Class_02 Sep 13 - Transformations

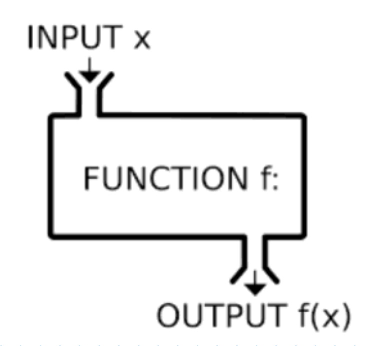
Sunday, September 11, 2022 3:48 PM

Tonight's Class:

- **Check-in and worksheets**
  - **Check-in based on email questions**
  - **First night Review Worksheet**
  - **Common Graphs Worksheet**
- **Translations**
- **Reflections & Stretches**
- **Combining transformations**

First Night Review Worksheet - #3, 6, 7, 8
Common Graphs Worksheet - base graphs

Last class....



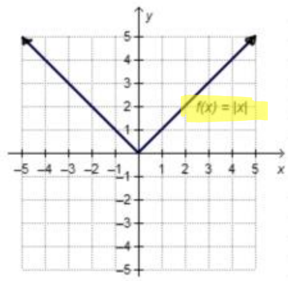
Domain
all allowable x-values. Can't use x-values that "BREAK" the function machine

Range
All y-values

Transformations

1.1 Horizontal and Vertical Translations

The graph of the base absolute value function is shown at right, and below are three transformed equations.



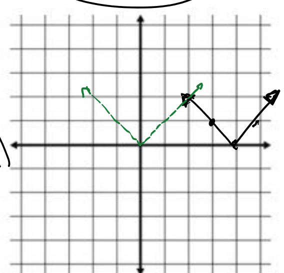
For each one:

- Sketch its graph on the grid.
- Describe, in words, the transformation that happened.
- Describe the transformation by giving its ***mapping***.
- State the domain and range.

x	y
3	$\|3-4\| = \|-1\| = 1$
4	$\|4-4\| = \|0\| = 0$
5	$\|5-4\| = \|1\| = 1$

$f(x) = |x|$ became $f(x) = |x-4|$

$f(x) = |x-4|$



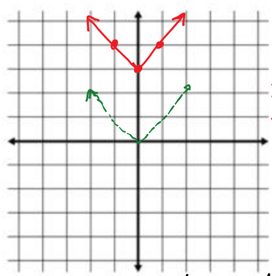
- moved 4 right

$(x, y) \rightarrow (x+4, y)$

$\{x \mid x \in \mathbb{R}\}$

$\{y \mid y \geq 0, y \in \mathbb{R}\}$

$f(x) = |x| + 3$



x	y
-1	4
0	3
1	4

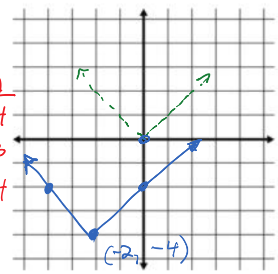
-moved original graph 3 up

$(x, y) \rightarrow (x, y+3)$

$\{x \mid x \in \mathbb{R}\}$

$\{y \mid y \geq 3, y \in \mathbb{R}\}$

$f(x) = |x+2| - 4$



x	y
-4	$\|-4+2\| - 4 = \|-2\| - 4 = 2 - 4 = -2$
-2	$\|-2+2\| - 4 = \|0\| - 4 = -4$
0	$\|0+2\| - 4 = \|2\| - 4 = -2$

- moved original graph 2 left, and 4 down

$(x, y) \rightarrow (x-2, y-4)$

$\{x \mid x \in \mathbb{R}\}$ $\{y \mid y \geq -4, y \in \mathbb{R}\}$

Points on an original graph correspond with points on a transformed graph, often called the ***image graph***. We say that each original point is ***mapped*** to an ***image point***.

$$y - \underbrace{k}_{\text{Vertical translation}} = f(x - \overbrace{h}^{\text{Horizontal translation}})$$

Often equations are arranged with the "y" term isolated:

$$y = f(x - \overbrace{h}^{\text{Horizontal translation}}) + \underbrace{k}_{\text{Vertical translation}}$$

Simplest equation: $y = f(x)$

$y = f(x-h)$ horizontal translation. Moves "h" units

$y = f(x+2)$ 2 left
$y = f(x-5)$ 5 right
$y = f(x+4)$ 4 left

$\left.\begin{array}{l} y-3 = f(x) \\ y = f(x)+3 \end{array}\right\}$ 3 up

On page 4, the original function can be written either $f(x) = | x |$ or $y = | x |$

When we change the equation, we get different graphs

- $y = | x - 4|$ moves the graph 4 RIGHT
  When x is replaced with x - 4, the graph moves 4 right
- $y = | x | + 3$ can also be written as $y - 3 = | x|$
  The nice thing about writing it the second way is it's easier to see that
  when y is replaced with y - 3, the graph moves 3 UP
- $y = |x + 2| - 4$ causes the graph to move 2 LEFT and also 4 DOWN
  If we want, we can re-write this as $y + 4 = |x + 2|$

$y = f(x - h)$ results in a horizontal translation

$y - h = f(x)$ OR $y = f(x) + h$ result in a vertical translation

TRANSLATIONS – sliding graphs left/right/up/down

Some specific examples:

- when x is replaced with $x-8$, the graph will move 8 right.
- when x is replaced with $x+6$, the graph will move 6 left.
- when y is replaced with $y-4$, the graph will move 4 up.
- when y is replaced with $y+7$, the graph will move 7 down.

$y=f(x)$ becomes $y=f(x-8)$

Base Function Equation	Transformed Equation	Mapping	Point on original graph	Its image point
$y=x^2$	$y-4=x^2$ up 4	$(x,y)\to(x, y+4)$	$(-3, 9)$ x y	$(-3, 13)$
$y=x+5$	$y=(x-3)+5$ right 3	$(x,y)\to(x+3, y)$	$(2,7)$	$(5,7)$
$y=\log_5 x$	$y=\log_5(x-2)+3$ 2 right, 3 up	$(x,y)\to(x+2, y+3)$	$(25, 2)$	$(27, 5)$
$y=2^x$	$y=2^{x-3}+8$ right 3, up 8	$(x,y)\to(x+3, y+8)$	$\left(-1, \frac{1}{2}\right)$	$(2, 8\frac{1}{2})$ or $(2, 8.5)$
$y=\frac{2}{x-4}$	$y=\frac{2}{(x+3)-4}+6$ left 3, up 6	$(x,y)\to(x-3, y+6)$	$\left(8, \frac{1}{2}\right)$	$(5, 6\frac{1}{2})$
$x^2+y^2=16$	$(x-5)^2+(y+3)^2=16$ 5 right, 3 down	$(x,y)\to(x+5, y-3)$	$(-4, 0)$	$(1, -3)$

Notice for this one, the "5" was already there in the original equation. The ONLY change, is the graph is shifted right 3.

moved $\frac{2}{3}$ up

$(-1, \frac{1}{2}) \to (-1, \frac{3}{3}\cdot\frac{1}{2} + \frac{2}{3}\cdot\frac{2}{2})$

$= (-1, \frac{3}{6} + \frac{4}{6})$

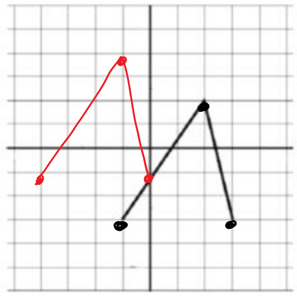
$= (-1, \frac{7}{6})$

To Try
Shown is the graph of $y = f(x)$.

a) Identify the transformations that result when the equation is changed to: $y - 2 = f(x+3)$

$y = f(x+3) + 2$ up 2 left 3

b) Make a table of key points on the original graph and the corresponding image points on the image graph.



Base

x	y
-1	-3
2	2
3	-3

Image

$x-3$	$y+2$
-4	-1
-1	4
0	-1

c) Sketch the image graph.

$(x, y) \rightarrow (x-3, y+2)$

d) State the domain and range of the image graph. (Assume that the line segments stop.)

$\{x \mid -4 \leq x \leq 0,\ x \in \mathbb{R}\}$
$\{y \mid -1 \leq y \leq 4,\ y \in \mathbb{R}\}$

Example
Given the mapping notation for a transformation, we can write the transformed equation.

a) Mapping notation $(x, y) \rightarrow (x-8, y+3)$ up 3 left 8
Original function $y = f(x)$
New function

$y = f(x+8) + 3$
or $y - 3 = f(x+8)$

* mappings give direct instructions

b) Mapping notation $(x, y) \rightarrow (x+4, y-9)$ 9 down 4 right
Original function $y = f(x)$
New function

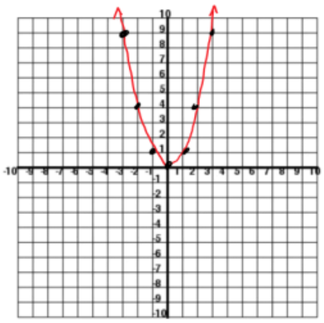
$y = f(x-4) - 9$
$y + 9 = f(x-4)$

[Common Graphs Worksheet](), translated graphs

Quadratic functions (BASE)

$y = x^2$

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

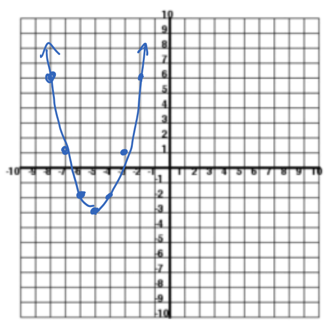


vertex: (0, 0)

Quadratic functions

$y = (x+5)^2 - 3$

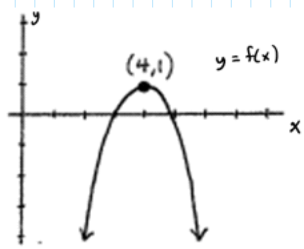
Moves 5 left and 3 down
(x, y) maps to (x-5, y-3)



x-5	y-3
-8	6
-7	1
-6	-2
-5	-3
-4	-2
-3	1
-2	6

vertex: (-5, -3)

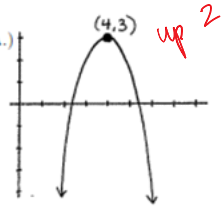
Translations Review - talk with your group, agree on answers.



Given the graph of $y = f(x)$ shown above, match the following four function equations with their graphs (A, B, C or D)

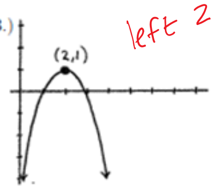
1. $y = f(x)+2$ graph: A
2. $y = f(x)-2$ graph: D
   down
3. $y = f(x+2)$ graph: B
   left
4. $y = f(x-2)$ graph: C

A.)



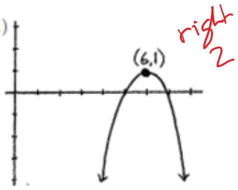
up 2

B.)



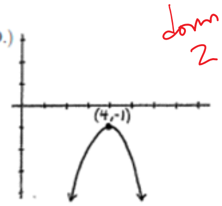
left 2

C.)

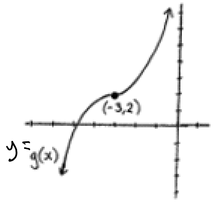


right 2

D.)



down 2

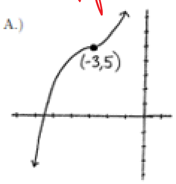


Given the graph of $y = g(x)$ shown above, match the following four function equations with their graphs (A, B, C, or D)

1. $y = g(x) + 3$ graph: A
   up 3
2. $y = g(x) - 3$ graph: C
   dwn
3. $y = g(x+3)$ graph: D
   left
4. $y = g(x-3)$ graph: B
   rgt

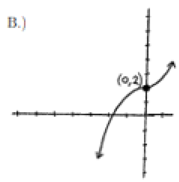
up 3

A.)

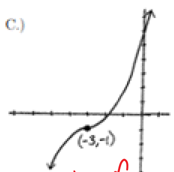


rts r 2

B.)

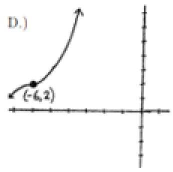


C.)



down 3

D.)



left 3

1.2 Reflections and Stretches

Reflections

Across the x-axis

Original key points

x	y
−4	3
−3	2
1	4
3	0

Reflected key points

x	$-y$
−4	−3
−3	−2
1	−4
3	0

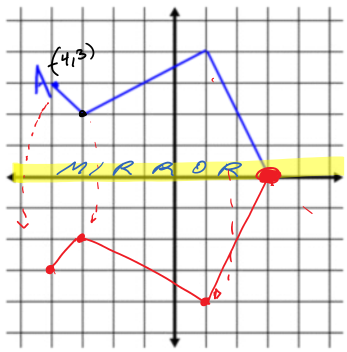


Image point for point A: $(-4,-3)$
Original equation: $y = f(x)$
New equation: $-y = f(x)$ or $y = -f(x)$
Mapping: $(x, y) \to (x, -y)$

Across the y-axis

Original key points

x	y
−4	3
−3	2
1	4
3	0

Reflected key points

x	y
4	3
3	2
−1	4
−3	0

all x-values changed sign

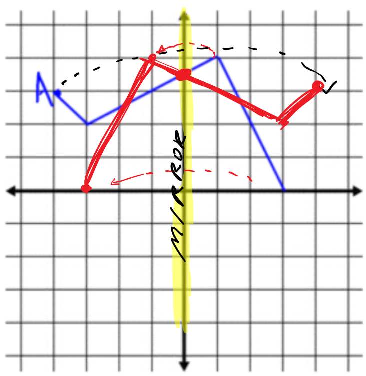


Image point for point A: $(4,3)$
Original equation: $y = f(x)$
New equation: $y = f(-x)$
Mapping: $(x, y) \to (-x, y)$

Points that do not change under a given transformation are called *invariant points*.
Which points are invariant in the reflections above?

vary or variable

Invariant means does NOT change!!

If reflecting across x-axis, points on it do not move. They are invariant.

If reflecting across y-axis, points on the y-axis are invariant.

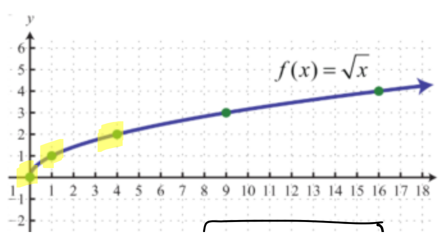
REFLECTIONS – reflecting graph across either y-axis or x-axis

Some specific examples:
- when x is replaced with $-x$, the graph will be reflected across the y-axis.
- when y is replaced with $-y$, the graph will be reflected across the x-axis.
- If instead of $y = f(x)$ we have $\underbrace{y = -f(x)}_{\text{means}}$, the graph is reflected across x-axis.

$-y = f(x)$

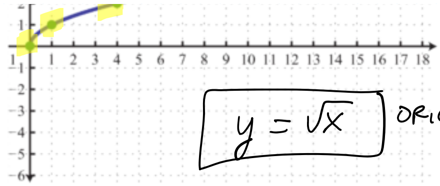
The graph of the base radical function is shown.

For each transformed equation below
- Sketch its graph on the grid.
- Give its domain and range, using set notation.
- Describe, in words, what change occurred.
- Describe the transformation by giving



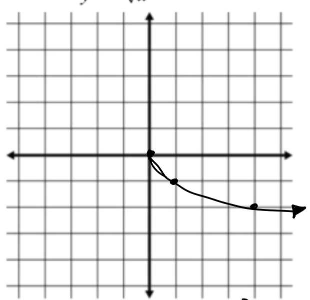
notation.

- Describe, in words, what change occurred.
- Describe the transformation by giving its ***mapping***.



$\boxed{y = \sqrt{x}}$ ORIGINAL

$y = -\sqrt{x}$ reflect across x-axis

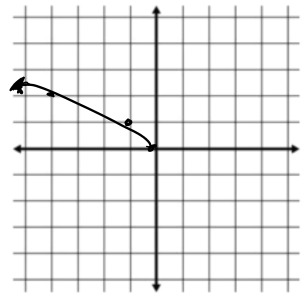


$\{x \mid x \geq 0, x \in \mathbb{R}\}$
$\{y \mid y \leq 0, y \in \mathbb{R}\}$

$(x, y) \rightarrow (x, -y)$

reflect across x-axis

$y = \sqrt{-x}$

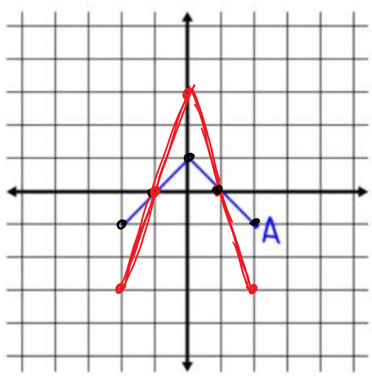


$(x, y) \rightarrow (-x, y)$

reflect across y-axis

$\{x \mid x \leq 0, x \in \mathbb{R}\}$
$\{y \mid y \geq 0, y \in \mathbb{R}\}$

Stretches



Vertical – all y-values are multiplied by a number, the stretch factor

let's multiply each y-value by 3

Vertical expansion

Key points

x	y
-2	-1
-1	0
0	1
1	0
2	-1

Image points

x	3y
-2	-3
-1	0
0	3
1	0
2	-3

Mapping: $(x, y) \rightarrow (x, 3y)$

Horizontal – all x-values are multiplied by a number, the stretch factor

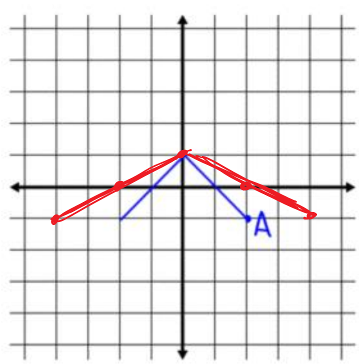
horizontal expansion by a factor of 2

Key points

x	y
-2	-1
-1	0
0	1
1	0
2	-1

Image points

2x	y
-4	-1
-2	0
0	1
2	0
4	-1



Mapping: $(x, y) \rightarrow (2x, y)$

2	-1

4	-1

Mapping: $(x, y) \rightarrow (2x, y)$

Which points are invariant in the stretches above

horizontal expansion/compression – invariant points are on y-axis
vertical expansion/compression – invariant points are on x-axis

STRETCHES – horizontal and vertical stretches

When $y = f(x)$ is changed to $y = a\,f(x)$, each point on the original graph has its y-value multiplied by "a."
This is a ***vertical stretch***, by a factor of a.

When $y = f(x)$ is changed to $y = f(bx)$, each point on the original graph has its x-value multiplied by the <u>reciprocal</u> of b. This is a ***horizontal stretch*** by a factor of $\frac{1}{b}$.

When the stretch factor is a number between -1 and 1, we call it a ***compression***. Otherwise, we call it an ***expansion.***

Examples

a) Identify each change, when $y = f(x)$ is changed to:

$y = 8f(x)$ VExp by 8

$y = f(2x)$ HComp. $\frac{1}{2}$ mult by $\frac{1}{2}$ HC, $\frac{1}{2}$

$y = \frac{1}{2}f(x)$ VC $\frac{1}{2}$

$y = f\left(\frac{1}{4}x\right)$ HExp by 4 HE, 4

$4y = f(x)$ mult by $\frac{1}{4}$ VC by $\frac{1}{4}$

$\frac{1}{2}y = f(x)$ VE by 2

could do this:

$y = \cancel{\tfrac{1}{2}}f(x)$

$\frac{y}{\frac{1}{2}} = f(x)$

$y \div \frac{1}{2} = f(x)$

$y \times \frac{2}{1} = f(x)$

$2y = f(x)$

b) Write the new equation that causes $y = f(x)$ to be stretched as follows:

Vertical stretch, by $\frac{2}{3}$

Horizontal stretch, by $\frac{5}{2}$

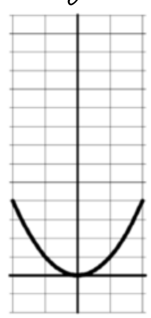
<u>To Try</u>

The graph of $y = f(x)$ is shown at right. When changed to $y = 3f(x)$,

- identify the transformation
- complete the table and mapping
- sketch the graph of $y = 3f(x)$

x	y
-2	4
-1	1
0	0
1	1
2	4

Image points



$(x, y) \rightarrow$

<u>For next class</u>

Complete: Common Graphs Worksheet

More practice available in textbook

- **p 12: 2, 3cd, 4ac, 5, 8, 11**
- **p 28: 3b, 4b, 5-7, 9, 12**
- **Also, you can look at these sites**
  https://www.mathsisfun.com/sets/function-transformations.html

  https://www.khanacademy.org/math/algebra2/manipulating-functions/stretching-functions/e/shifting_and_reflecting_functions

  https://www.purplemath.com/modules/fcntranq.htm

Please erase your whiteboard area, and return the whiteboards, erasers, pens and calculators. Thanks!!