Class_03 Sep 15 - More Transformations

Sunday, September 11, 2022   2:36 PM

Tonight's Class:

- **Wifi**
- **Learning Center & Career Advisor info**
- **Check-in**
- **More about Transformations**
- **Inverses**
- **Polynomial Functions**

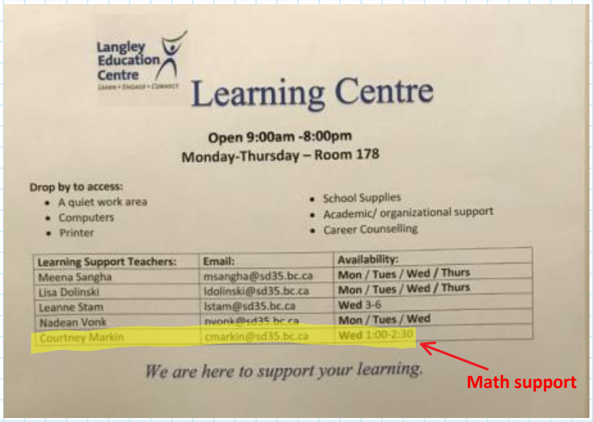
Wifi

SD35 -Secured-Students

username: your pupil number

password: $\underbrace{\_\ \_}_{\text{First 2 letters of your first name}}\ \underbrace{\_\ \_\ \_\ \_}_{\text{First 4 numbers of your pupil number}}$ $ $

Learning Center Info



Langley Education Centre

Learning Centre

Open 9:00am -8:00pm
Monday-Thursday – Room 178

Drop by to access:

- A quiet work area
- Computers
- Printer
- School Supplies
- Academic/ organizational support
- Career Counselling

Learning Support Teachers:	Email:	Availability:
Meena Sangha	msangha@sd35.bc.ca	Mon / Tues / Wed / Thurs
Lisa Dolinski	ldolinski@sd35.bc.ca	Mon / Tues / Wed / Thurs
Leanne Stam	lstam@sd35.bc.ca	Wed 3-6
Nadean Vonk	nvonk@sd35.bc.ca	Mon / Tues / Wed
Courtney Markin	cmarkin@sd35.bc.ca	Wed 1:00-2:30

We are here to support your learning.

Math support

- **Career Advisor**
  Leyla Baluoch, at school on Wednesdays, 9-4:30
  https://lec.sd35.bc.ca/

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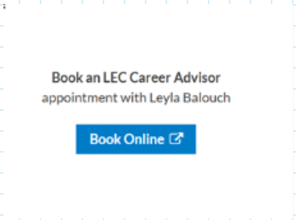
- 

Check-in

-fill in the table in your notes, page 12

-compare your answers with someone else

1.3 Combining Transformations

Summary of Transformations. Original Equation, $y = f(x)$

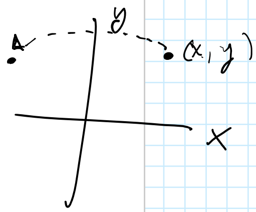
Translations

$y = f(x) - 4$

	Graph moves…	Mapping
$y + 4 = f(x)$	down 4	$(x, y) \rightarrow (x, y-4)$
$y - 5 = f(x)$	up 5	$\rightarrow (x, y+5)$
$y = f(x+2)$	left 2	$\rightarrow (x-2, y)$
$y = f(x-6)$	right 6	$\rightarrow (x+6, y)$

Stretches (expansions + compressions)

	Graph is stretched…	Mapping $(x,y) \rightarrow$
$y = 5f(x)$	VE by 5	$\rightarrow (x, 5y)$
$\frac{3}{2}y = f(x)$	VC by $\frac{2}{3}$	$\rightarrow (x, \frac{2}{3}y)$
$y = f(4x)$ $\frac{4}{1}$	horizontal compression $\frac{1}{4}$ $\boxed{HC}$	$\rightarrow (\frac{1}{4}x, y)$
$y = f\left(\frac{1}{3}x\right)$ 3	HE, by 3	$\rightarrow (3x, y)$



Reflections

means $-y = f(x)$

	Reflects across…	Mapping
$y = -f(x)$	x-axis	$(x,y) \rightarrow (x,-y)$
$y = f(-x)$	y-axis	$(x,y) \rightarrow (-x,y)$

x value change + -+

Vertical stretch, factor *a*

Horizontal translation

$$y = a\,f(b(x-h)) + k$$

Horizontal stretch, factor *1/b*

Vertical translation

Started this last time.......

STRETCHES – horizontal and vertical stretches

When $y = f(x)$ is changed to $y = a\,f(x)$, each point on the original graph has its y-value multiplied by "a."
This is a ***vertical stretch***, by a factor of a.

When $y = f(x)$ is changed to $y = f(bx)$, each point on the original graph has its x-value multiplied by the *reciprocal* of b. This is a ***horizontal stretch*** by a factor of $\frac{1}{b}$.

When the stretch factor is a number between -1 and 1, we call it a ***compression***. Otherwise, we call it an ***expansion.***

Examples

a) Identify each change, when $y = f(x)$ is changed to:

$y = 8f(x)$ VExp by 8	$y = f(2x)$ HComp. $\frac{1}{2}$	$y = \frac{1}{2}f(x)$ VC $\frac{1}{2}$
$y = f\left(\frac{1}{4}x\right)$ HE, 4	$4y = f(x)$ VC by $\frac{1}{4}$	$\frac{1}{2}y = f(x)$ VE by 2

Could do this:

$y = \frac{1}{2}f(x)$

$\frac{y}{\frac{1}{2}} = f(x)$

$y \div \frac{1}{2} = f(x)$

$y \times \frac{2}{1} = f(x)$

$2y = f(x)$

b) Write the new equation that causes $y = f(x)$ to be stretched as follows:

Vertical ~~stretch~~ compression, by $\frac{2}{3}$

$\boxed{y = \frac{2}{3}f(x)}$ or $\boxed{\frac{3}{2}y = f(x)}$

Horizontal ~~stretch~~ expansion, by $\frac{5}{2}$

$y = f\left(\frac{2}{5}x\right)$

To Try

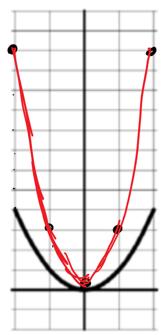
The graph of $y = f(x)$ is shown at right. When changed to $y = 3f(x)$,

- identify the transformation VE, 3
- complete the table and mapping
- sketch the graph of $y = 3f(x)$

x	y
-2	4
-1	1
0	0
1	1
2	4

Image points

x	3y
-2	12
-1	3
0	0
1	3
2	12



$(x, y) \rightarrow (x, 3y)$

To Try

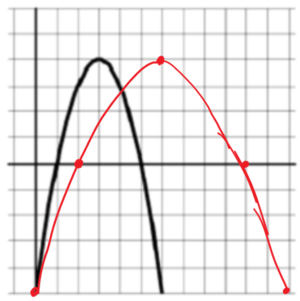
The graph of $y = f(x)$ is shown at right. When changed to $y = f\left(\frac{1}{2}x\right)$,

- identify the transformation HE by 2
- complete the table and mapping
- sketch the graph of $y = f\left(\frac{1}{2}x\right)$

x	y
0	-5
1	0
3	4
5	0
6	-5

Image points

2x	y
0	-5
2	0
6	4
10	0
12	-5



$(x, y) \rightarrow (2x, y)$

To Try

OR: VC by $-\frac{1}{2}$

The graph of $y = f(x)$ is shown at right. When changed to $y = -\frac{1}{2}f(x)$,

- identify the transformation VC by ½, and reflect across x-axis
- complete the table and mapping
- sketch the graph of $y = -\frac{1}{2}f(x)$

x	y
0	-5
1	0
3	4
5	0
6	-5

Image points

X	$-\frac{1}{2}y$
0	$\frac{5}{2}$ $=2\frac{1}{2}$
1	0
3	-2
5	0
6	$\frac{5}{2}$

$(x, y) \rightarrow (x, -\frac{1}{2}y)$

Textbook p 28: 3b, 4b, 5-7, 9, 12

Already filled in, above!

1.3 <u>Combining Transformations</u>

Summary of Transformations. Original Equation, $y = f(x)$

Translations		
	Graph moves…	**Mapping**
$y+4=f(x)$		$(x,\ y)\rightarrow$
$y-5=f(x)$		
$y=f(x+2)$		
$y=f(x-6)$		
Stretches		
	Graph is stretched…	**Mapping**
$y=5f(x)$		
$\frac{3}{2}y=f(x)$		
$y=f(4x)$		
$y=f\left(\frac{1}{3}x\right)$		
Reflections		
	Reflects across…	**Mapping**
$y=-f(x)$		
$y=f(-x)$		

Vertical stretch, factor *a*

Horizontal translation

$$y = a\,f(b(x-h)) + k$$

Horizontal stretch, factor 1/*b*

Vertical translation

Question…….

If more than one transformation is applied to a graph, does the ***order*** in which the transformations are done change the final graph?

***

Worksheet - each person will receive a copy to fill in.
Compare/discuss with others in your group.

$y = f(x)$ is shown on the grid

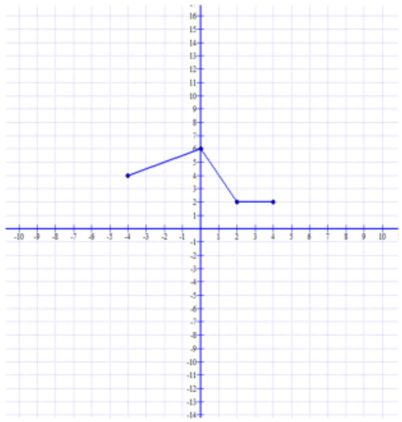
- Reflect across the x-axis and sketch the result.
- Take that graph and translate it 4 units up to get your FINAL graph

Original

x	y
–4	4
0	6
2	2
4	2

Reflected

FINAL



$y = f(x)$ is shown on the grid

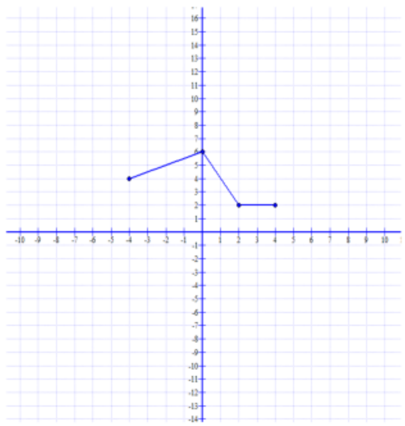
- Translate 4 units up and sketch the result
- Take that graph and reflect it across the x-axis to get your FINAL graph

Original

x	y
–4	4
0	6
2	2
4	2

Translated

FINAL



Conclusions: Yes, it makes a difference. The order in which we do a reflection and a translation changes the final result.

$y = f(x)$ is shown on the grid

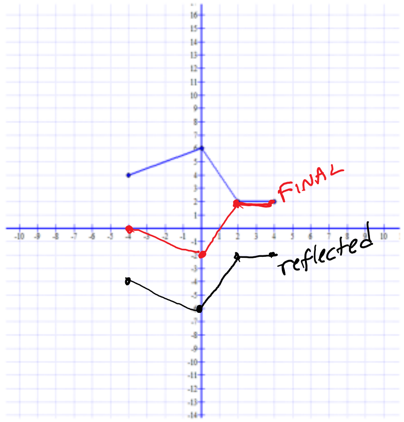
- Reflect across the x-axis and sketch the result.
- Take that graph and translate it 4 units up to get your FINAL graph

Original

x	y
−4	4
0	6
2	2
4	2

Reflected

x	−y
−4	−4
0	−6
2	−2
4	−2



FINAL

x	−y+4
−4	0
0	−2
2	2
4	2

$(x, y) \to (x, -y+4)$

$y = f(x)$ is shown on the grid

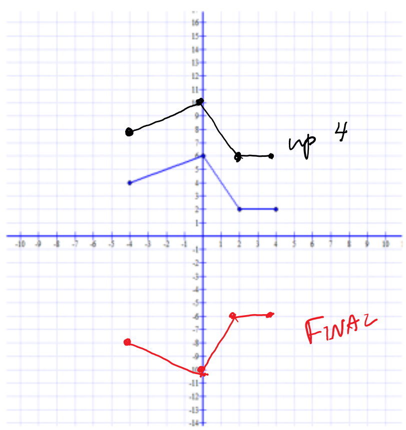
- Translate 4 units up and sketch the result
- Take that graph and reflect it across the x-axis to get your FINAL graph

Original

x	y
−4	4
0	6
2	2
4	2

Translated

x	y+4
−4	8
0	10
2	6
4	6



FINAL

x	−(y+4)
−4	−8
0	−10
2	−6
4	−6

$(x, y) \to (x, -(y+4))$

OR, simplifying:

$(x, y) \to (x, -y-4)$

Question…….
If more than one transformation is applied to a graph, does the ***order*** in which the transformations are done change the final graph?

- Yes, if they are both acting on the same variable.

$

← getting rich is not easy

Apply transformations in this order, to get the final graph:
1) all reflections & stretches FIRST ← multiplying
2) all translations AFTER ← adding/subtracting

BEDMAS → add/subtracting
brackets, exponents, division/mult.

Example List all the transformations, then give the mapping.

a) $y=-4f\left(\frac{1}{2}(x-3)\right)+6$

reflect across x-axis
OR
VE by −4
VE by −4
HE by 2
right 3
up 6

$(x,y)\rightarrow(2x+3,\ -4y+6)$

b) $y=2f(3x-6)+5$

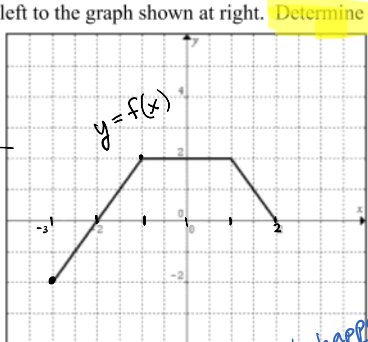
$y=2f[3(x-2)]+5$

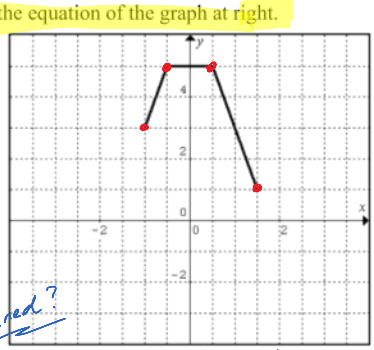
FACTOR first!! If there's a horizontal stretch and translation

$(x,y)\rightarrow(\frac{1}{3}x+2,\ 2y+5)$

Example
Identify the transformations that need to happen, to change the graph of $y=f(x)$ on the left to the graph shown at right. Determine the equation of the graph at right.

x	y
−3	−2
−1	2
1	2
2	0





$-\frac{1}{2}x$	$y+3$
$\frac{3}{2}$	1
$\frac{1}{2}$	5
$-\frac{1}{2}$	5
−1	3

What happened?
- reflect across y-axis
- HC by $\frac{1}{2}$
- up 3

$(x,y)\rightarrow(-\frac{1}{2}x,\ y+3)$

$$\boxed{y=f(-2x)+3}$$

Combining Transformations – Radicals
The changes we have discussed work with any function. Here are some questions to try, relating to the base radical function $y=\sqrt{x}$.

FACTOR first!

1) List the transformations that occur when the base radical function is changed to:
$y+9=\frac{1}{2}\sqrt{-3x-12}$. Give the r

$y+9=\frac{1}{2}\sqrt{-3(x+4)}$

VC $\frac{1}{2}$
down 9
left 4
HC $-\frac{1}{3}$ — OR (HC $\frac{1}{3}$ and reflect across y-axis)

$(x,y)\rightarrow(-\frac{1}{3}x-4,\ \frac{1}{2}y-9)$

$y+9=\frac{1}{2}\sqrt{-3(x+4)}$

dom 4
left 4
HC $-\frac{1}{3}$ — OR HC $\frac{1}{3}$ and reflect across y-axis

2) Given the mapping, which acts on the base radical function, write the new, transformed equation.

$\boxed{y=\sqrt{x}}$

$$(x,y)\rightarrow\underbrace{\left(5x+2,\frac{1}{2}y-4\right)}$$

HE by 5 right 2
VC by $\frac{1}{2}$ down 4

$$y=\frac{1}{2}\sqrt{\frac{1}{5}(x-2)}-4$$

"We need inner brackets"

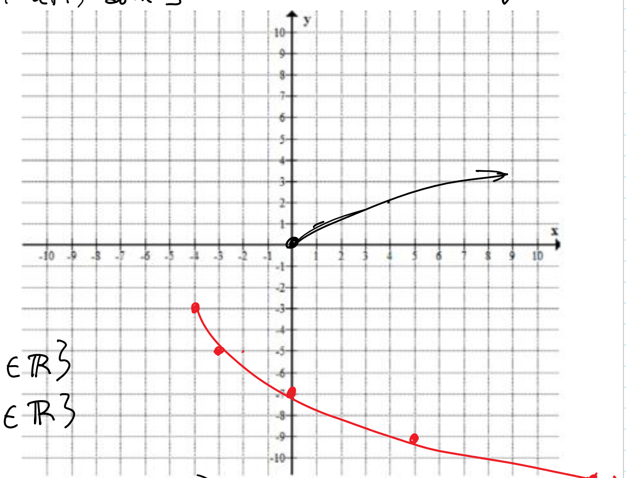
3) Complete the table of values for the base function, $y=\sqrt{x}$, and for the transformed equation, $y=-2\sqrt{x+4}-3$. Give the mapping, and the final graph's domain and range.

$(x,y)\rightarrow(x-4,-2y-3)$

$y=\sqrt{x}$ VE by -2, 4 left, down 3

x	y
0	0
1	1
4	2
9	3
16	4

x-4	-2y-3
-4	-3
-3	-5
0	-7
5	-9
12	-11



domain: $\{x \mid x\geq -4, x\in\mathbb{R}\}$

range: $\{y \mid y\leq -3, y\in\mathbb{R}\}$

mapping:

$(x,y)\rightarrow(x-4,-2y-3)$

1.4 Inverses

Inverse Operations

The operation that reverses the effect of another operation.
Examples of inverse operations?

adding / subtracting
multiply / divide
square-root / squaring

Inverse of a Relation

The inverse of a relation is found by interchanging the x-coordinates and y-coordinates of each of the ordered pairs in the relation. Each ordered pair in the relation, (x,y), is changed to the ordered pair (y,x) to form a point on the inverse of the relation.

Example For the relation $\{(0,0)\ (1,1)\ (2,4)\ (3,9)\}$, what ordered pairs form its inverse?

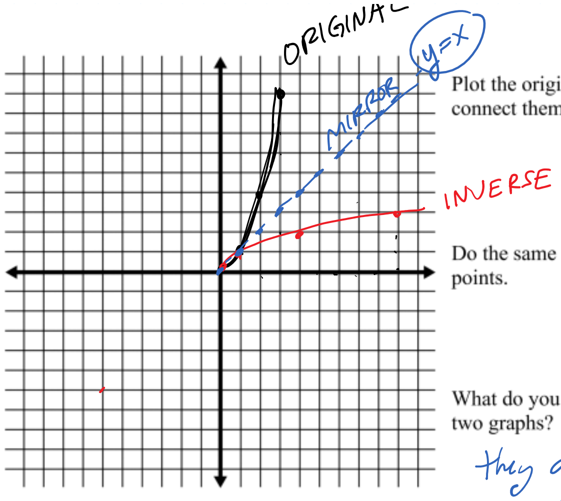
$\rightarrow \{(0,0)\ (1,1)\ (4,2)\ (9,3)\}$

ORIGINAL

$y=x$

Plot the original points and

→ $\{(0,0)\ (1,1)\ (4,2)\ (9,3)\}$



Plot the original points and connect them.

Do the same for the inverse points.

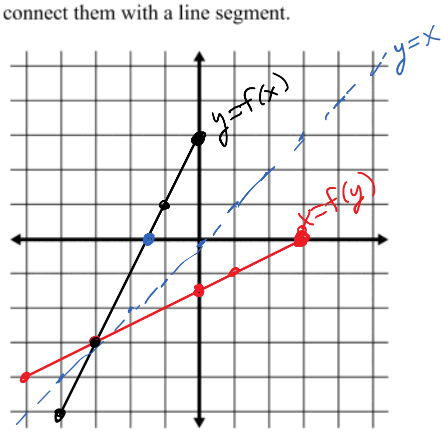
What do you notice about the two graphs?

they are reflections of each other across the line $y=x$

Example

a) Complete the table below for the equation $f(x) = 2x + 3$. Plot the points on the grid and connect them with a line segment.

b) Complete the table for the ***inverse*** of $f(x)$. Plot these new points on the same grid and connect them with a line segment.



function

$y = f(x)$	
$y = 2x + 3$	
x	y
-4	$2(-4)+3$ $= -5$
-3	$2(-3)+3$ $= -3$
-1	$2(-1)+3$ $= 1$
0	$2(0)+3$ $= 3$

inverse

$x = f(y)$	
x	y
-5	-4
-3	-3
1	-1
3	0

c) Use the graphs to complete the table below.

	Original function	*Inverse*
domain	$\{x \mid -4 \leq x \leq 0,\ x \in \mathbb{R}\}$	$\{x \mid -5 \leq x \leq 3,\ x \in \mathbb{R}\}$
range	$\{y \mid -5 \leq y \leq 3,\ y \in \mathbb{R}\}$	$\{y \mid -4 \leq y \leq 0,\ y \in \mathbb{R}\}$
x-intercept	Using the equation $y = 2x+3$ let $y=0$, $0 = 2x+3$; -3 -3; $-3 = 2x$; $x = -3/2$; $(-3/2, 0)$	$(3, 0)$

	let $y=0$, $x=-3/2$ $(-3/2, 0)$ $0=2x+3$ $-3 \quad -3$ $\frac{-3}{2} = \frac{\cancel{2}x}{\cancel{2}}$	$(3, 0)$
y-intercept	$(0, 3)$	$(0, -3/2)$

Worksheet - each person will receive a copy to fill in. (back of the worksheet done earlier)
Compare/discuss with others in your group.

INVERSES

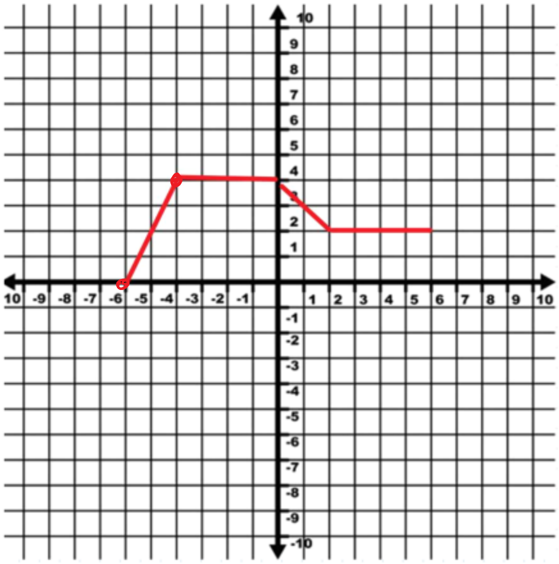
Consider the graph of the relation shown.

a) Sketch the graph of the inverse relation.

b) State the domain, range and intercepts of the relation and its inverse.

c) Determine whether the relation and its inverse are functions.

d) State the coordinates of any invariant points.



	Original Relation	Inverse Relation
Domain		
Range		
x-intercept		
y-intercept		
Is it a function?		
invariant points		

C_03 INVERSES

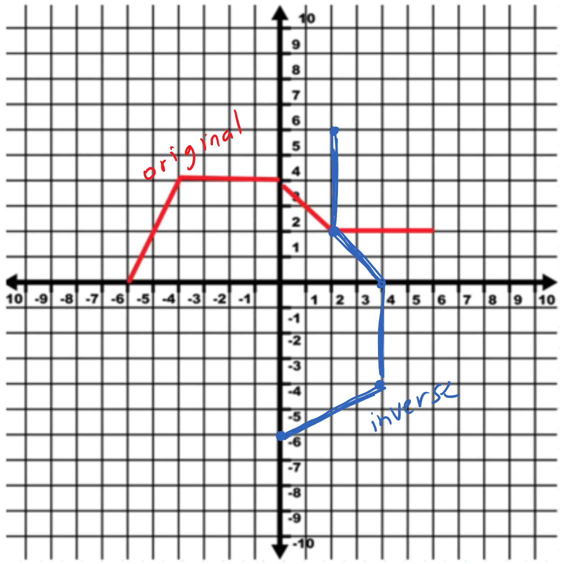
Consider the graph of the relation shown.

a) Sketch the graph of the inverse relation.

b) State the domain, range and intercepts of the relation and its inverse.

c) Determine whether the relation and its inverse are functions.

d) State the coordinates of any invariant points.



x	y
-6	0
-4	4
0	4
2	2
6	2

inverse

0	-6
4	-4
4	0
2	2
2	6

	Original Relation	Inverse Relation
Domain	$\{x \mid -6 \le x \le 6,\ x \in \mathbb{R}\}$	$\{x \mid 0 \le x \le 4,\ x \in \mathbb{R}\}$
Range	$\{y \mid 0 \le y \le 4,\ y \in \mathbb{R}\}$	$\{y \mid -6 \le y \le 6,\ y \in \mathbb{R}\}$
x-intercept	$(-6, 0)$	$(4, 0)$
y-intercept	$(0, 4)$	$(0, -6)$
Is it a function?	yes	No
invariant points	$(2, 2)$	

The graph of a relation and its inverse are always reflections of each other across the line $y = x$.

Example Find the equation of the inverse for the function on the previous page, $f(x) = 2x + 3$.

ORIGINAL
$y = 2x + 3$

1) trade x's and y's in the equation
2) Solve that new equation, isolating the "y" term

trade: $x = 2y + 3$
$-3 \qquad -3$

solve: $\frac{x-3}{2} = \frac{\cancel{2}y}{\cancel{2}}$

$\frac{x-3}{2} = y$

$$\boxed{y = \frac{x-3}{2}}$$

notation for inverse

$f^{-1}(x) = \frac{x-3}{2}$

f <u>inverse</u> of x

Example Find the equation of the inverse for $\underbrace{f(x) = (2x-1)^2 + 4}$.

$y = (2x-1)^2 + 4$

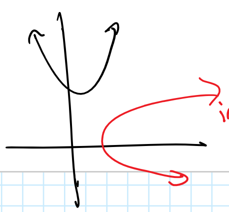
1) trade: $x = (2y-1)^2 + 4$
$-4 \qquad -4$

2) solve for y: $\pm\sqrt{x-4} = \sqrt{(2y-1)^2}$

remember this!

$\pm\sqrt{x-4} = 2y - 1$
$+1 \qquad +1$

$\frac{\pm\sqrt{x-4} + 1}{2} = \frac{\cancel{2}y}{\cancel{2}}$



inverse is NOT a function

$$y = \frac{\pm\sqrt{x-4} + 1}{2} \quad \text{OR} \quad y = \frac{1 \pm \sqrt{x-4}}{2}$$

We do NOT use $f^{-1}(x)$ notation this time, because the inverse is NOT a function.

Why do we put in the $\pm$?
We need it when solving an equation by square-rooting, otherwise we wouldn't get all the solutions:

$\sqrt{x^2} = \pm\sqrt{25}$

$x = \pm 5$

$x = 5$

$x = -5$

$y = \sqrt{4x-5}$

Example Find the equation of the inverse for $f(x) = \sqrt{4x-5}$.

1) trade $(x)^2 = \left(\sqrt{4y-5}\right)^2$

2) solve $x^2 = 4y - 5$

$+5 \qquad +5$

$\frac{x^2+5}{4} = \frac{\cancel{4}y}{\cancel{4}}$

$$\boxed{y = \frac{x^2+5}{4} \quad \text{OR} \quad f^{-1}(x) = \frac{x^2+5}{4}}$$

Example Find the equation of the inverse for $f(x) = \frac{8x+1}{2x-5}$.

$y = \frac{8x+1}{2x-5}$

1) trade $x = \frac{8y+1}{2y-5}$

2) solve: $(2y-5)(x) = \left(\frac{8y+1}{\cancel{2y-5}}\right)\cancel{(2y-5)}$

$(2y-5)(x) = 8y+1$

$2xy - 5x = 8y + 1$

$-1 \qquad -1$

$\cancel{2xy} - 5x - 1 = 8y$

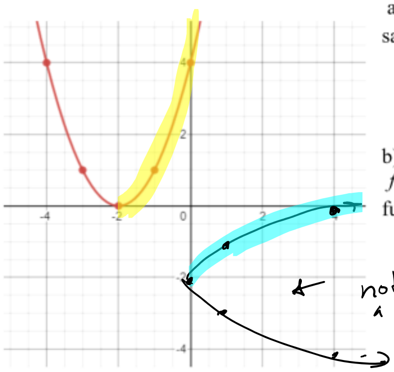
$-2xy \qquad -2xy$

$-5x - 1 = 8y - 2xy$

$\frac{-5x-1}{8-2x} = \frac{y\cancel{(8-2x)}}{\cancel{8-2x}}$

$$\boxed{y = \frac{-5x-1}{8-2x}} \quad \text{OR} \quad \boxed{y = \frac{5x+1}{2x-8}}$$

Example Shown is the graph of $f(x) = (x+2)^2$.



not a function

a) Graph the inverse of $f(x)$ on the same grid.

b) How can we restrict the domain of $f(x)$ so that the inverse graph is a function?

Use only one half of the original graph:

$\{x \mid x \geq -2,\ x \in \mathbb{R}\}$

For next class

Due: Chapter 1 Hand-in due Tuesday, Sep 20

Prepare for Chapter 1 Test on Tuesday, Sep 20

- **Make sure you understand how to do all questions on it. Complete the assignment, then check your work against the posted solutions.**

- Additional review worksheets (with solutions) are posted on the class website.

More Chapter 1 Practice available in textbook

- **(1.1) p 12: 2, 3cd, 4ac, 5, 8, 11**
- **(1.2) p 28: 3b, 4b, 5-7, 9, 12**
- **(1.3) p 38: 4, 5a, 6, 7abcd, 8, 9ce, 10ab**
- **(1.4) p 51: 1b, 2a, 3ac, 5ae, 8b, 12a**