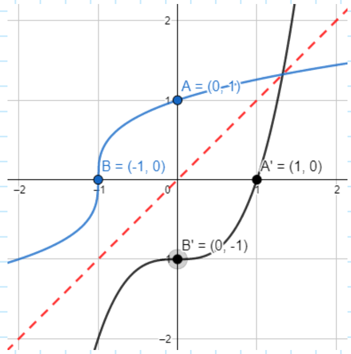
Plan For Today:

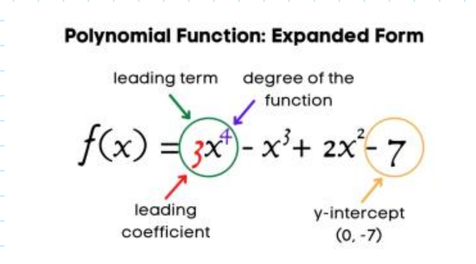
1. Question about anything?
2. Finish working on Chapter 1
    - 1.3 Combining Transformations Review
    - 1.4 Inverse of a Relation
3. Work on practice questions from Textbook

Page 38: #4, 5a, 6, 7abcd, 8, 9ce, 10ab
Page 51: #1b, 2a, 3ac, 5ae, 3b, 12a

4. ~~Start Chapter 3: Polynomial Functions~~
    - ~~3.1: Characteristics of Polynomial Functions~~
5. ~~Work on practice questions from Textbook~~

Page 114: #1-3, 4ace, 6, 7, 9





Plan Going Forward:

1. Finish going through practice question from 1.4 and 3.1 in textbook. Focus on the chapter 1 assignment and 3.1 section of the chapter 3 assignment to help you prepare for the first test.

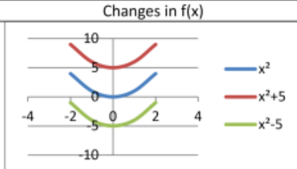
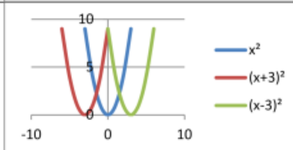
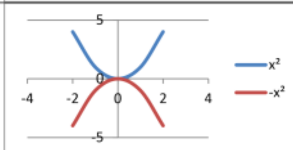
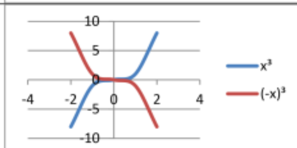
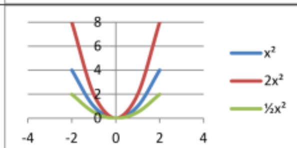
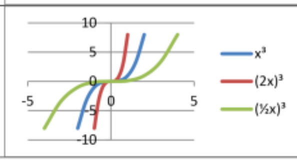
- **TEST 1 ON 1.1-1.4 TOMORROW, TUESDAY, MAY 9TH**
- **CHAPTER 1 ASSIGNMENT DUE TOMORROW, TUESDAY, MAY 9TH**

2. You will go through 3.1-3.2 on Tuesday after the test. Have a look through these sections to prepare for tomorrow.

Please let us know if you have any questions or concerns about your progress in this course.
The notes from today will be posted at egolfmath.weebly.com after class.
Anurita Dhiman = adhiman@sd35.bc.ca
Susana Egolf = segolf@sd35.bc.ca

1.3 Review

Summary of Transformations

Graph	Draw the graph of f(x) and:	Changes in f(x)
Vertical shift $y = f(x) + c$ $y = f(x) - c$	Raise the graph of f(x) by c units -add c to y coordinate Lower the graph of f(x) by c units -subtract c from y coordinate	
Horizontal shift $y = f(x + c)$ $y = f(x - c)$	Shift the graph f(x) to the left c units -subtract c from x coordinate Shift the graph f(x) to the right c units -add c to x coordinate	
Reflection about the x-axis $y = -f(x)$	Reflect the graph of f(x) about the x-axis -multiply each y coordinate by -1	
Reflection about the y-axis $y = f(-x)$	Reflect the graph of f(x) about the y-axis -multiply each x coordinate by -1	
Vertical stretching and compression $y = cf(x), c > 1$ $y = cf(x), 0 < c < 1$	Vertically stretching the graph of f(x) $(c > 1)$ Vertically compressing the graph of f(x) $(0 < c < 1)$ -multiply each y coordinate by c	
Horizontal stretching and compression $y = f(cx), c > 1$ $y = f(cx), 0 < c < 1$	Horizontally compressing the graph of f(x) $(c > 1)$ Horizontally stretching the graph of f(x) $(0 < c < 1)$ -divide each x coordinate by c	
$y = \frac{1}{f(x)}$	Take the reciprocal of each y coordinate of f(x)	

Order of operations for transformations: 1) horizontal shifts 2) stretches/compressions 3) reflections 4) vertical shifts

March 2017 MVCC Learning Commons Math Lab

Perform the transformations in this order

1. Vertical scale by a

$$(x, y) \rightarrow (x, ay)$$

If a is negative, reflects across x-axis

2. Vertical shift

+k: shift up k

-k : shift down k

$$a \cdot f\big(b(x-h)\big)+k$$

1. Horizontal scale by $1/b$

$$(x, y) \rightarrow \left(\frac{1}{b}x, y\right)$$

If b is negative, reflects across y-axis

2. Horizontal shift

-h : shift to right

+h : shift to left

Vertical stretch by a factor of 3

Horizontal translation 4 units left

$$f(x) = -3\sqrt{2(x+4)} - 1$$

Vertical translation 1 unit down

Reflection in the x-axis

Horizontal compression by a factor of $\frac{1}{2}$

Question…….

If more than one transformation is applied to a graph, does the ***order*** in which the transformations are done change the final graph?

$

YES!

Apply transformations in this order, to get the final graph:
1) reflections & expansions /compressions
2) translations

Example List all the transformations, then give the mapping.

a) $y=-4f\left(\frac{1}{2}(x-3)\right)+6$ HE 2, 3 right, reflect across x-axis, VE 4, 6 up

$(x,y)\rightarrow(2x+3,-4y+6)$

May 8/23

b) $y=2f(3x-6)+5$

MUST factor first!!

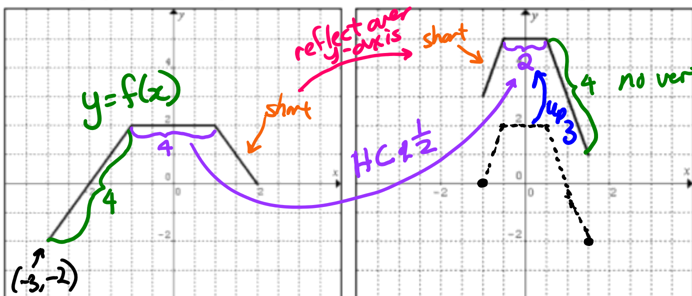
$y=2f(3(x-2))+5$

$(x,y)\xrightarrow{\text{stretches + reflections}}\left(\frac{1}{3}x,2y\right)\xrightarrow{\text{translations}}\left(\frac{1}{3}x+2,2y+5\right)$

1) vert. exp. of 2 (VE 2)
horiz. comp. of $\frac{1}{3}$ (HC $\frac{1}{3}$)
2) 2 right, 5 up.

Example

Identify the transformations that need to happen, to change the graph of $y=f(x)$ on the left to the graph shown at right. Determine the equation of the graph at right.

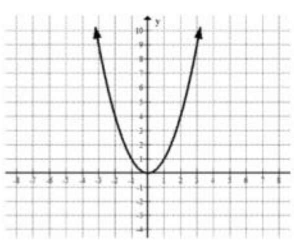


refl over y-axis
$-b$
HC of $\frac{1}{2}\Rightarrow b=2$
up 3

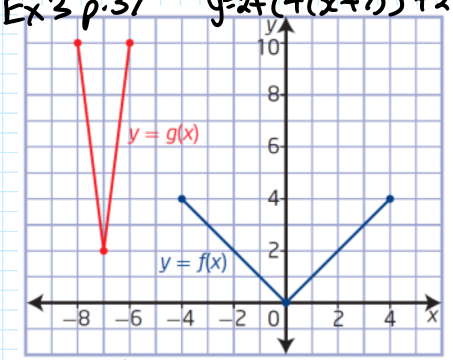
$y=af(b(x-h))+k.$

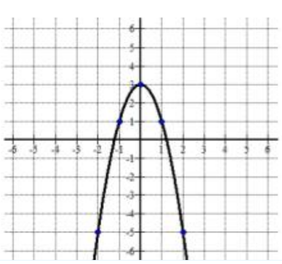
$\boxed{y=f(-2x)+3}$

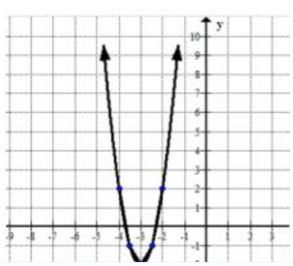
Given the following graph of the function $y=x^2$, determine the equation of the transformed functions below.



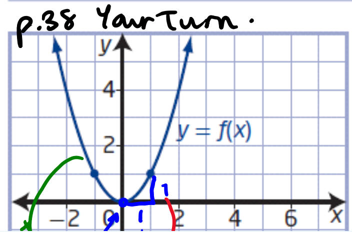
Ex 3 p.37 $y=2f(4(x+7))+2$



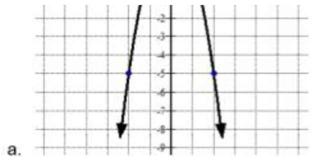




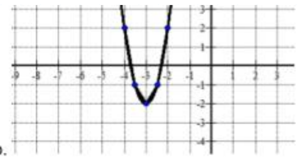
p.38 Your Turn.



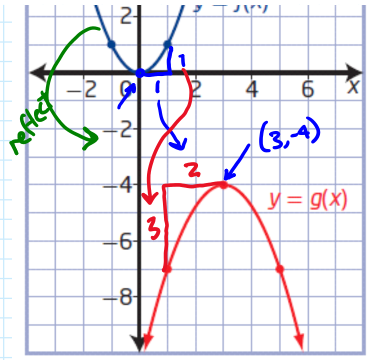
HE of 2



a.



b.



HE of 2
VE of 3
$-a$ = ref. over x-axis
3 right, 4 down.

$\boxed{y = -3g\left(\tfrac{1}{2}(x-3)\right) - 4}$

Combining Transformations – Radicals

The changes we have discussed work with any function. Here are some questions to try, relating to the base radical function $y = \sqrt{x}$.

1) List the transformations that occur when the base radical function is changed to: $y + 9 = \frac{1}{2}\sqrt{-3x - 12}$. Give the mapping.

transformation form $y = \frac{1}{2}\sqrt{-3(x+4)} - 9$

① VC of $\frac{1}{2}$, HC of $\frac{1}{3}$ ref. over y-axis

② 4 left, 9 down.

Mapping $(x, y) \rightarrow (-\frac{1}{3}x - 4, \frac{1}{2}y - 9)$

2) Given the mapping, which acts on the base radical function, write the new, transformed equation.

$(x, y) \rightarrow \left(5x + 2, \frac{1}{2}y - 4\right)$

① HE of 5, VC of $\frac{1}{2}$

② 2 right, 4 down

$y = \sqrt{x} \rightarrow \boxed{y = \frac{1}{2}\sqrt{\frac{1}{5}(x-2)} - 4}$

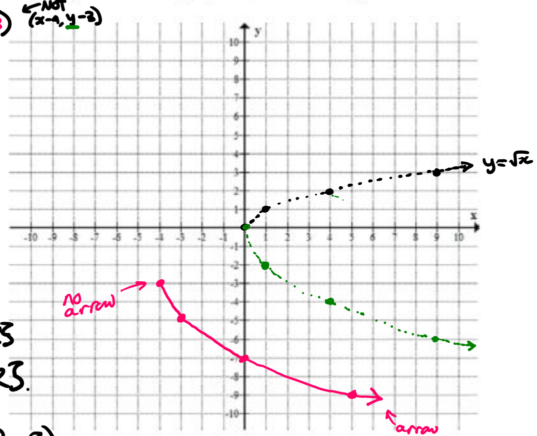
3) Complete the table of values for the base function, $y = \sqrt{x}$, and for the transformed equation, $y = -2\sqrt{x+4} - 3$. Give the mapping, and the final graph's domain and range.

① -2 VE of 2 + ref. in x-axis

② 4 left 3 down.

$(x, y) \rightarrow (x, -2y) \rightarrow (x-4, -2y-3)$ ← NOT $(x-4, y-3)$

	x	y					
$\sqrt{0}$	0	0	0	0		-4	-3
$\sqrt{1}$	1	1	1	-2		-3	-5
$\sqrt{4}$	4	2	4	-4		0	-7
$\sqrt{9}$	9	3	9	-6		5	-9
$\sqrt{16}$	16	4	16	-8		12	-11



domain: $\{x \mid x \geq -4, x \in \mathbb{R}\}$

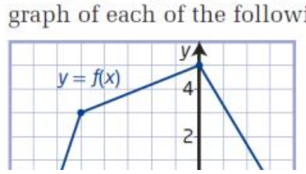
range: $\{y \mid y \leq -3, y \in \mathbb{R}\}$

mapping: $(x, y) \rightarrow (x-4, -2y-3)$

p.40 #9 f)

9. The graph of $y = f(x)$ is given. Sketch the graph of each of the following functions.

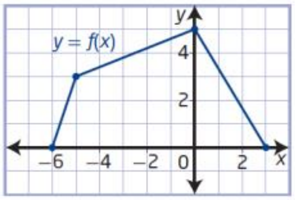
f) $y = \frac{1}{2}f\left(-\frac{1}{2}(x + 2)\right) - 1$



p.40 #9 f)

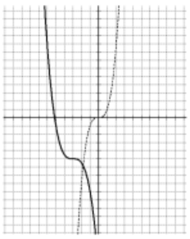
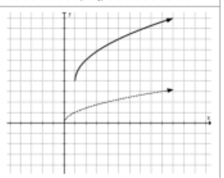
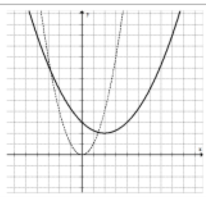
f) $y = \frac{1}{2}f\left(-\frac{1}{2}(x+2)\right) - 1$

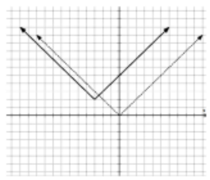
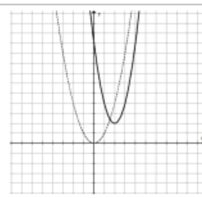
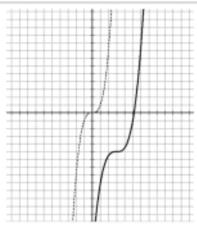
9. The graph of $y = f(x)$ is given. Sketch the graph of each of the following functions.

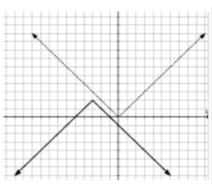
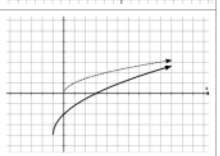


Group Matching Activity

Matching Transformations Students

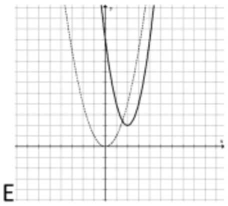
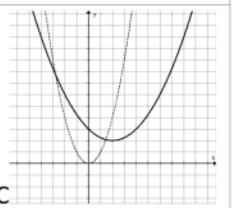
1. $y=2(x-2)^2+2$	A.	
2. $y=\left(\frac{1}{2}(x-2)\right)^2+2$	B.	
3. $y=\lvert x+3\rvert+2$	C.	

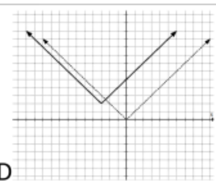
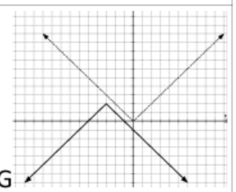
4. $y=-\lvert x+3\rvert+2$	D.	
5. $y=2\sqrt{x-1}+4$	E.	
6. $y=2\sqrt{x+1}-4$	F.	

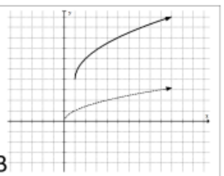
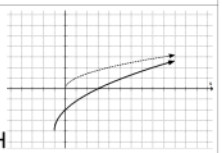
7. $y=\frac{1}{2}(x-3)^3-5$	G.	
8. $y=-\frac{1}{2}(x+3)^3-5$	H.	

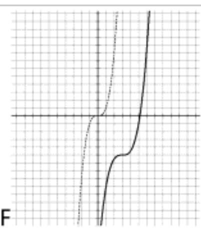
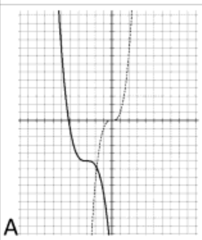
Group Matching Activity KEY

Matching Transformations

1. $y=2(x-2)^2+2$	 E
2. $y=\left(\frac{1}{2}(x-2)\right)^2+2$	 C

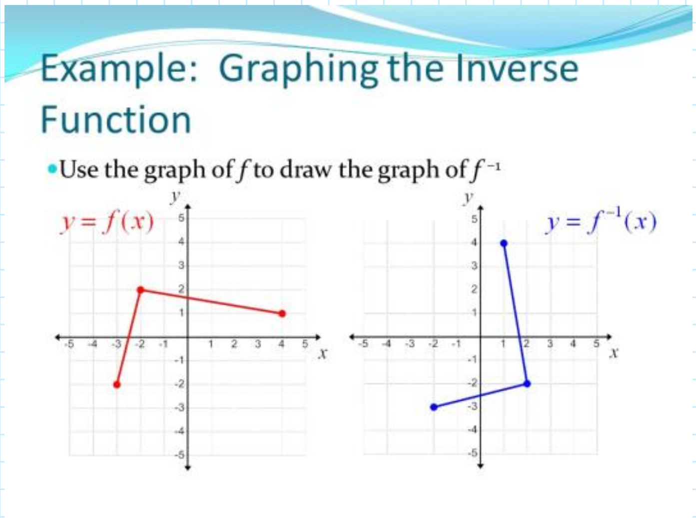
3. $y=\lvert x+3\rvert+2$	 D
4. $y=-\lvert x+3\rvert+2$	 G

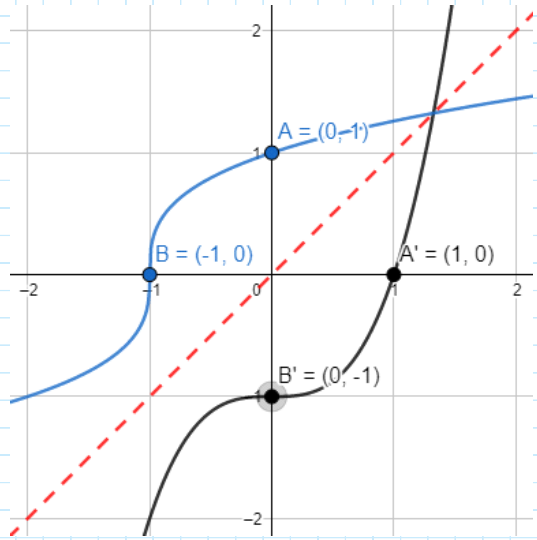
5. $y=2\sqrt{x-1}+4$	 B
6. $y=2\sqrt{x+1}-4$	 H

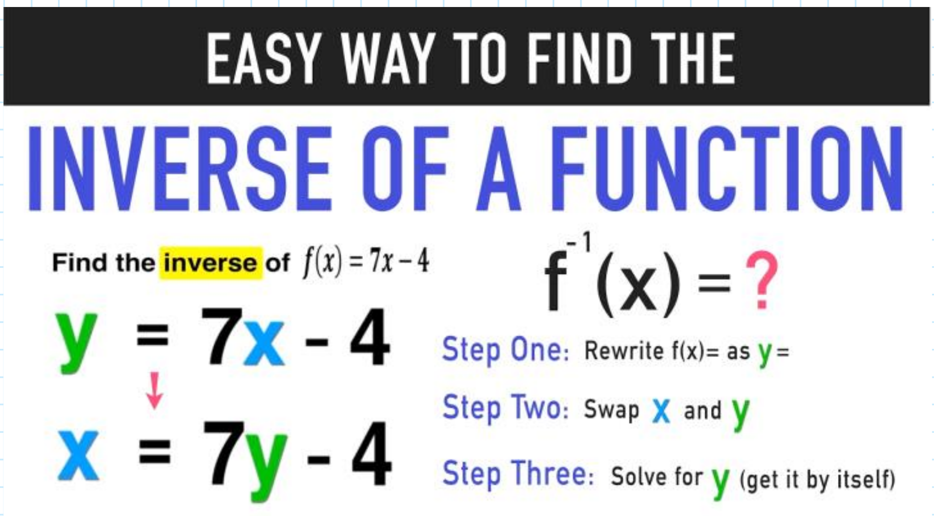
7. $y=\frac{1}{2}(x-3)^3-5$	 F
8. $y=-\frac{1}{2}(x+3)^3-5$	 A

1.4 Inverse of a Function

$$f^{-1}(x)$$







Original Function	Inverse Function
$f(x)$	$f^{-1}(x)$
Domain: $x \geq 5$	Domain: $x \leq 0$
Range: $y \leq 0$	Range: $y \geq 5$

Find the Inverse of a Function

1. Replace f(x) with y
2. Interchange x and y
3. Solve the equation for y
4. Replace y with $f^{-1}(x)$

Example:

Given $f(x)=\dfrac{4x+2}{5}$ find the inverse of $f(x)$

$f(x)=\dfrac{4x+2}{5}$	
$y=\dfrac{4x+2}{5}$	Replace f(x) with y
$x=\dfrac{4y+2}{5}$	Interchange x and y
$5x=4y+2$	
$5x-2=4y$	Solve the equation for y
$y=\dfrac{5x-2}{4}$	
$f^{-1}(x)=\dfrac{5x-2}{4}$	Replace y with $f^{-1}(x)$

1.4 Inverses

Inverse Operations

The operation that reverses the effect of another operation.
Examples of inverse operations?

Inverse of a Relation

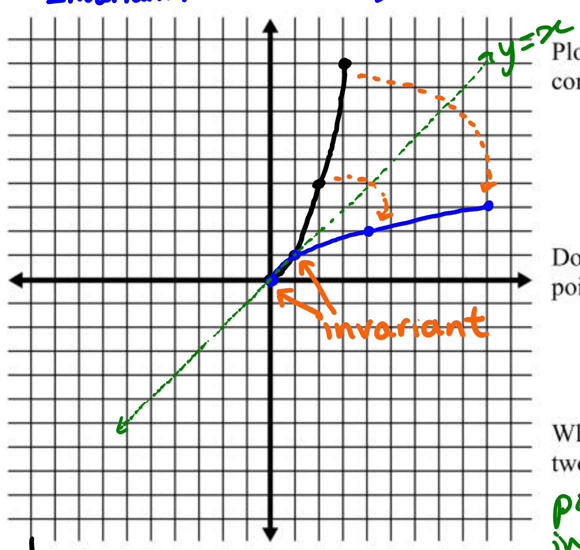
The inverse of a relation is found by interchanging the x-coordinates and y-coordinates of each of the ordered pairs in the relation. Each ordered pair in the relation, (x, y), is changed to the ordered pair (y, x) to form a point on the inverse of the relation. $(x,y) \rightarrow (y,x)$

Example For the relation $\{(0,0)\ (1,1)\ (2,4)\ (3,9)\}$, what ordered pairs form its inverse?

$\{(0,0)\ (1,1)\ (4,2)\ (9,3)\}$

Invariant Points: (0,0) (1,1)

∗ the inverse reflects on the $y=x$ line ∴ the invariant points are ON $y=x$



Plot the original points and connect them.

Do the same for the inverse points.

What do you notice about the two graphs?

points on $y=x$ are invariant, + graph reflects over $y=x$

original

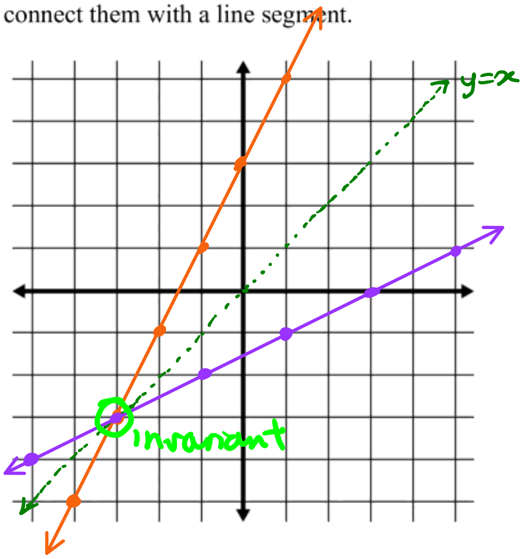
$\{x \mid 0 \le x \le 3, x \in \mathbb{R}\}$ $\rightarrow$ $\{x \mid 0 \le x \le 9, x \in \mathbb{R}\}$

$\{y \mid 0 \le y \le 9, y \in \mathbb{R}\}$ $\rightarrow$ $\{y \mid 0 \le y \le 3, y \in \mathbb{R}\}$

Example

a) Complete the table below for the equation $f(x)=2x+3$. Plot the points on the grid and connect them with a line segment.

b) Complete the table for the ***inverse*** of $f(x)$. Plot these new points on the same grid and connect them with a line segment.



function

$y=f(x)$

$y=2x+3$

input | output

x	y
–4	-5
–3	-3
–1	1
0	3

2(-4)+3

2( )+3

inverse

$x=f(y)$

x	y
-5	-4
-3	-3 ← invariant
1	-1
3	0

c) Use the graphs to complete the table below.

	Original function	*Inverse*
domain	$\{x \mid x \in \mathbb{R}\}$	$\{x \mid x \in \mathbb{R}\}$
range	$\{y \mid y \in \mathbb{R}\}$	$\{y \mid y \in \mathbb{R}\}$.
x-intercept	$(-1.5, 0)$ [$x=-1.5$ ok]	$(3,0)$
y-intercept	$(0,3)$ [usu. don't write $y=3$]	$(0,-1.5)$

The graph of a relation and its inverse are always reflections of each other across the line $y = x$.

Example Find the equation of the inverse for the function on the previous page, $f(x) = 2x + 3$.

① rewrite as $y = 2x + 3$

② switch x + y $x = 2y + 3$

$-3 \qquad -3$

③ solve for y

$\frac{x-3}{2} = \frac{2y}{2}$

$y = \frac{x-3}{2}$ or $y = \frac{1}{2}x - \frac{3}{2}$

④ rewrite with $f^{-1}(x) \rightarrow \underline{f^{-1}(x) = \frac{1}{2}x - \frac{3}{2}}$

Example Find the equation of the inverse for $f(x) = (2x-1)^2 + 4$.

① $y = (2x-1)^2 + 4$

② $x = (2y-1)^2 + 4$

$-4 \qquad -4$

③ $\sqrt{x-4} = \sqrt{(2y-1)^2}$

$\pm\sqrt{x-4} = 2y - 1$

$+1 \qquad +1$

$\frac{\pm\sqrt{x-4} + 1}{2} = \frac{2y}{2}$

$\pm\frac{1}{2}\sqrt{x-4} + \frac{1}{2} = y$

$\boxed{f^{-1}(x) = \pm\frac{1}{2}\sqrt{x-4} + \frac{1}{2}}$ or $f^{-1}(x) = \frac{\pm\sqrt{x-4}+1}{2}$

Example Find the equation of the inverse for $f(x)=\sqrt{4x-5}$.

1. $y=\sqrt{4x-5}$
2. $(x)^2=\left(\sqrt{4y-5}\right)^2$
3. $x^2=4y-5$

$+5 \qquad +5$

$\div 4 \qquad \div 4$

$\frac{x^2+5}{4}=y \rightarrow \boxed{f^{-1}(x)=\frac{1}{4}x^2+\frac{5}{4}}$ or $f^{-1}(x)=\frac{x^2+5}{4}$

Example Find the equation of the inverse for $f(x)=\dfrac{8x+1}{2x-5}$.

1. $y=\frac{8x+1}{2x-5}$
2. $(2y-5)\,x=\frac{8y+1}{2y-5}\,(2y-5)$
3. $x(2y-5)=8y+1$

$2xy-5x=8y+1$

$+5x \qquad -8y$

$2xy-8y=1+5x$

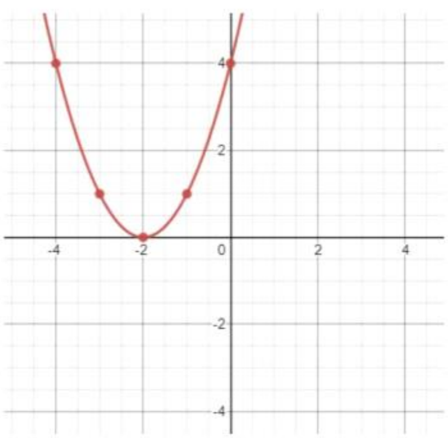
factor

$y(2x-8)=1+5x$

$\div(2x-8) \qquad \div(2x-8)$

$\boxed{f^{-1}(x)=\frac{1+5x}{2x-8}}$

Example Shown is the graph of $f(x)=(x+2)^2$.



a) Graph the inverse of $f(x)$ on the same grid.

b) How can we restrict the domain of $f(x)$ so that the inverse graph is a function?

Skip