WELCOME TO PRE-CALCULUS 12 Spring 2023

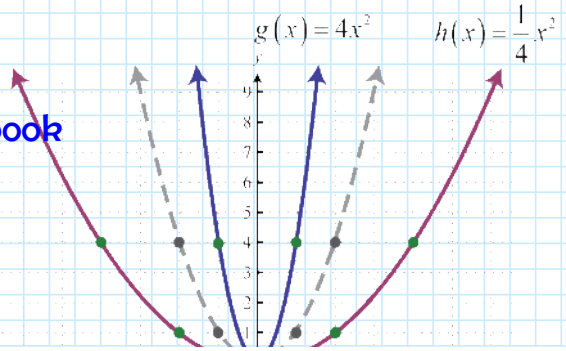
- **Monday-Thursday, 6:30-9:30 p.m.**
- **May 3 - June 21**
- **Meets in Portable 26**
- **Instructors: Anurita Dhiman and Susana Egolf**

As we work together for student success through LEARNING, ENGAGEMENT, and CONNECTION to inspire all learners (including ourselves) to reach their full potential and create a positive legacy for the future, we do so on the traditional, unceded territory of the Matsqui, Kwantlen, Katzie, and Semiamhoo first nations

Plan For Today:

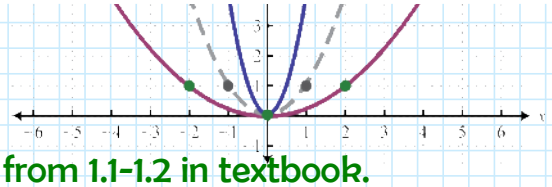
1. Intro to course: Course Outline & Calendar
2. Review Basic Graphing of Functions and Domain & Range
3. Start working on Chapter 1
   - ✓ 1.1 Horizontal and Vertical Translations
   - ✓ 1.2 Reflections and Stretches - INTRO
4. Work on practice questions from Textbook

Page 12: #2, 3cd, 4ac, 5, 8, 11
Page 28: #3b, 4b, 5-7, 9, 12



Plan Going Forward:

| #3b, 4b, 5-7, 9, 12 |



Plan Going Forward:

1. Finish going through practice question from 1.1-1.2 in textbook.
2. You will continue practicing 1.2 and will go through 1.3 on Thursday. Have a look through these sections to prepare for tomorrow.

Please let me know if you have any questions or concerns about your progress in this course. The notes from today will be posted at egolfmath.weebly.com after class.
Let us know if you have any questions or will be away.
Anurita Dhiman = adhiman@sd35.bc.ca
Susana Egolf = segolf@sd35.bc.ca

Do you need a textbook, or need to see the office for something else?

Wednesday, May	9:30 - 6:30 pm
Thursday, May 4	9:30 - 6:30 pm

**Office hours REVISED beginning week of MAY 8th 2023:

Mon/Tue/Wed/Th	9:30am-4:30pm
Fri	9:30am-2:30pm (closed 12:00-1:00pm)

Course Outline

PREC 12 Outline May 2023

Pre-Calculus 12

Teachers: Anurita Dhiman adhiman@sd35.bc.ca
Susana Egolf segolf@sd35.bc.ca

Website: https://egolfmath.weebly.com/

Meets in
Portable 26
Mon-Tues-Wed-Thurs
6:30-9:30 PM

(28 classes)

MAY

Su	M	Tu	W	Th	F	Sa
	1	2	first class	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	no class	23	24	25	26	27
28	29	30	31			

JUNE

Su	M	Tu	W	Th	F	Sa
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	last class	22	23	24
25	26	27	28	29	30	

Portable 26
Mon-Tues-Wed-Thurs
6:30-9:30 PM

(28 classes)

PC12 Topics

Chapter	Content
1	Function Transformations
3	Polynomial Functions
4	Trigonometry and the Unit Circle, Equations
5	Trigonometric Functions and Graphs
6	Trigonometric Identities
7	Exponential Functions and Equations
8	Logarithmic Functions and Equations
9	Rational Functions and Equations
G	Geometric Sequences and Series

Curricular Competencies

Reason and model
Communicate and represent
Understand and solve
Connect and reflect

Evaluation

CATEGORY	PERCENT OF COURSE
Chapter Assignments	9%
Weekly Tests	91%
TOTAL	100%

Important Information

Tests There will be 7 tests, each covering all the material completed in class in the previous week. Tests will be closed book and supervised in class. There is no final exam for this course.

Test Dates:

Tuesday, May 9	Monday, June 5
Monday, May 15	Monday, June 12
Tuesday, May 23	Tuesday, June 20
Monday, May 29	Wednesday, June 21 – Retest Day

On the last day of class, June 21, there will be the opportunity to do a rewrite test on any TWO tests. If you rewrite a test, only the higher mark of the two attempts will be recorded towards your final grade.

Assignments There will be an assignment for each chapter, marked based on completion. Doing math questions helps you develop confidence with the math we are learning. Partial solutions to these assignments will be posted on the class website, **http://egolfmath.weebly.com/**

In-class Notes You will receive notes packages which we use in class. You will also be able to access filled-in notes on the class website.

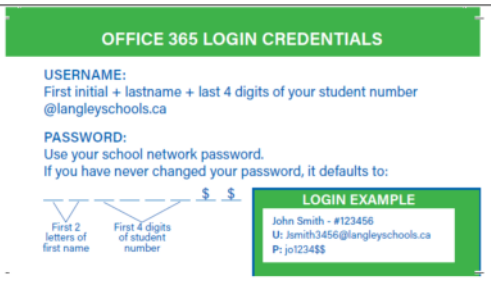
Expectations

- If in good health, attend every class, arriving on time.
- If you are sick, use the posted in-class notes and stay caught up with the course schedule.
- **No smoking is permitted on school property.**
- Show consideration and courtesy.
  - Be attentive and focused when class is in session.
  - Use in-class time productively; participate in group activities.
- Ask for help with questions you find difficult. If you are struggling, watch the video for that section. Video links are provided on the class website for every section in the textbook.
- Set aside extra time to prepare for tests.
-

Wireless access:
SD35 – Visitors, password is VisitorSD35

SD35-Secured-Students
Username: pupil number
Password: as shown at right

Class website:
http://egolfmath.weebly.com/



Class website
https://egolfmath.weebly.com/

TB, page 2

Unit 1

Transformations and Functions

Functions help you make sense of the world around you. Many ordinary measuring devices are based on mathematical functions:

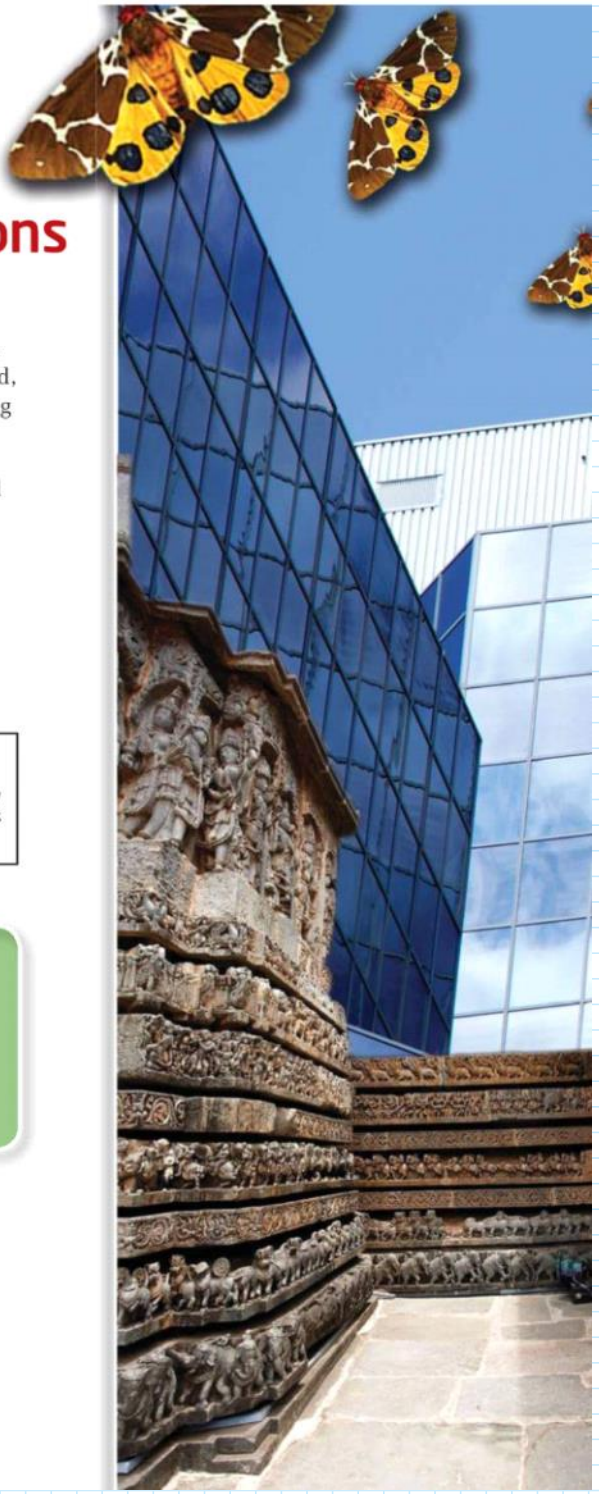
- Car odometer: The odometer reading is a function of the number of rotations of the car's transmission drive shaft.
- Display on a barcode reader: When the screen displays the data about the object, the reader performs an inverse function by decoding the barcode image.

Many natural occurrences can be modelled by mathematical functions:

- Ripples created by a water droplet in a pond: You can model the area spanned by the ripples by a polynomial function.
- Explosion of a supernova: You can model the time the explosion takes to affect a volume of space by a radical function.

CHAPTER 1

Function Transformations



Mathematical shapes are found in architecture, bridges, containers, jewellery, games, decorations, art, and nature. Designs that are repeated, reflected, stretched, or transformed in some way are pleasing to the eye and capture our imagination.

In this chapter, you will explore the mathematical relationship between a function and its transformed graph. Throughout the chapter, you will explore how functions are transformed and develop strategies for relating complex functions to simpler functions.

Did You Know?

Albert Einstein (1879–1955) is often regarded as the father of modern physics. He won the Nobel Prize for Physics in 1921 for "his services to Theoretical Physics, and especially for his discovery of the law of the photoelectric effect." The Lorentz transformations are an important part of Einstein's theory of relativity.

Key Terms

transformation
mapping
translation
image point
reflection
invariant point
stretch
inverse of a function
horizontal line test

Chapter 1: Function Transformations

1.0 Review

A ***relation*** is a set of ordered pairs (x, y).

For example: {(−1, 6) (2, 8) (5, 10) (8, 12)}

A ***function*** is a special type of relation.

- A function is like a machine. For each x-value, the function follows a rule to create exactly ONE y-value that goes with that x-value.
- **Vertical Line Test:** Function graphs contain NO points that are directly above one another.

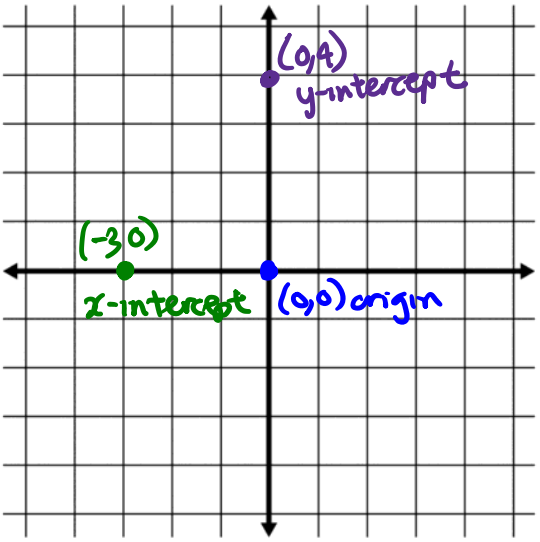
  For example, a function CANNOT contain both $(2,8)$ and $(2,5)$

Graphing

Ordered pairs are graphed on a coordinate system: (x, y)

x*-coordinates** tell how far to move left or right from the ***origin, $(0,0)$

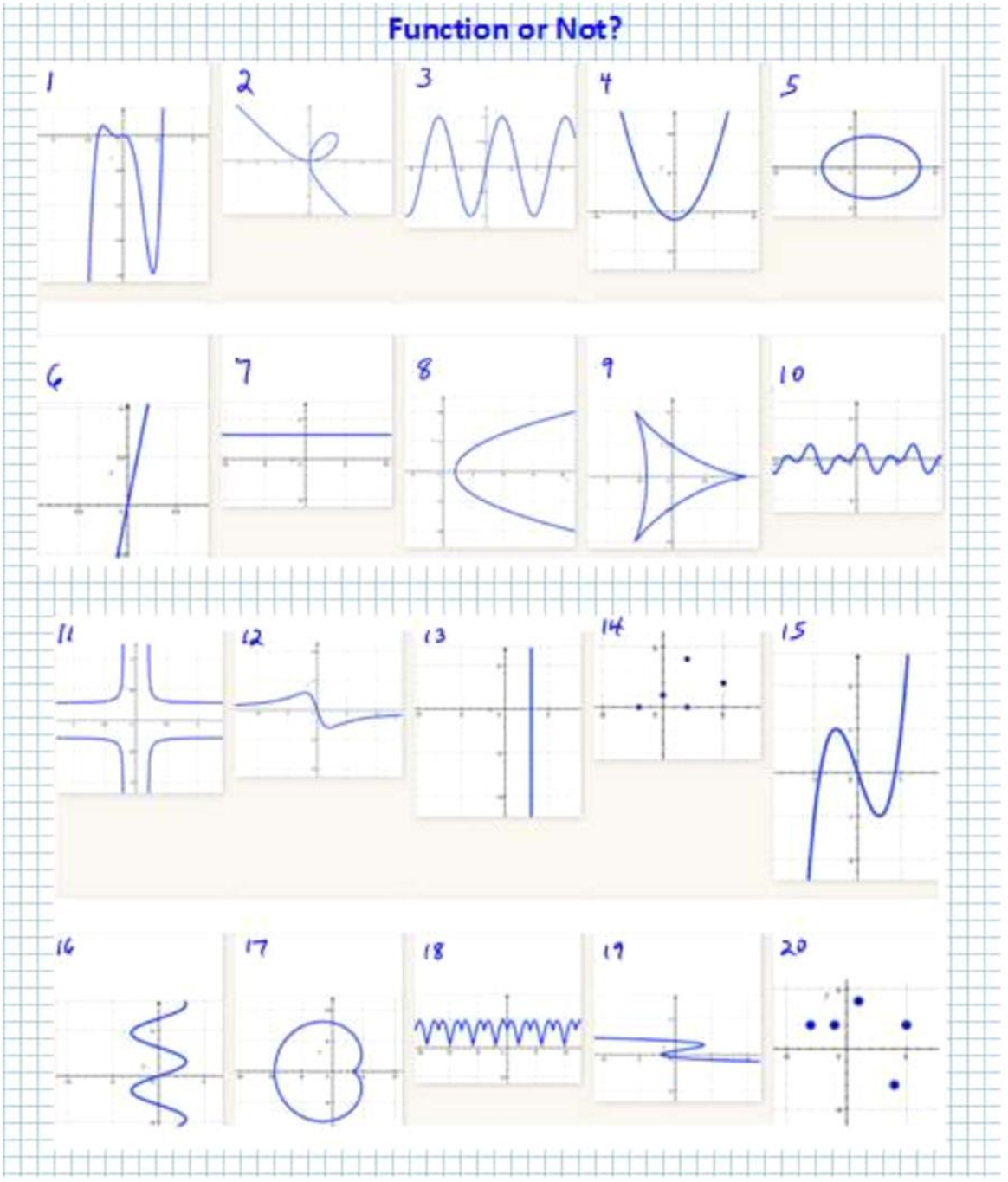
***y*-coordinates** tell how far to move up or down from the origin.



Plotting points.

Plotting graphs,

**On this page we use the vertical line test to decide whether or not the pictured graphs are functions.



Function Notation

$y = f(x)$

- means y is a function of x, so the y-value depends on the x-value we choose
- is read "y equals f of x"

To Try

1) Given the function $f(x) = 3 - 4x$, find the value of $f(-4)$

x-input, output

$f(-4) = 3 - 4(-4)$
$= 3 + 16$ → $f(-4) = \boxed{19}$

2) Given the function $g(x) = -2x^2 + 5$, find the value of $g(2)$

BEDMAS (1) (2) (3)

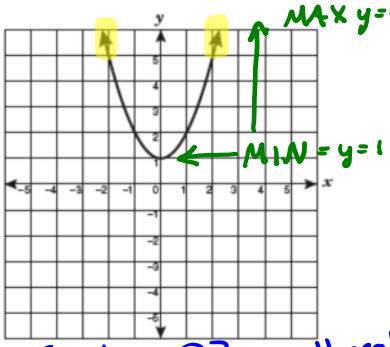
$g(2) = -2(2)^2 + 5$
$= -2(4) + 5$
$= -8 + 5$
$g(2) = \boxed{-3}$

Domain all the possible x-values of a function or relation $\{x \mid x \_\_, x \in \mathbb{R}\}$ ($\in$ = in the set of; $\mathbb{R}$ = real numbers)

Range all the possible y-values of a function or relation $\{y \mid y \_\_, y \in \mathbb{R}\}$

Find the domain and range for each graph below, and write it using set notation.

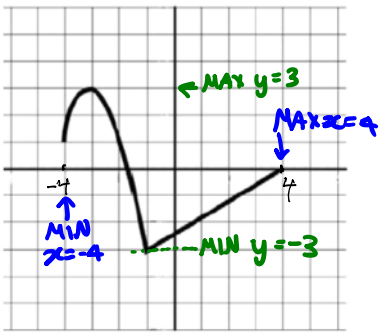
a)



Domain $\{x \mid x \in \mathbb{R}\}$ = all real numbers

Range $\{y \mid y \geq 1, y \in \mathbb{R}\}$ = y is greater than or equal to 1 + all real numbers

b)



$\{x \mid -4 \leq x \leq 4, x \in \mathbb{R}\}$

$\{y \mid -3 \leq y \leq 3, y \in \mathbb{R}\}$

We can find domain restrictions even without graphing, by looking at a function's equation.

Remember, we:

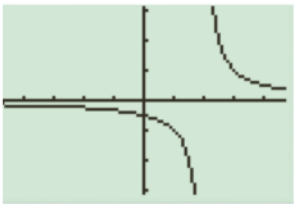
- **Can't divide by zero**
- **Can't take square-roots of negatives**

To Try

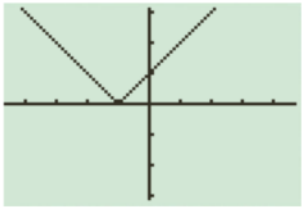
We can use technology to create the graph of a function.

- Use a graphing calculator to graph the following functions. Your graphs should match the graphs shown below.
- Determine the ***domain*** and ***range*** for each one.

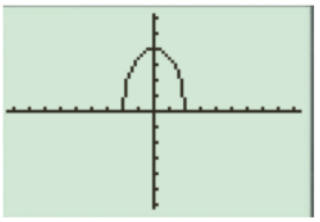
a) $f(x) = \dfrac{1}{x-2}$



b) $f(x) = |x+1|$



c) $f(x) = \sqrt{16-4x^2}$



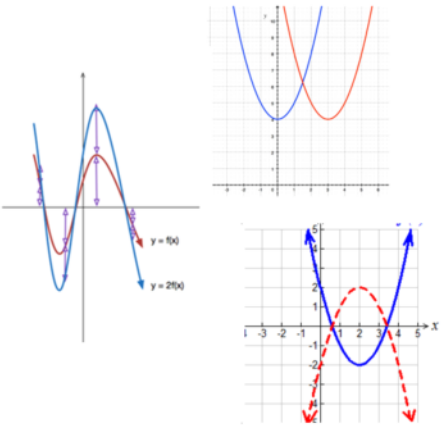
Used [-9.4, 9.4] [-6.2, 6.2]

Transformations

We often sketch the graphs of functions. If we change a function's equation, the new equation produces a new, TRANSFORMED, graph.

Transformations include:

translations
reflections
stretches



When a graph is transformed, each point on the graph is affected by the transformation.

Suppose that the *x*-coordinates for all the points on a graph are increased by three units. Here is a way to show how the points are changed.

MAPPING $(x, y) \rightarrow (x+3, y)$

p. 7 textbook.

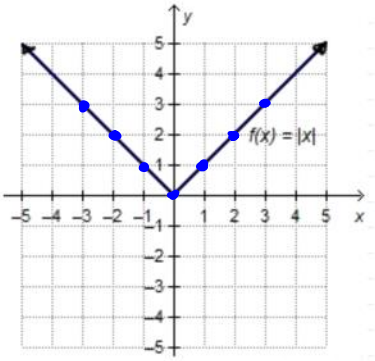
mapping

- the relating of one set of points to another set of points so that each point in the original set corresponds to exactly one point in the image set

(Textbook, page 7)

1.1 Horizontal and Vertical Translations

The graph of the base absolute value function is shown at right, and below are three transformed equations.

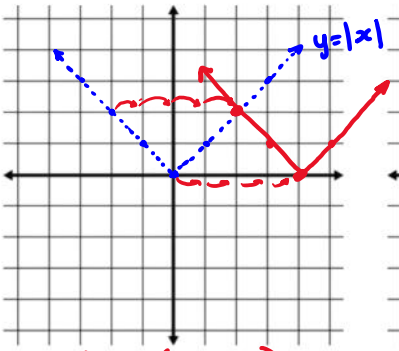


For each one:

- Sketch its graph on the grid.
- Describe, in words, the transformation that happened.
- Describe the transformation by giving its ***mapping***.
- State the domain and range.

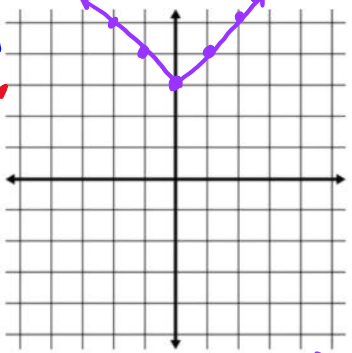
$f(x) = |x-4|$

4 right



$(x,y) \rightarrow (x+4, y)$

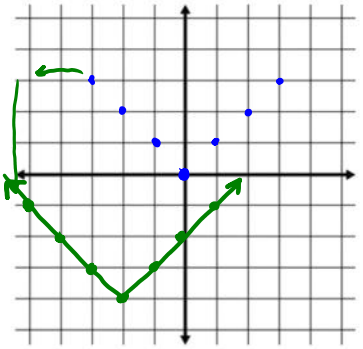
$f(x) = |x| + 3$ 3 up



$(x,y) \rightarrow (x, y+3)$

$f(x) = |x+2| - 4$

2 left 4 down



$(x,y) \rightarrow (x-2, y-4)$

Points on an original graph correspond with points on a transformed graph, often called the ***image graph***. We say that each original point is ***mapped*** to an ***image point***.

$$y - k = f(x - h)$$

(Horizontal translation: h; Vertical translation: k)

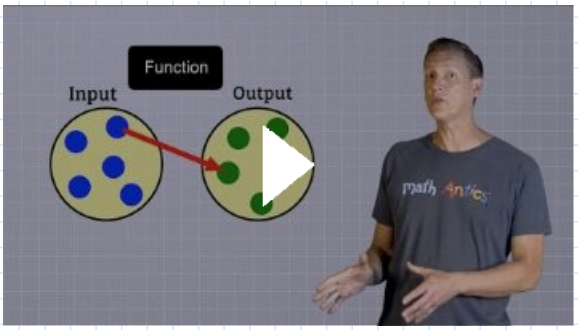
Often equations are arranged with the "y" term isolated:

$$y = f(x - h) + k$$

(Horizontal translation: h; Vertical translation: k)

Want more explanation about functions, domain and range? Watch this:

Algebra Basics: What Are Functions? - Math Antics


Function
Input
Output

TRANSLATIONS – sliding graphs left/right/up/down

Some specific examples:
- when x is replaced with $x-8$, the graph will move 8 right.
- when x is replaced with $x+6$, the graph will move 6 left.
- when y is replaced with $y-4$, the graph will move 4 up.
- when y is replaced with $y+7$, the graph will move 7 down.

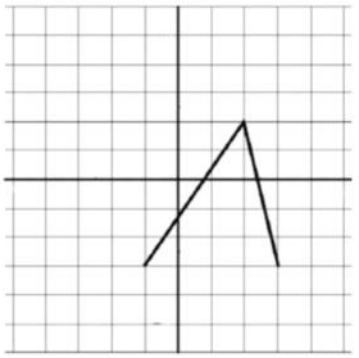
Base Function Equation	Transformed Equation	Mapping	Point on original graph	Its image point
$y=x^2$	$y-4=x^2$		$(-3, 9)$	
$y=x+5$	$y=(x-3)+5$		$(2,7)$	
$y=\log_5 x$	$y=\log_5(x-2)+3$		$(25, 2)$	
$y=2^x$	$y=2^{x-3}+8$		$\left(-1, \frac{1}{2}\right)$	
$y=\frac{2}{x-4}$	$y=\frac{2}{(x+3)-4}+6$		$\left(8, \frac{1}{2}\right)$	
$x^2+y^2=16$	$(x-5)^2+(y+3)^2=16$		$(-4, 0)$	

To Try

Shown is the graph of $y = f(x)$.

a) Identify the transformations that result when the equation is changed to: $y - 2 = f(x+3)$

b) Make a table of key points on the original graph and the corresponding image points on the image graph.



Base

x	y

Image

c) Sketch the image graph.

d) State the domain and range of the image graph. (Assume that the line segments stop.)

Example

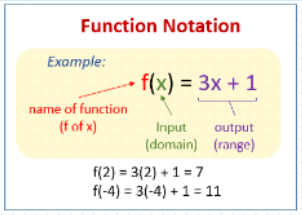
Given the mapping notation for a transformation, we can write the transformed equation.

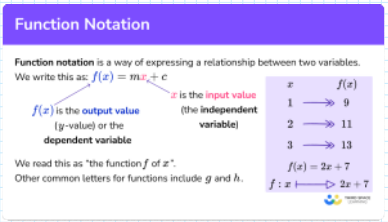
a) Mapping notation $(x, y) \to (x-8, y+3)$
Original function $y = f(x)$
New function

b) Mapping notation $(x, y) \to (x+4, y-9)$
Original function $y = f(x)$
New function

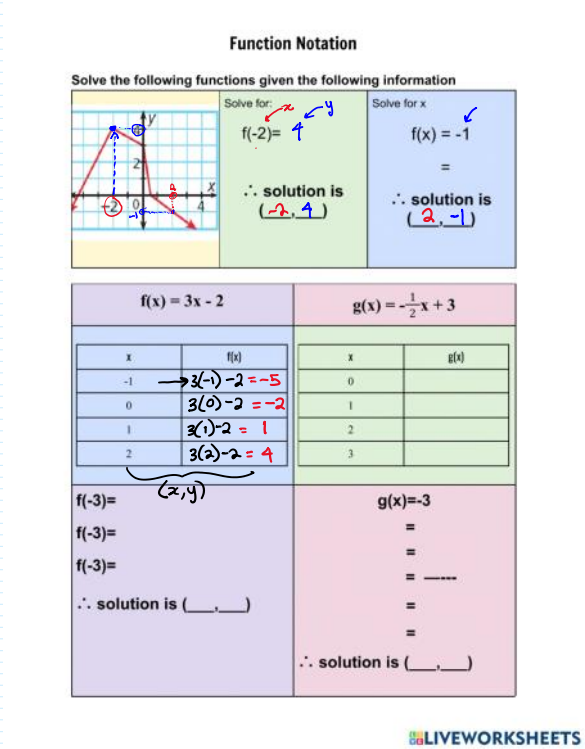
Review: Functions and Graphs

What is a function? *y = f(x)*
Vertical line test
Domain and Range

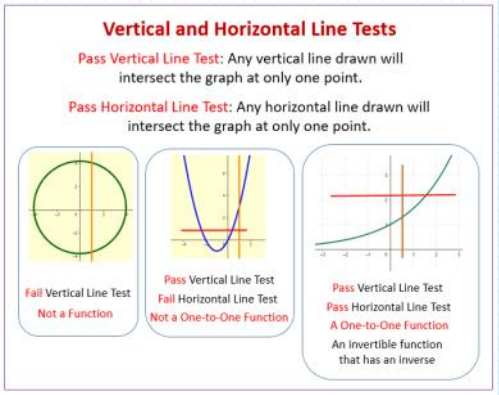




https://www.liveworksheets.com/ia2907456em



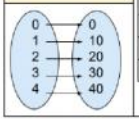
https://www.liveworksheets.com/worksheets/en/Math/Functions/Function_or_Relation_tc2853874lz



What is a Relation? What is a Function?

A **relation** is a rule that associates an x-value with y-value(s)

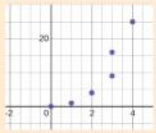
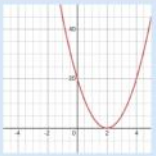
Question 1: Name the method of expression of the relation in the row above the examles

{(0,0),(1,10),(2,20),(3,30)}		y = 10x	
			

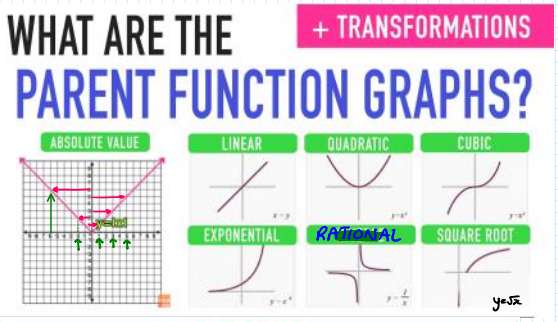
# of Lawns	Revenue ($)
0	0
1	10
2	20
3	30

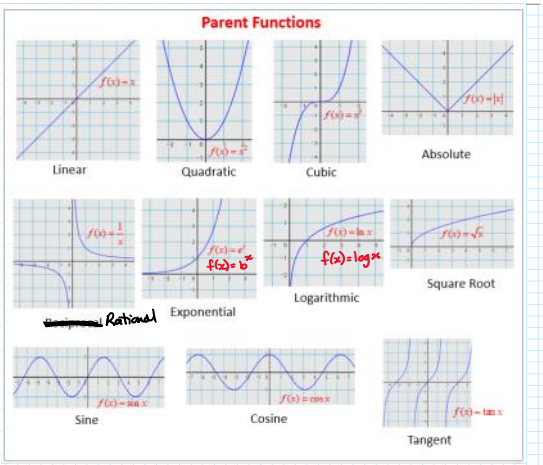
A **function** is a special type of relation in which every independent value (x-value) has **only one** dependent value (y-value). In other words, a relation is a function when everytime we plug a value in for x we only get one y value

Question 2: Determine whether the following relations are functions:

{ (4,3),(5,6),(6,9),(7,12),(8,15)}	{ (4,3),(4,6),(5,9),(5,12),(6,15)}
	

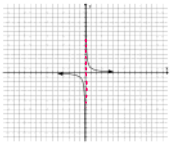
LIVEWORKSHEETS



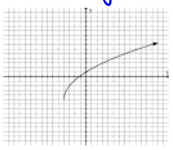


Try to name these base/parent functions:

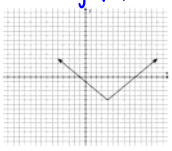
Rational Function
$y = \frac{1}{x}$



Square Root Function
$y = \sqrt{x}$

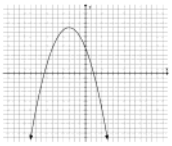


Absolute Value Function
$y = |x|$



Cubic Function
(Polynomial Function) $y = x^3$

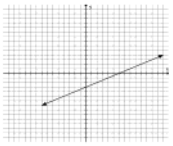
Quadratic Function
$y = x^2$

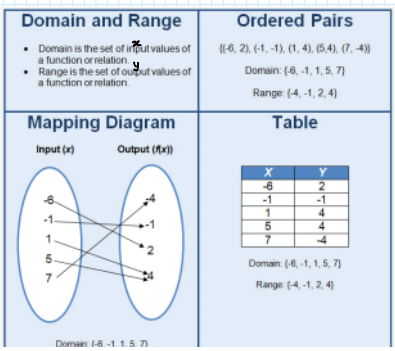


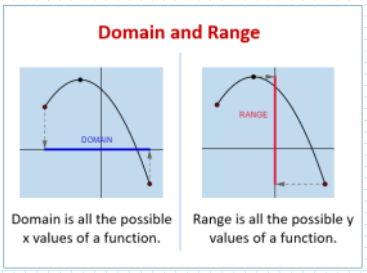
Linear Function
$y = x$

$y = mx + b$

slope, y-intercept







Domain and Range

- Domain is the set of input values of a function or relation.
- Range is the set of output values of a function or relation.

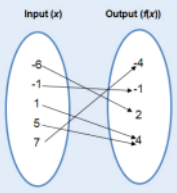
Ordered Pairs

{(-6, 2), (-1, -1), (1, 4), (5,4), (7, -4)}

Domain: {-6, -1, 1, 5, 7}

Range: {-4, -1, 2, 4}

Mapping Diagram



Domain: {-6, -1, 1, 5, 7}

Range: {-4, -1, 2, 4}

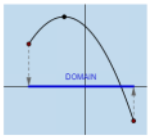
Table

X	Y
-6	2
-1	-1
1	4
5	4
7	-4

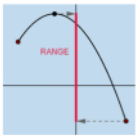
Domain: {-6, -1, 1, 5, 7}

Range: {-4, -1, 2, 4}

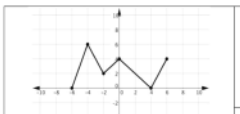
Domain and Range



Domain is all the possible x values of a function.



Range is all the possible y values of a function.



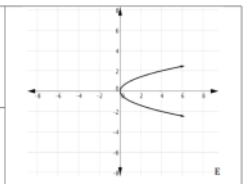
A

Domain: $\{-6 \le x \le 6\}$

Range: $\{0 \le y \le 6\}$

Function: YES

1



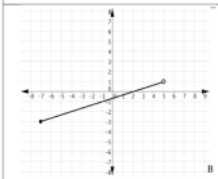
E

Domain: $\{-4 \le x \le 4\}$

Range: $\{-4 \le y \le 4\}$

Function: NO

2



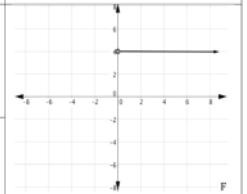
B

Domain: $\{-4 \le x \le 2\}$

Range: $\{-2 \le y \le 4\}$

Function: YES

3



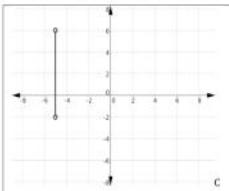
F

Domain: $\{x = -5\}$

Range: $\{-2 < y < 6\}$

Function: NO

4



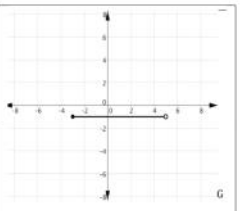
C

Domain: $\{-3 \le x < 5\}$

Range: $\{y = -1\}$

Function: YES

5



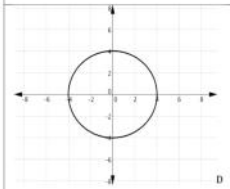
G

Domain: $\{x > 0\}$

Range: $\{y = 4\}$

Function: YES

6



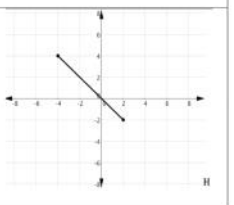
D

Domain: $\{x \ge 0\}$

Range: {all real numbers}

Function: NO

8



H

Domain: $\{-7 \le x < 5\}$

Range: $\{-3 \le y < 1\}$

Function: YES

11

Basics of Graphing Review 2019

Basic Graphing Review – Know these base functions and their graphs so you are able to apply transformations on them in the course.

- Label the x- and y- axis
- Make a table of values
- Plot the point on your grid
- Draw a line or smooth curve
- Domain: the set of x values valid in the equation
- Range: the set of y values valid in the equation

Use Set Notation for writing domain and range:
$\{x \mid x \in R\}$ means x is in the set of real numbers
$\{y \mid y \in R\}$ means y is in the set of real numbers
Use the following symbols:
$\leq$ for less than and equal to; $<$ for only less than
$\geq$ for greater than and equal to: $>$ for only greater than
$\neq$ for not equal to

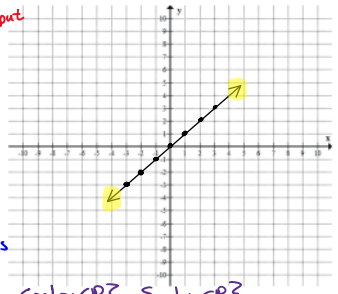
When graphing, start with a table of values. Look at restrictions and use your graphing calc to verify.

1. Graph: $y = x$

input, output

x	y
-3	-3
-2	-2
-1	-1
0	0
1	1
2	2
3	3

(x, y) coordinates



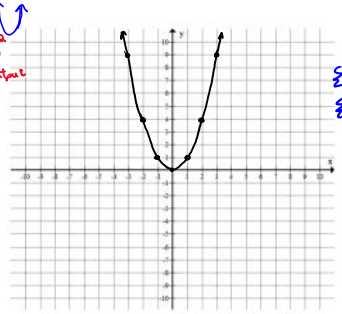
$\{x \mid x \in \mathbb{R}\}$ $\{y \mid y \in \mathbb{R}\}$

2. Graph: $y = x^2$

input $y=(x)^2$ output

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

$y=(-3)^2=9$
$y=-3^2=-9$



$\{x \mid x \in \mathbb{R}\}$
$\{y \mid y \geq 0, y \in \mathbb{R}\}$

3. Graph: $y = \sqrt{x}$

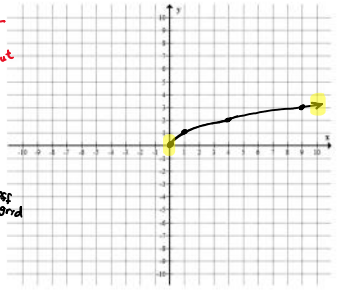
$y=\sqrt{-1}$ undefined

input $y=\sqrt{x}$ output

x	y
0	0
1	1
4	2
9	3
16	4
25	5
36	6

off grid

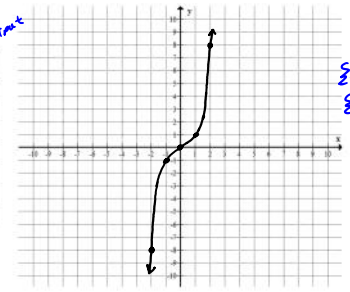
$y=\sqrt{0}$
$y=\sqrt{1}$
$y=\sqrt{4}$
$y=\sqrt{9}$



$\{x \mid x \geq 0, x \in \mathbb{R}\}$
$\{y \mid y \geq 0, y \in \mathbb{R}\}$

4. Graph: $y = x^3$

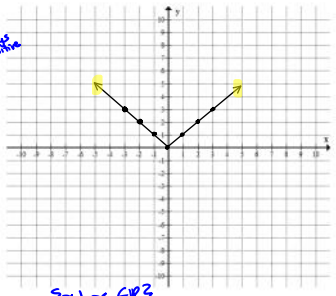
x	y
-3	-27
-2	-8
-1	-1
0	0
1	1
2	8
3	27



$\{x \mid x \in \mathbb{R}\}$
$\{y \mid y \in \mathbb{R}\}$

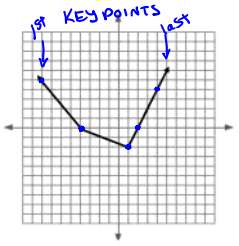
5. Graph: $y = |x|$

x	y
-3	3
-2	2
-1	1
0	0
1	1
2	2
3	3



$\{x \mid x \in \mathbb{R}\}$
$\{y \mid y \geq 0, y \in \mathbb{R}\}$

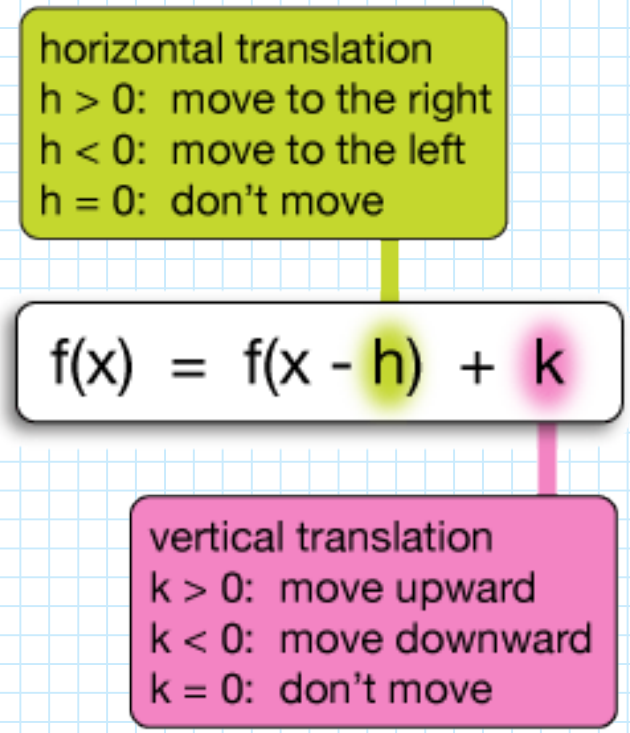
6. This is the graph of $y = f(x)$. List 4 or 5 points on this graph in the table of values.



x	y
-8	5
-4	0
1	-2
2	0
4	4

1.1 Translations

$$y = f(x) \rightarrow y = f(x - h) + k$$



The following table shows the coordinates of the point on the different curves after translation:

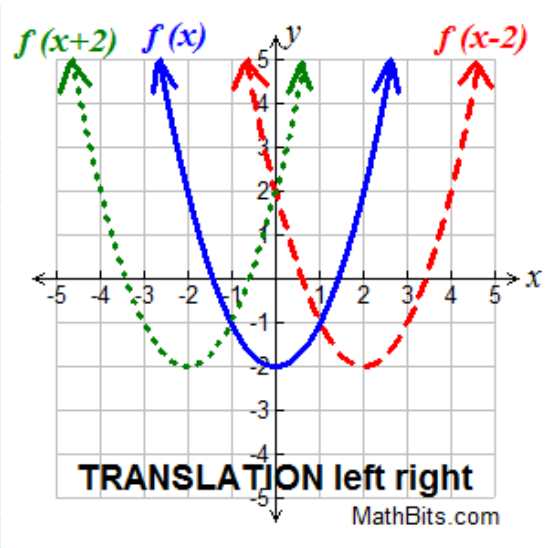
The point on $f(x)$	The point on $f(x+k)$	The point on $f(x-k)$
(x, y)	$(x-k, y)$	$(x+k, y)$

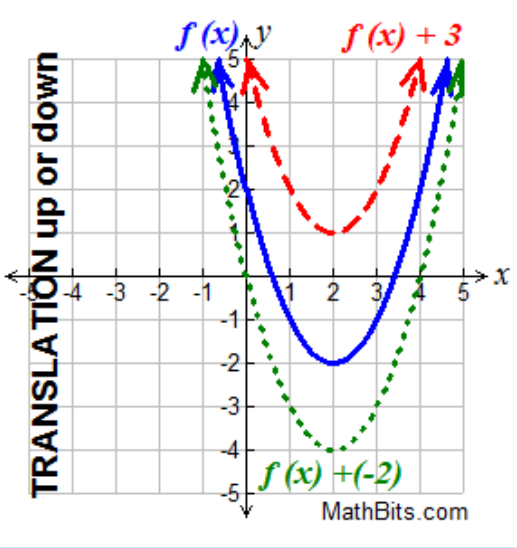
Meanwhile, the shape of the function and domain of the function remains the same.

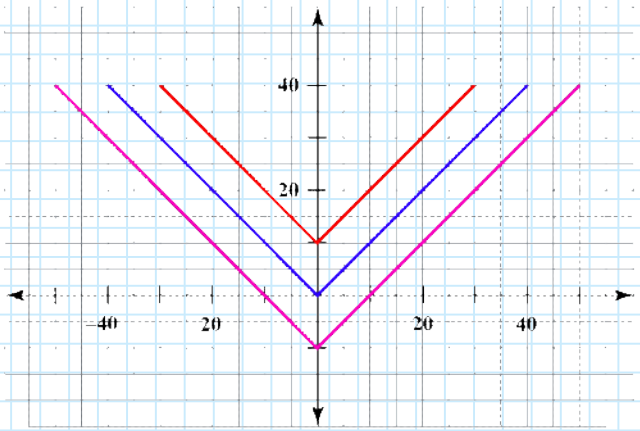
The following table shows the coordinates of the point on the different curves after translation:

The point on f(x)	The point on f(x) + C	The point on f(x) - C
(x, y)	$(x, y+C)$	$(x, y-C)$

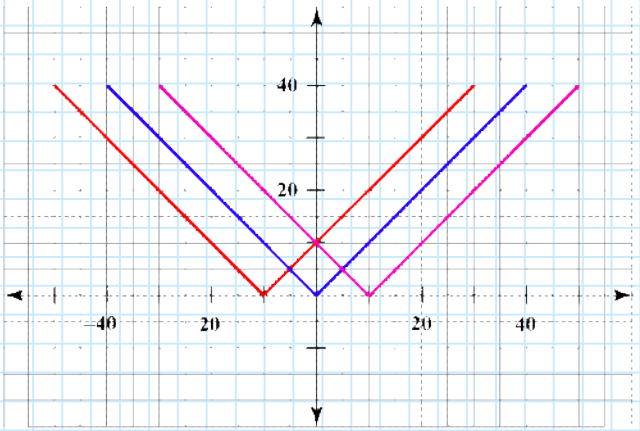
Meanwhile, the shape of the function and domain of the function remains the same.



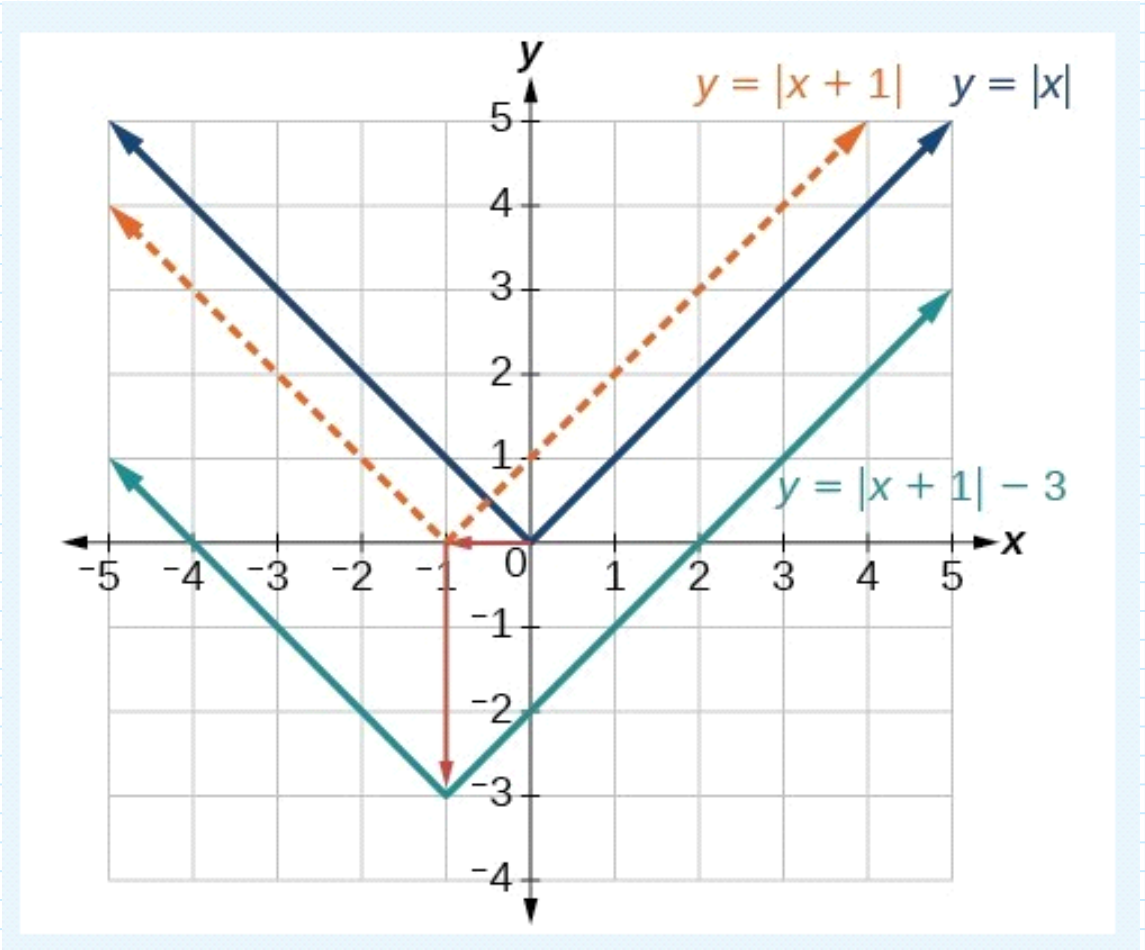


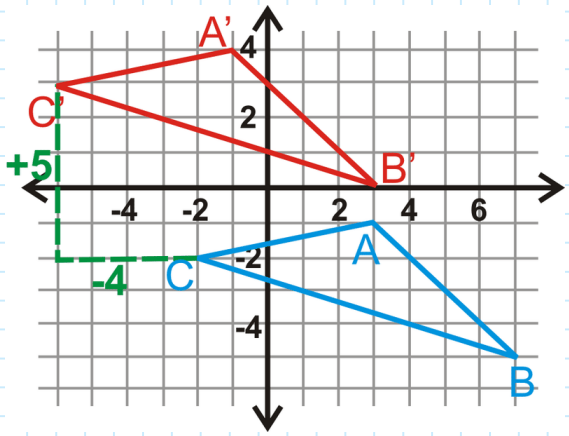


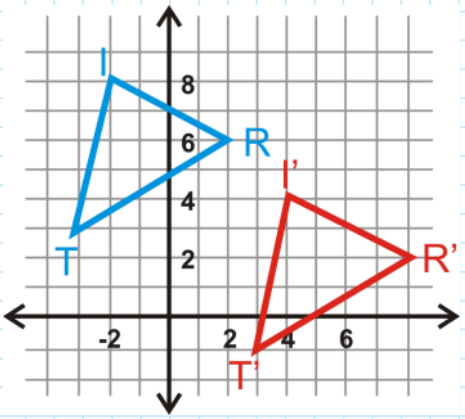
Vertical Translation



Horizontal Translation


y
x
y = |x + 1|
y = |x|
y = |x + 1| − 3


A'
C'
B'
+5
-4
C
A
B


I
R
T
I'
R'
T'