Class_03 Jan 12 - Negative Rational Exponents

Wednesday, January 11, 2023 5:42 PM

Tonight's Class:

- **Recap #2**
- **Questions from 1.3?**
- **Working through sections 1.4-1.5**
  - **Powers with positive fractional exponents (continued)**
  - **Powers with negative fractional exponents**
- **Work on practice questions from worktext**

From section 1.3

To change a mixed radical to an entire radical
- Identify the index of the radicand, *n*
- Raise the coefficient of the radical to the *n*th power, put it inside the radical sign
- Multiply the numbers to create the new radicand
- If the coefficient is negative, be careful!
  - odd index, put the negative sign either inside OR outside the radical
  - even index, leave negative sign out in front of radical

p27, #8 a) $\sqrt{32} = \sqrt{4 \cdot 8}$

perfect squares method

perfect squares: 1, 4, 9, 16, 25, 36, 49

$= 2\sqrt{8}$

always aim for the smallest radicand

$= 2\sqrt{4 \cdot 2}$

$= 2 \cdot 2\sqrt{2}$

$= \boxed{4\sqrt{2}}$

$\sqrt{32} = \sqrt{16 \cdot 2}$

$= \boxed{4\sqrt{2}}$

c) $\sqrt{22}$ not possible to change to mixed form (because radicand doesn't have any perfect square factors)

22 → 2, 11

p29 15. $\sqrt{15} \cdot \sqrt[3]{35}$ No, as indexes are different.

16. $\sqrt{2} = 1.414213562 \ldots$

a) $\sqrt{200} = \sqrt{2 \cdot 100}$

$= \sqrt{2}\sqrt{100}$

$= (\sqrt{2})(10)$

$= 10\sqrt{2} \quad = 10(1.414213562\ldots)$

$= 14.14213562\ldots$

p27 7c) $\sqrt{96} = \sqrt{2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 3}$

$= 2\cdot 2\sqrt{2\cdot 3}$

$= 4\sqrt{6}$

prime factors

96 → 2, 48; 48 → 2, 24; 24 → 2, 12; 12 → 2, 6; 6 → 2, 3

Example 4 **Evaluating Powers with Rational Exponents II**

Write each power as a radical, then evaluate the radical.

a) $(-8)^{\frac{4}{3}}$ **b)** $\left(\frac{16}{54}\right)^{\frac{2}{3}}$ **c)** $-32^{1.2}$

a)

$\sqrt[3]{(-8)}^{\,4}$

$= (-2)^4$

$= 16$

b)

$\sqrt[3]{\frac{16^{\div 2}}{54^{\div 2}}}^{\,2}$ can I reduce?

$= \left(\frac{\sqrt[3]{8}}{\sqrt[3]{27}}\right)^2$

$= \left(\frac{2}{3}\right)^2$

$= \frac{4}{9}$

c)

$-32^{1.2}$ change the exponent into fraction form

$1.2 = 1\frac{2}{10}$

$= \frac{12}{10}$

reduce it, if possible

$= \frac{6}{5}$

$= -32^{6/5}$

$= -\sqrt[5]{(32)}^{\,6}$

$= -(2)^6$

$= -64$

Try

- **1.4 questions, WT P 41-45**

1.4 Powers with Positive Rational Exponents	Page 41-47, #3,4,5,6aceg,7aceg,8,9,10aceg,11acf,12aceh,14,16,19,20

- **Checkpoint, WT p 31-34** $\underbrace{}_{1.1-1.3}$

Preview 3

1.5 Powers with Negative Rational Exponents

Focus: develop understanding of powers with negative rational exponents

What do we mean by a negative exponent?

$(\text{any number}_{\neq 0})^0 = 1$

$0^0 =$ "indeterminate"

$\frac{1}{4}$

$2^4 = 2\cdot2\cdot2\cdot2 = 16$ $\div 2$

$2^3 = 2\cdot2\cdot2 = 8$ $\div 2$

$2^2 = 2\cdot2 = 4$ $\div 2$

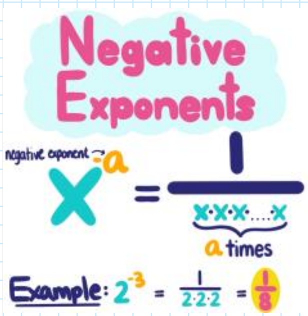
$2^1 = \quad = 2$ $\div 2$

$2^0 = \quad = 1$

$2^{-1} = \quad = 0.5 = \frac{1}{2}$

$2^{-2} = \quad = 0.25 = \frac{1}{4} = \frac{1}{2\cdot2} = \frac{1}{2^2}$

$2^{-3} = \quad = \frac{1}{2\cdot2\cdot2} = \frac{1}{2^3}$



$5^{-4} = \frac{1}{5\cdot5\cdot5\cdot5} = \frac{1}{5^4}$

Negative Exponents

$$a^{-n} = \frac{1}{a^n}$$

For $a \neq 0$

a^{-n} is a reciprocal of a^n

Example:

$3^{-2} = \frac{1}{3^2}$ $\left(\frac{2}{5}\right)^{-6} = \left(\frac{5}{2}\right)^{6}$

$$\left(\frac{2}{5}\right)^{-6} = \frac{1}{\left(\frac{2}{5}\right)^6}$$

$$= \frac{1}{\left(\frac{2^6}{5^6}\right)}$$

$$= 1 \div \frac{2^6}{5^6}$$

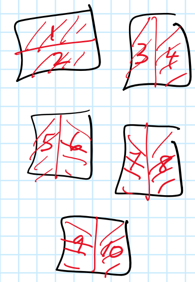
$$= 1 \times \frac{5^6}{2^6}$$

$$= \frac{5^6}{2^6} = \left(\frac{5}{2}\right)^6$$

$$5 \div \frac{1}{2}$$

$$= 5 \times \frac{2}{1}$$

$$= 10$$



Powers with a Negative Integer Exponent and a Rational Base

$\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^{m}$, where a and b are integers; $a, \neq 0$, $b \neq 0$

p50

Example 1 Evaluating Powers with Negative Integer Exponents

Evaluate each power.

a) 6^{-2}

$= \frac{1}{6^2}$

$= \frac{1}{36}$

b) $(-5)^{-2}$

$= \frac{1}{(-5)^2}$

$= \frac{1}{25}$

c) $(1.5)^{-4}$

if the base is a decimal, we change it to a fraction

$= \left(\frac{15}{10}\right)^{-4}$

reduce the base, if possible

$= \left(\frac{3}{2}\right)^{-4}$

$= \left(\frac{2}{3}\right)^{+4}$

$= \frac{2^4}{3^4}$

$= \frac{16}{81}$

d) $\left(\frac{8}{10}\right)^{-2}$

$= \left(\frac{10}{8}\right)^{+2}$

$= \left(\frac{5}{4}\right)^{2}$

$= \frac{25}{16}$

1) Do reciprocal of the base, & make the exponent positive.

2) evaluate & simplify

Try "check for understanding", p 50

Negative Rational Exponents

Definition of $a^{-\frac{m}{n}}$: $\dfrac{1}{a^{m/n}}$ or $\dfrac{1}{\sqrt[n]{a^m}}$ or $\dfrac{1}{(\sqrt[n]{a})^m}$

Example 2 **Evaluating Powers with Negative Rational Exponents and Integer Bases**

a) Evaluate each power after writing it in the form $\dfrac{1}{\sqrt[n]{a^m}}$.

i) $4^{-\frac{3}{2}}$ **ii)** $(-1000)^{-\frac{2}{3}}$

b) Evaluate each power after writing it in the form $\dfrac{1}{(\sqrt[n]{a})^m}$.

i) $9^{-1.5}$ **ii)** $-32^{-\frac{4}{5}}$

a) i) $4^{-3/2} = \dfrac{1}{4^{3/2}} = \dfrac{1}{\sqrt{4}^{\,3}} = \dfrac{1}{(2)^3} = \dfrac{1}{8}$

ii) $(-1000)^{-2/3} = \dfrac{1}{(-1000)^{2/3}} = \dfrac{1}{\sqrt[3]{-1000}^{\,2}} = \dfrac{1}{(-10)^2} = \dfrac{1}{100}$

b) i) $9^{-1.5} = 9^{-3/2} = \dfrac{1}{9^{3/2}} = \dfrac{1}{\sqrt{9}^{\,3}} = \dfrac{1}{3^3} = \dfrac{1}{27}$

$(-1.5)\times\frac{10}{10}$

$= \frac{-15}{10}$

$= -3/2$

Try - page 51: 2ab, page 56: #6-10

(check your understanding)

PS3

Example 3 **Evaluating Powers with Negative Rational Exponents and Rational Bases**

a) Evaluate each power after writing it in the form $\sqrt[n]{\left(\frac{b}{a}\right)^m}$.

i) $\left(\frac{1}{4}\right)^{-\frac{3}{2}}$ ii) $\left(-\frac{27}{8}\right)^{-\frac{2}{3}}$

b) Evaluate each power after writing it in the form $\left(\sqrt[n]{\left(\frac{b}{a}\right)}\right)^m$.

i) $\left(\frac{48}{243}\right)^{-\frac{3}{4}}$ ii) $\left(-\frac{27}{125}\right)^{-\frac{2}{3}}$

a) i) $\left(\frac{1}{4}\right)^{-3/2} = \left(\frac{4}{1}\right)^{+3/2}$

$= \sqrt[2]{4}^{\,3}$

$= (2)^3$

$= 8$

1) reciprocal of the base and make the exponent POSITIVE
2) evaluate, using radicals

ii) $\left(\frac{-27}{8}\right)^{-2/3} = \left(\frac{8}{-27}\right)^{+2/3}$

$= \sqrt[3]{\frac{-8}{27}}^{\,2}$

$= \left(\frac{-2}{3}\right)^2$

$= \frac{4}{9}$

b) i) $\left(\frac{48}{243}\right)^{-3/4} = \left(\frac{243}{48}\right)^{+3/4}$

$= \left(\sqrt[4]{\frac{243 \div 3}{48 \div 3}}\right)^3$

reduce!

$= \left(\sqrt[4]{\frac{81}{16}}\right)^3$

$= \left(\frac{3}{2}\right)^3$

$= \frac{27}{8}$

Remember, if digits add up to a number that's divisible by 3, the original number will also be divisible by 3:
2+4+3 = 9, which is divisible by 3
=> 243 is, too

ii) $\left(\ldots\right)^{-2/3} \ldots \left(\ldots\right)^{+2/3}$

$= \frac{27}{8}$

ii) $\left(\frac{-27}{125}\right)^{-2/3} = \left(-\frac{125}{27}\right)^{+2/3}$

$= \sqrt[3]{\frac{-125}{27}}^{\,2}$

$= \left(\frac{\sqrt[3]{-125}}{\sqrt[3]{27}}\right)^2$

$= \left(-\frac{5}{3}\right)^2$

$= \frac{25}{9}$

For next class

- **Complete the "Recap" from tonight!**
  - **Do all you can without looking at examples/worktext. Then, switch to a different color of pen/pencil and complete the rest of it. This way you can see what you might need to spend more time on.**
- **Finish worktext questions 1.4 and 1.5. Doing them helps you learn/practice the concepts.**
- **Start on the Chapter 1 Hand-in, which is due Thursday, Jan 19**