Class_02 Jan 10 - Mixed and Entire Radicals, Rational Exponents

Tonight's Class:
- **Questions from 1.1-1.2?**
- **Preview 1.3-1.4**
- **Working through sections 1.3-1.4**
  - **Converting between mixed and entire radicals**
  - **Powers with positive and negative fractional exponents**
- **Work on practice questions from worktext**

RECAP

verdad

Estimate the value of each radical to 1 decimal place. (Verify)

a) $\sqrt{18}$ b) $\sqrt[3]{50}$

$\sqrt{16} = 4$ $\sqrt{18}$ $\sqrt{25} = 5$

4.2

$\sqrt{18} \approx 4.24...$

$(4.2)^2 = 17.64$

$(4.3)^2 = 18.49$

$\sqrt[3]{27} = 3$ $\sqrt[3]{50} = 3.7$ $\sqrt[3]{64} = 4$

$\sqrt[3]{50} \doteq 3.68$

$(3.7)^3 = 50.653$

Identify the sets to which each of the following numbers belongs by marking an "X" in the appropriate boxes.

	Number	Natural Numbers	Whole Numbers	Integers	Rational Numbers	Irrational Numbers
1.	26	X	X	X	X	
2.	−5.24				X	
3.	$\sqrt{19}$					X
4.	$\frac{2}{11}$				X	
5.	$-\frac{8}{2} = -4$			X	X	
6.	0		X	X	X	
7.	$\sqrt{81} = 9$	X	X	X	X	
8.	$4.2\overline{7}$				X	
9.	2.101001...					X

rational: the number can be written as a ratio of two integers.

$-\frac{8}{2} = -\frac{8}{2} = -4$ OR -4

$\frac{-8}{2} = -4$

$\frac{8}{-2} = -4$

Is $1 \div 17$ rational or irrational?

Is $1 \div 17$ rational or irrational?

$\boxed{\frac{1}{17}}$ this is rational

$$17 \overline{)1.000000} = .0588235294117647 05$$

irrational: π, $\sqrt{2}$, $4.0100100010000 1\ldots$

p 20 $\sqrt[3]{\frac{54 \div 2}{200 \div 2}}$ rational / irrational ?

1) reduce radicand

$= \sqrt[3]{\frac{27}{100}}$

2) evaluate

$= \frac{\sqrt[3]{27}}{\sqrt[3]{100}}$ ← not a perfect cube

$= \frac{3}{\sqrt[3]{100}}$ not rational

$\Rightarrow$ irrational

Example 2 **Identifying Radicals as Representing Rational or Irrational Numbers**

Without determining the value of each radical, identify whether it represents a rational number or an irrational number.

a) $\sqrt{0.5625}$

b) $\sqrt{45}$

c) $\sqrt{5.8}$

d) $\sqrt[3]{1.728}$

a) $\sqrt{0.5625}$

1) write radicand as a fraction

$= \sqrt{\frac{5625 \div 5}{10000 \div 5}}$

2) reduce radicand

$= \sqrt{\frac{1125 \div 5}{2000 \div 5}}$

$= \sqrt{\frac{225 \div 5}{400 \div 5}}$

$$= \sqrt{\frac{225}{400}} \begin{matrix}\div 5\\ \div 5\end{matrix}$$

$$= \sqrt{\frac{45}{80}} \begin{matrix}\div 5\\ \div 5\end{matrix}$$

$$= \sqrt{\frac{9}{16}} = \frac{\sqrt{9}}{\sqrt{16}} = \frac{3}{4}$$ rational

Preview 2

1.3 Mixed and Entire Radicals

Focus: convert radicals between entire and mixed forms

The Product Rule for Square Roots and Cube Roots

For any real numbers A and B:

$$\sqrt{A \cdot B} = \sqrt{A} \cdot \sqrt{B}$$

$$\sqrt[3]{A \cdot B} = \sqrt[3]{A} \cdot \sqrt[3]{B}$$

multiply

$$\sqrt{2} \cdot \sqrt{8} = \sqrt{2 \cdot 8} = \sqrt{16} = 4$$

$$\sqrt[3]{40} = \sqrt[3]{5 \cdot 8} = \sqrt[3]{5} \cdot \sqrt[3]{8} = \sqrt[3]{5} \cdot 2 = 2 \cdot \sqrt[3]{5} = 2\sqrt[3]{5}$$

Terminology

Entire Radical: a radical in the form $\sqrt{x}$

Examples: $\sqrt{35}$, $\sqrt{68}$

Mixed Radical: a radical in the form $a\sqrt{x}$

Examples: $3\sqrt{5}$, $-4\sqrt{21}$

Changing entire radicals to mixed radicals

$$\sqrt{20} = \sqrt{2 \cdot 2 \cdot 5} = \sqrt{2^2 \cdot 5}$$

entire radical

1) write radicand in prime factored form

20

4 5

entire radical

$$= \sqrt{2^2 \cdot 5}$$
$$= \sqrt{2^2} \cdot \sqrt{5}$$
$$= 2 \cdot \sqrt{5}$$
$$= 2\sqrt{5}$$

mixed radical

20

4 5

2 2

We always make the radicand is as small as possible.

$$\sqrt{20} = \sqrt{4 \cdot 5}$$
$$= \sqrt{4}\sqrt{5}$$
$$= 2\sqrt{5}$$

Perfect Square Method

$$\sqrt{48} = \sqrt{16 \times 3}$$
$$= \sqrt{16} \times \sqrt{3}$$
$$= 4 \times \sqrt{3}$$
$$= 4\sqrt{3}$$

Prime Factors Method

$$\sqrt{48} = \sqrt{2 \times 2 \times 2 \times 2 \times 3}$$
$$= \sqrt{2 \times 2} \times \sqrt{2 \times 2} \times \sqrt{3}$$
$$= 2 \times 2 \times \sqrt{3}$$
$$= 4 \times \sqrt{3}$$
$$= 4\sqrt{3}$$

Perfect Cube Method

$$\sqrt[3]{24} = \sqrt[3]{8 \cdot 3}$$
$$= \sqrt[3]{8}\ \sqrt[3]{3}$$
$$= 2\sqrt[3]{3}$$

Prime Factors Method

$$\sqrt[3]{24} = \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 3}$$
$$= 2\sqrt[3]{3}$$

24

3 12

2 6

2 3

Try "Check Your Understanding" p 23, p 24

Check Your Understanding

2. Write each entire radical as a mixed radical.

a) $\sqrt{63}$ b) $-\sqrt[3]{80}$

c) $\sqrt[4]{243}$

2c) $\sqrt[4]{243}$ index

$$= \sqrt[4]{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}$$
$$= 3 \cdot \sqrt[4]{3}$$

243

3 81

3 27

3 9

243

2+4+3

= 9

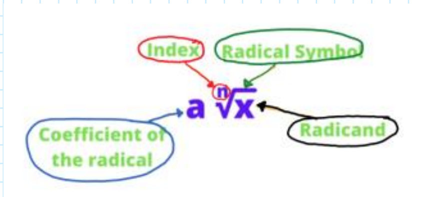
9 is divisible by 3

⇒ 243 is divisible by 3

3 9

3 3

Changing mixed radicals to entire radicals



$$6\sqrt{20} = \sqrt{6^2 \cdot 20} = \sqrt{720}$$

$$2\sqrt[3]{7} = \sqrt[3]{2^3 \cdot 7} = \sqrt[3]{56}$$

$$2\sqrt[4]{3} = \sqrt[4]{2^4 \cdot 3} = \sqrt[4]{48}$$

Can you figure out the process?

To change a mixed radical to an entire radical
- Identify the index of the radicand, *n*
- Raise the coefficient of the radical to the *n*th power, put it inside the radical sign
- Multiply the numbers to create the new radicand
- If the coefficient is negative, be careful!
  - odd index, include the negative sign as part of the radicand
  - even index, leave negative sign out in front of radical

change to entire form

$2\sqrt{3} = \sqrt{2^2 \cdot 3}$

index = 2

$= \sqrt{4 \cdot 3}$

$= \sqrt{12}$

check:

1) you could change the entire radical back to mixed form

$\sqrt{12} = \sqrt{2 \cdot 6}$

$= \sqrt{2 \cdot 2 \cdot 3}$

$= 2\sqrt{3}$

OR

2) evaluate $\sqrt{12}$, $2\sqrt{3}$ } Are they the same?

index = 2 even

$-6\sqrt{3} = -\sqrt{6^2 \cdot 3}$

$= -\sqrt{36 \cdot 3}$

$= -\sqrt{108}$

$2\sqrt[3]{5} = \sqrt[3]{2^3 \cdot 5}$

index = 3

$$= -\sqrt{36\cdot 3}$$
$$= -\sqrt{108}$$

index odd

$$-2\sqrt[3]{4} = \sqrt[3]{(-2)^3\cdot 4}$$
$$= \sqrt[3]{-8\cdot 4}$$
$$= \sqrt[3]{-32}$$

index=3

$$2\sqrt[3]{5} = \sqrt[3]{2^3\cdot 5} = \sqrt[3]{8\cdot 5} = \sqrt[3]{40}$$

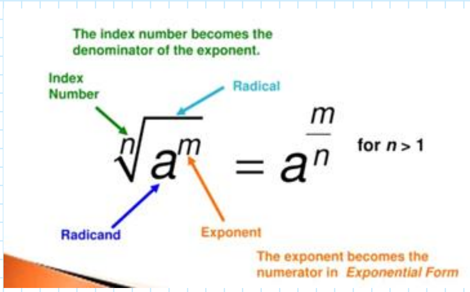
index=4

$$3\sqrt[4]{4} = \sqrt[4]{3^4\cdot 4}$$
$$= \sqrt[4]{81\cdot 4}$$
$$= \sqrt[4]{324}$$

- Worktext, check understanding p 25 #3
- Try some 1.3 questions, starting on p 26

1.4 Powers with Positive Rational Exponents

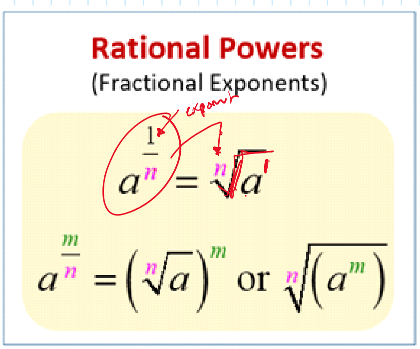
Focus: understand powers with positive rational exponents



radical form | exponential form

bottom | top

$$\sqrt{5^3} = 5^{3/2}$$



Worktext examples, pages 37-39

Example 1 Writing Powers with Rational Exponents as Radicals

Write each power as a radical, then evaluate the radical. = get number answer

a) $49^{\frac{1}{2}}$ **b)** $(-64)^{\frac{1}{3}}$ **c)** $\left(\frac{16}{25}\right)^{\frac{1}{2}}$ **d)** $81^{0.25}$

top, index

a) $49^{1/2} = \sqrt[2]{49^1}$

$= \sqrt{49}$

$= 7$

b) $(-64)^{1/3} = \sqrt[3]{-64^1}$

OR $= \sqrt[3]{-64}$

$= -4$

c) $\left(\frac{16}{25}\right)^{1/2} = \sqrt[2]{\frac{16}{25}^1}$

$= \sqrt{\frac{16}{25}} = \frac{4}{5}$

d) $81^{0.25} = 81^{1/4}$

$= \sqrt[4]{81^1} = 3$

Example 2 Writing Radicals as Powers with Rational Exponents

Write each entire or mixed radical as a power with a rational exponent.

a) $\sqrt[3]{1.2}$ **b)** $\sqrt[4]{\frac{9}{25}}$ **c)** $4\sqrt{3}$

denom = index

a) $\sqrt[3]{1.2^1} = 1.2^{1/3}$ index

b) $\sqrt[4]{\frac{9}{25}} = \left(\frac{9}{25}\right)^{1/4}$ use brackets to make it clear!

c) $4\sqrt{3}$

1) first, we'll change it to entire form

2) then, write as a rational exponent

index = 2

$4\sqrt{3} = \sqrt{4^2 \cdot 3}$

$4\sqrt{3} = \sqrt{4^2 \cdot 3}$

$= \sqrt{16 \cdot 3}$

$= \sqrt{48}$

$48^{1/2}$

Example 3	**Evaluating Powers with Rational Exponents I**

Write each power as a radical, then evaluate the radical.

a) $4^{\frac{3}{2}}$ **b)** $\left(-\frac{1}{8}\right)^{\frac{2}{3}}$ **c)** $100^{1.5}$

a) $4^{3/2} = \sqrt[2]{4^3}$

$= \sqrt[2]{64}$

$= 8$

b) $\left(-1/8\right)^{2/3} = \sqrt[3]{\left(-1/8\right)^2}$

$= \sqrt[3]{\frac{1}{64}}$

$= \frac{1}{4}$

$\left(-\frac{1}{8}\right)\left(-\frac{1}{8}\right)$

$= +\frac{1}{64}$

c) $100^{1.5} = 100^{3/2}$

$= \sqrt[2]{(100)^3}$

$= \sqrt[2]{1{,}000{,}000}$

$= 1000$

$\left(\sqrt[2]{(100)}\right)^3$

$= (10)^3$

$= 1000$

$\boxed{1.5 = 1\tfrac{5}{10} = \frac{15 \div 5}{10 \div 5} = \frac{3}{2}}$

For next class

- Complete the "Recap" from tonight!
  - Do all you can without looking at examples/worktext. Then, switch to a different color of pen/pencil and complete the rest of it. This way you can see what you might need to spend more time on.
- Finish worktext questions 1.3, and you can start with those from 1.4. You don't get marks for doing these. Doing them helps you learn/practice the concepts.