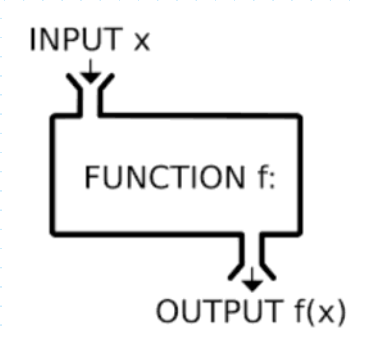
Class_02 May 4 - Transforming Functions

Thursday, May 4, 2023 12:43 PM

Tonight's Class:

- **Translations, wrapping up**
- **Reflections & Stretches**
- **Combining transformations**

Last class....



$$y = \frac{2}{x+3} \qquad \{x \mid x \neq -3,\ x \in \mathbb{R}\}$$

What x-value can't be used?

Domain

all allowable x-values. Can't use x-values that "BREAK" the function machine

Range

All y-values

WB - domain/range

TRANSLATIONS – sliding graphs left/right/up/down

Some specific examples:
- when x is replaced with $x-8$, the graph will move 8 right.
- when x is replaced with $x+6$, the graph will move 6 left.
- when y is replaced with $y-4$, the graph will move 4 up.
- when y is replaced with $y+7$, the graph will move 7 down.

$y = f(x)$ changed to $y = f(x-8)$ } moves original graph 8 units RIGHT

Base Function Equation	Transformed Equation	Mapping	Point on original graph	Its image point
$y=x^2$	$y-4=x^2$ up 4	$(x,y)\to(x,y+4)$	$(-3, 9)$	$(-3,13)$
$y=x+5$	$y=(x-3)+5$ 3 right	$(x,y)\to(x+3,y)$	$(2,7)$	$(5,7)$
$y=\log_5 x$	$y=\log_5(x-2)+3$ 2 right 3 up	$(x,y)\to(x+2,y+3)$	$(25, 2)$	$(27,5)$
$y=2^x$	$y=2^{x-3}+8$ 3 right up 8	$(x,y)\to(x+3,y+8)$	$\left(-1,\frac{1}{2}\right)$	$\left(2,8\frac{1}{2}\right)$
$y=\frac{2}{x-4}$	$y=\frac{2}{(x+3)-4}+6$ 3 left 6 up	$(x,y)\to(x-3,y+6)$	$\left(8,\frac{1}{2}\right)$	$\left(5,6\frac{1}{2}\right)$
$x^2+y^2=16$	$(x-5)^2+(y+3)^2=16$ 5 right 3 down	$(x,y)\to(x+5,y-3)$	$(-4, 0)$	$(1,-3)$

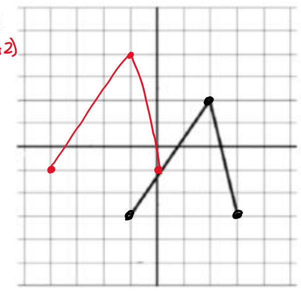
where a point ends up, after it is transformed

Notice for this one, the "5" was already there in the original equation. The ONLY change, is the graph is shifted right 3.

To Try
Shown is the graph of $y = f(x)$.

a) Identify the transformations that result when the equation is changed to: $y-2 = f(x+3)$
2 up 3 left $(x,y) \rightarrow (x-3, y+2)$

b) Make a table of key points on the original graph and the corresponding image points on the image graph.



Base

x	y
-1	-3
2	2
3	-3

Image

$x-3$	$y+2$
-4	-1
-1	4
0	-1

c) Sketch the image graph.

d) State the domain and range of the image graph. (Assume that the line segments stop.)

$\{x \mid -4 \le x \le 0,\ x \in \mathbb{R}\}$

$\{y \mid -1 \le y \le 4,\ y \in \mathbb{R}\}$

Example
Given the mapping notation for a transformation, we can write the transformed equation.

a) Mapping notation $(x, y) \rightarrow (x-8, y+3)$ 8 left and 3 up
Original function $y = f(x)$
New function

$y = f(x+8) + 3$

OR $y - 3 = f(x+8)$

b) Mapping notation $(x, y) \rightarrow (x+4, y-9)$ 4 right 9 down
Original function $y = f(x)$
New function

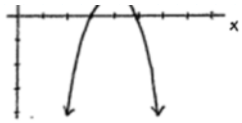
$y = f(x-4) - 9$

OR

$y + 9 = f(x-4)$

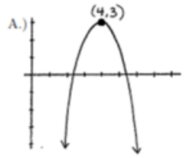
WB - domain/range, looking at graphs

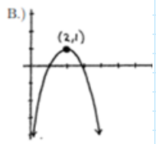
Translations Review - talk to each other, agree on your answers

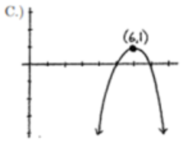


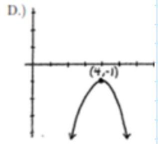
Given the graph of $y = f(x)$ shown above, match the following four function equations with their graphs (A, B, C or D)

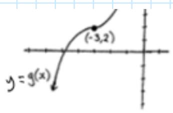
1. $y = f(x) + 2$ graph: A
2. $y = f(x) - 2$ graph: D
3. $y = f(x + 2)$ graph: B
4. $y = f(x - 2)$ graph: C





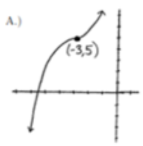


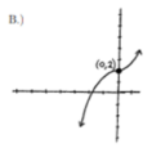


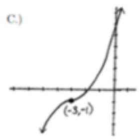


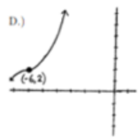
Given the graph of $y = g(x)$ shown above, match the following four function equations with their graphs (A, B, C, or D)

1. $y = g(x) + 3$ graph: A
2. $y = g(x) - 3$ graph: C
3. $y = g(x + 3)$ graph: D
4. $y = g(x - 3)$ graph: B









1.2 Reflections and Stretches

Reflections

Across the x-axis

$y \to -y$

$\frac{-y}{-1} = \frac{f(x)}{-1}$

$y = -f(x)$

Original key points

x	y
-4	3
-3	2
1	4
3	0

Reflected key points

x	$-y$
-4	-3
-3	-2
1	-4
3	0

all y-values changed to its opposite (multiplied by -1)

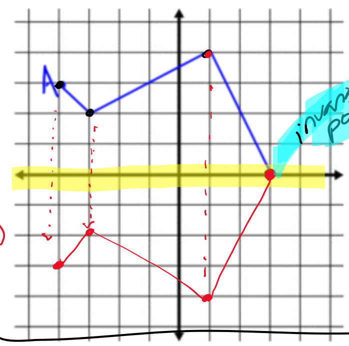


Image point for point A: $(-4, -3)$
Original equation: $y = f(x)$
New equation: $-y = f(x)$ but we usually write that instead as $y = -f(x)$
Mapping: $(x, y) \to (x, -y)$

Across the y-axis

Original key points

x	y
-4	3
-3	2
1	4
3	0

Reflected key points

x	y
4	3
3	2
-1	4
-3	0

all x-values are multiplied by -1

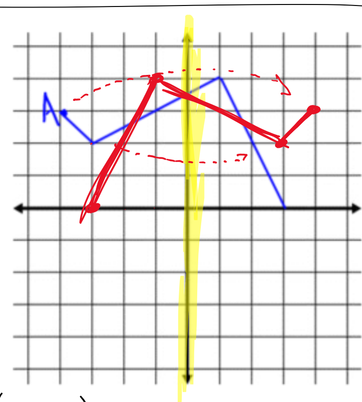
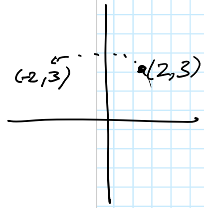


Image point for point A: $(4, 3)$
Original equation: $y = f(x)$
New equation: $y = f(-x)$
Mapping: $(x, y) \to (-x, y)$

Points that do not change under a given transformation are called ***invariant points***.
Which points are invariant in the reflections above?

They will be on the "mirror", the line we are reflecting across.



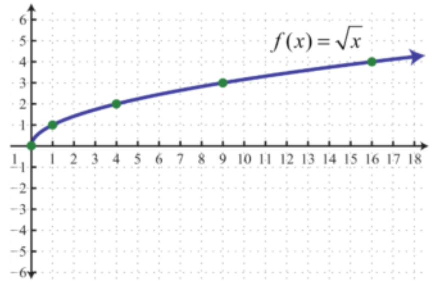
REFLECTIONS – reflecting graph across either *y*-axis or *x*-axis

Some specific examples:

- when x is replaced with $-x$, the graph will be reflected across the y-axis.
- when y is replaced with $-y$, the graph will be reflected across the x-axis.
- If instead of $y = f(x)$ we have $y = -f(x)$, the graph is reflected across X-axis

same as $\boxed{-y = f(x)}$

The graph of the base radical function is shown.



$\boxed{y = \sqrt{x}}$

x	y
0	0
1	1
4	2
9	3
16	4

For each transformed equation below

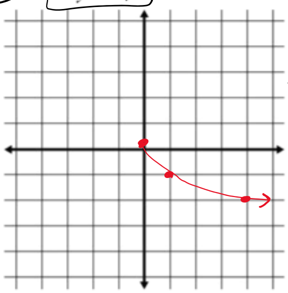
- Sketch its graph on the grid.
- Give its domain and range, using set notation.
- Describe, in words, what change occurred.
- Describe the transformation by giving its ***mapping***.

$y = \sqrt{x}$

x	y
0	0
1	1
4	2

$\boxed{y = -\sqrt{x}}$ this is the same thing as $-y = \sqrt{x}$

x	-y
0	0
1	-1
4	-2



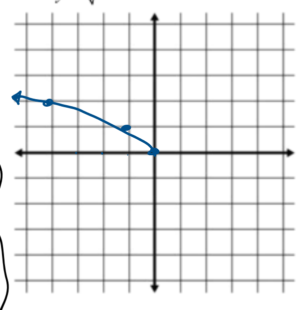
- reflect across x-axis

$(x, y) \rightarrow (x, -y)$

- domain: $\{x \mid x \geq 0, x \in \mathbb{R}\}$

range: $\{y \mid y \leq 0, y \in \mathbb{R}\}$

$y = \sqrt{-x}$



x got changed to $-x$, so:
- reflect across y-axis

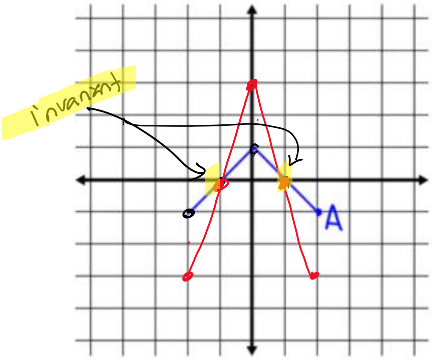
$(x, y) \rightarrow (-x, y)$

$-x$	y
0	0
-1	1
-4	2

$\{x \mid x \leq 0, x \in \mathbb{R}\}$

$\{y \mid y \geq 0, y \in \mathbb{R}\}$

Stretches [expansions / compressions]



Vertical – all y-values are multiplied by a number, the stretch factor

Let's do Vertical expansion by a factor of 3

VE, 3

- multiply every y-value by 3

Key points x	y
-2	-1
-1	0
0	1
1	0
2	-1

Image points x	3y
-2	-3
-1	0
0	3
1	0
2	-3

Mapping: $(x, y) \rightarrow (x, 3y)$

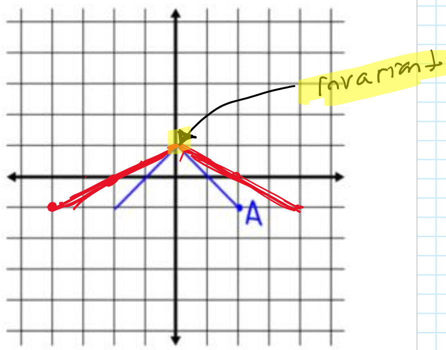
Horizontal – all x-values are multiplied by a number, the stretch factor

Let's do horizontal expansion, factor 2

HE, 2

Key points x	y
-2	-1
-1	0
0	1
1	0
2	-1

Image points 2x	y
-4	-1
-2	0
0	1
2	0
4	-1



Mapping: $(x, y) \rightarrow (2x, y)$

Which points are invariant in the stretches above ?

vertical stretch – any point on the x-axis
horizontal stretch – any point on the y-axis

STRETCHES – horizontal and vertical stretches

When $y = f(x)$ is changed to $y = a\,f(x)$, each point on the original graph has its y-value multiplied by "a."
This is a ***vertical stretch***, by a factor of a.

When $y = f(x)$ is changed to $y = f(bx)$, each point on the original graph has its x-value multiplied by the *reciprocal* of b. This is a ***horizontal stretch*** by a factor of $\frac{1}{b}$.

When the stretch factor is a number between -1 and 1, we call it a ***compression***. Otherwise, we call it an ***expansion.***

Examples

a) Identify each change, when $y = f(x)$ is changed to:

$y = 8f(x)$ VE, 8 $\quad$ $y = f(2x)$ HC, $\frac{1}{2}$ $\quad$ $y = \frac{1}{2}f(x)$ VC, $\frac{1}{2}$

$y = f\left(\frac{1}{4}x\right)$ HE, 4 $\quad$ $4y = f(x)$ VC, $\frac{1}{4}$ $\quad$ $\frac{1}{2}y = f(x)$ VE, 2

b) Write the new equation that causes $\boxed{y = f(x)}$ to be stretched as follows:

Vertical stretch, by $\frac{2}{3}$ $\quad$ Horizontal stretch, by $\frac{5}{2}$ $\quad$ $y = f\left(\frac{2}{5}x\right)$

$y = \frac{2}{3}f(x)$ OR $\frac{3}{2}y = f(x)$

To Try

The graph of $y = f(x)$ is shown at right. When changed to $y = 3f(x)$, VE, 3

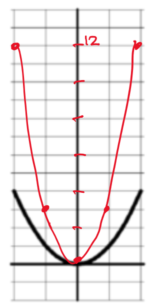
- identify the transformation
- complete the table and mapping
- sketch the graph of $y = 3f(x)$

$(x, y) \rightarrow (x, 3y)$

x	**y**
-2	4
-1	1
0	0
1	1
2	4

Image points

x	3y
-2	12
-1	3
0	0
1	3
2	12



$(x, y) \rightarrow$

Remember, in translations, if the change is IMMEDIATELY next to the variable, we have to "reverse" what it says:

Translation

$y = f(x+5)$
5 left

$y - 2 = f(x)$
2 up

A similar thing happens with expansions/compressions:

$2y = f(x)$
VC by $\frac{1}{2}$

$y = f(3x)$
$\frac{1}{3}$ HC by $\frac{1}{3}$ horizontal compression

$y = f\left(\frac{2}{5}x\right)$
HE $\frac{5}{2}$

Here, we do NOT need to use the reciprocal:

$y = 5f(x)$ VE by 5

To Try

The graph of $y = f(x)$ is shown at right. When changed to $y = f\left(\frac{1}{2}x\right)$,

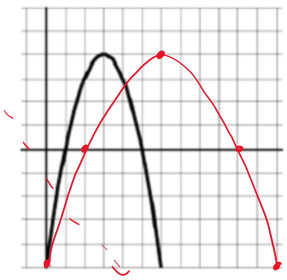
HE, 2

- identify the transformation
- complete the table and mapping
- sketch the graph of $y = f\left(\frac{1}{2}x\right)$

x	y
0	-5
1	0
3	4
5	0
6	-5

Image points

2x	y
0	-5
2	0
6	4
10	0
12	-5



$(x, y) \rightarrow (2x, y)$

To Try

The graph of $y = f(x)$ is shown at right. When changed to $y = -\frac{1}{2}f(x)$,

VC $\frac{1}{2}$, and reflect across x-axis

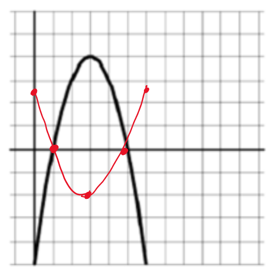
~~OR~~, we could say VC $-\frac{1}{2}$

- identify the transformation
- complete the table and mapping
- sketch the graph of $y = -\frac{1}{2}f(x)$

x	y
0	-5
1	0
3	4
5	0
6	-5

Image points

x	$-\frac{1}{2}y$
0	$5/2 = 2\frac{1}{2}$
1	0
3	-2
5	0
6	5/2



$(x, y) \rightarrow (x, -\frac{1}{2}y)$

Check-in

-fill in the table in your notes, page 12

-compare your answers with someone else

1.3 Combining Transformations

Summary of Transformations. Original Equation. $\boxed{y = f(x)}$

Translations

	Graph moves…	Mapping
$y+4=f(x)$	4 down	$(x, y) \rightarrow (x, y-4)$
$y-5=f(x)$	5 up	$(x,y) \rightarrow (x, y+5)$
$y=f(x+2)$	2 left	$(x,y) \rightarrow (x-2, y)$
$y=f(x-6)$	6 right	$(x,y) \rightarrow (x+6, y)$

Stretches

	Graph is stretched…	Mapping
$y=5f(x)$	VE, 5	$(x,y) \rightarrow (x, 5y)$
$\frac{3}{2}y=f(x)$	VC, $\frac{2}{3}$	$(x,y) \rightarrow (x, \frac{2}{3}y)$
$y=f(4x)$	HC, $\frac{1}{4}$	$(x,y) \rightarrow (\frac{1}{4}x, y)$
$y=f\left(\frac{1}{3}x\right)$	HE 3	$(x,y) \rightarrow (3x, y)$

Reflections

	Reflects across…	Mapping
$y=-f(x)$	x-axis	$(x,y) \rightarrow (x, -y)$
$y=f(-x)$	y-axis	$(x,y) \rightarrow (-x, y)$

$$y = \overbrace{a}^{\text{Vertical stretch, factor } a} f(\underbrace{b}_{\text{Horizontal stretch, factor } 1/b}(x - \overbrace{h}^{\text{Horizontal translation}})) + \underbrace{k}_{\text{Vertical translation}}$$

Question…….

If more than one transformation is applied to a graph, does the ***order*** in which the transformations are done change the final graph?

***

WB - explore this

$y = f(x)$ is shown on the grid

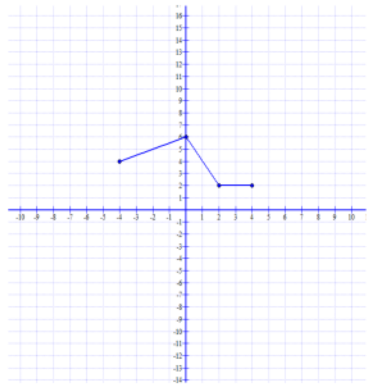
- Reflect across the x-axis and sketch the result.
- Take that graph and translate it 4 units up to get your FINAL graph

Original

x	y
–4	4
0	6
2	2
4	2

Reflected

FINAL



$y = f(x)$ is shown on the grid

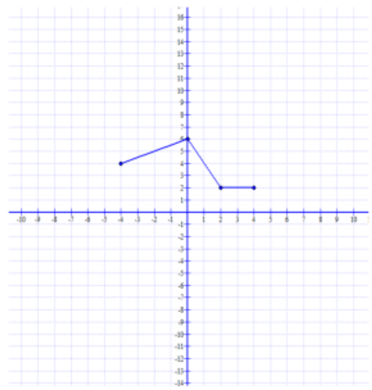
- Translate 4 units up and sketch the result
- Take that graph and reflect it across the x-axis to get your FINAL graph

Original

x	y
–4	4
0	6
2	2
4	2

Translated

FINAL



Conclusions:
Yes, it makes a difference. The order in which we do a reflection and a translation changes the final result.

$y = f(x)$ is shown on the grid

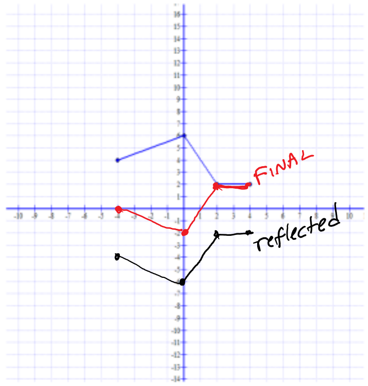
- Reflect across the x-axis and sketch the result.
- Take that graph and translate it 4 units up to get your FINAL graph

Original

x	y
–4	4
0	6
2	2
4	2

Reflected

x	-y
-4	-4
0	-6
2	-2
4	-2



FINAL

x	-y+4
-4	0
0	-2
2	2
4	2

$(x, y) \rightarrow (x, -y+4)$

$y = f(x)$ is shown on the grid

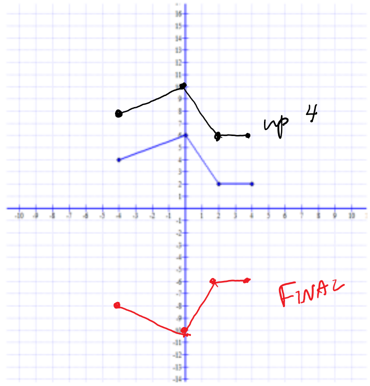
- Translate 4 units up and sketch the result
- Take that graph and reflect it across the x-axis to get your FINAL graph

Original

x	y
–4	4
0	6
2	2
4	2

Translated

x	y+4
-4	8
0	10
2	6
4	6



FINAL

x	-(y+4)
-4	-8
0	-10
2	-6
4	-6

$(x, y) \rightarrow (x, -(y+4))$

OR, simplifying:

$(x, y) \rightarrow (x, -y-4)$

Question…….

If more than one transformation is applied to a graph, does the ***order*** in which the transformations are done change the final graph?

$

YES!

Apply transformations in this order, to get the final graph:

1) reflections & expansions / compressions
2) translations

Example List all the transformations, then give the mapping.

a) $y = -4f\left(\frac{1}{2}(x-3)\right)+6$

HE 2

3 right

6 up

reflect across x-axis, VE 4

$(x, y) \rightarrow (2x+3, -4y+6)$

b) $y = 2f(3x-6)+5$

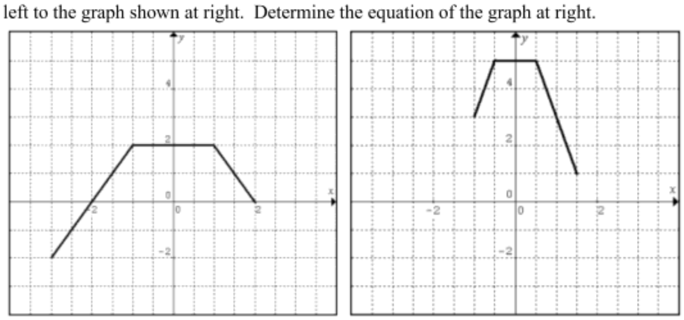
MUST factor first!!

$y = 2f(3(x-2)) + 5$

didn't finish this yet!

Example

Identify the transformations that need to happen, to change the graph of $y = f(x)$ on the left to the graph shown at right. Determine the equation of the graph at right.



For next class

Complete:

- **First Night Review questions**
- **Chapter 1 HW, #1-3, 6-7**

More practice available in textbook

- **Also, you can look at these sites**

  https://www.mathsisfun.com/sets/function-transformations.html

  https://www.khanacademy.org/math/algebra2/manipulating-functions/stretching-functions/e/shifting_and_reflecting_functions https://www.purplemath.com/modules/fcntranq.htm

Please erase your whiteboard area, and return the whiteboards, erasers, pens and calculators. Thanks!!