree is 34m from the skier and the cliff is 42m from her. Find the distance between the tree and the cliff rounded to the nearest meter.

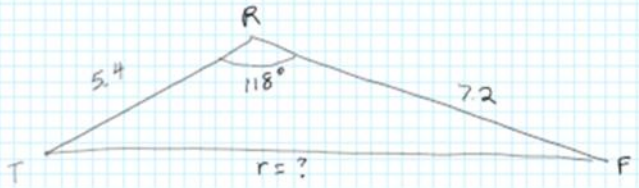


$s^2 = 34^2 + 42^2 - 2(34)(42)\cos 74°$

$s^2 = 2132.779712$

$s \doteq$ 46 m

4. A radar tracking station locates a fishing trawler at a distance of 5.4km, and a passenger ferry at a distance of 7.2km. At the station, the angle between the two boats is 118°. How far apart are they? Round your answer to the nearest kilometer.



$r^2 = 5.4^2 + 7.2^2 - 2(5.4)(7.2)\cos 118°$

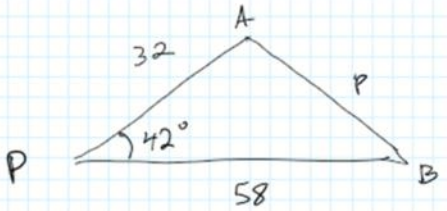
$r^2 = 117.5061087$

$r \doteq 10.84 \doteq$ 11 km

5. Two ships leave a port, sailing 16km/h and 29km/h. The angle between their directions of travel from the port is 42°. How far apart to the nearest kilometer are the ships after 2 hours?

$16 \times 2 = 32$

$29 \times 2 = 58$



$p^2 = 32^2 + 58^2 - 2(32)(58)\cos 42^\circ$

$p^2 = 1629.446408$

$p \doteq 40$ km