Class_04 Jan 17 - Order of Operations and Exponent Laws

Tonight's Class:

- **Recap #3**
- **Any questions from 1.4 or 1.5?**
- **Working through section 1.6**
  - **Order of operations**
  - **Exponent Laws**
- **Work on practice questions from worktext**

p46

b) $\left(\frac{1}{32}\right)^{0.6} - \left(\frac{9}{25}\right)^{3/2} + \sqrt[3]{\frac{8}{27}}$

$= \left(\frac{1}{32}\right)^{\frac{6\div2}{10\div2}} - \left(\sqrt{\frac{9}{25}}\right)^3 + \frac{2}{3}$

$= \left(\frac{1}{32}\right)^{3/5} - \left(\frac{3}{5}\right)^3 + \frac{2}{3}$

$= \left(\sqrt[5]{\frac{1}{32}}\right)^3 - \left(\frac{27}{125}\right) + \frac{2}{3}$

$= \left(\frac{1}{2}\right)^3 - \frac{27}{125} + \frac{2}{3}$

$= \frac{1}{8}\cdot\frac{(125\times3)}{(125\times3)} - \frac{27}{125}\cdot\frac{(8\times3)}{(8\times3)} + \frac{2}{3}\cdot\frac{(1000)}{(1000)}$

$(8\times125\times3 = 3000)$

$= \frac{375}{3000} - \frac{648}{3000} + \frac{2000}{3000}$

$= \boxed{\frac{1727}{3000}}$

p 59 #19 a) $3^x > 1$

? is x positive, negative, or zero

$3^0 = 1$

$3^{-1} = \frac{1}{3}$

$3^{-2} = \frac{1}{3^2}$

} exp negative, result is fraction < 1

p 61 21) $C(t) = A(2)^{-t/5}$ (time; amount ingested)

a) $\frac{50}{100} = \frac{100}{100}(2)^{-t/5}$

$\frac{1}{2} = (2)^{-t/5}$

$2^{-1} = 2^{-t/5}$

this means $5 \times -1 = -\frac{t}{5} \times 5$

$\frac{-5}{-1} = \frac{-t}{-1}$

$5 = t$

$t = 5$ hours

p 58 #14

c) $-\left(\frac{243}{32}\right)^{-0.8} = -\left(\frac{32}{243}\right)^{+0.8}$

$0.8 = \frac{8}{10} = \frac{4}{5}$

$= -\left(\frac{32}{243}\right)^{4/5}$

$\sqrt[5]{32}^4$

$$= -\sqrt[5]{\left(\frac{32}{243}\right)^4}$$

$$= -\left(\frac{2}{3}\right)^4$$

$$= -\frac{16}{81}$$

---

g) $-\left(\frac{81}{10000}\right)^{-0.25} = -\left(\frac{10\,000}{81}\right)^{+0.25}$

$0.25 = \frac{25}{100} = \frac{1}{4}$

$$= -\left(\frac{10\,000}{81}\right)^{1/4}$$

$$= -\sqrt[4]{\frac{10\,000}{81}}$$

$$= -\frac{10}{3}$$

Preview 4

1.6 Order of Operations and Exponent Laws

Focus: apply order of operations and exponent laws to

- **evaluate numerical expressions**
- **simplify algebraic expressions**

Order of Operations

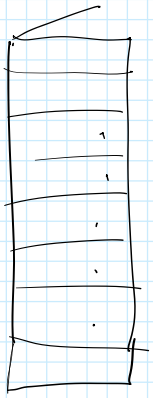
BEDMAS

B	Brackets, ()
E	Exponents, a^n
D M	Division or Multiplication (Left to right)
A S	Addition or Subtraction (Left to Right)

Exponent Rules for $a \neq 0, b \neq 0$	
Multiplication Property/ Product Rule	$a^m \cdot a^n = a^{m+n}$
Division Property/Quotient Rule	$\frac{a^m}{a^n} = a^{m-n}$
Power of a Power	$\left(a^m\right)^n = a^{m \cdot n}$
Power of a Product	$(ab)^m = a^m b^m$
Power of a Fraction	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
Zero Exponent	$a^0 = 1$
Negative Exponent	$a^{-n} = \frac{1}{a^n}, \quad \frac{1}{a^{-n}} = a^n, \quad \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$
Fractional Exponent	$a^{1/n} = \sqrt[n]{a}, \quad a^{m/n} = \sqrt[n]{\left(a^m\right)}$ or $\left(\sqrt[n]{a}\right)^m$

[Exponent Rules Review - create a foldable, summarizing them]

Exponent Rules Review - create a foldable, summarizing them



Multiplication Rule	$a^m \cdot a^n = a^{m+n}$	$4^2 \cdot 4^7 = 4^{2+7} = 4^9$ $(3x^2)(5x^4) = 15x^6$
Division Rule	$\frac{a^m}{a^n} = a^{m-n}$ $\quad a^m \div a^n = a^{m-n}$	$5^8 \div 5^2 = 5^{8-2} = 5^6$ $\frac{10x^3y^9}{2xy^3} = 5x^2y^6$
Power of a Power	$(a^m)^n = a^{mn}$	$(6^2)^5 = 6^{10}$ $(x^2)^7 = x^{14}$
Power of a Product	$(ab)^n = a^n b^n$	$(4x^2y)^3 = 4^3 x^{2\cdot3} y^{1\cdot3} = 64x^6y^3$
Power of a Fraction	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\left(\frac{3x^2}{y^4}\right)^2 = \frac{3^2x^{2\cdot2}}{y^{4\cdot2}} = \frac{9x^4}{y^8}$ (OR $9\frac{x^4}{y^8}$)
Zero Exponent	$a^0 = 1$	$(7x^2y)^0 = 1$
Negative Exponent	$a^{-n} = \frac{1}{a^n}$ $\quad \frac{1}{a^{-n}} = a^n$ $\quad \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{+n}$	$\frac{x^3y^5}{x^7y^2} = x^{-4}y^3 = \frac{1}{x^4}\cdot y^3 = \frac{y^3}{x^4}$ $\left(\frac{8x}{5}\right)^{-2} = \left(\frac{5}{8x}\right)^{+2} = \frac{25}{64x^2}$
Fractional Exponent	$a^{1/n} = \sqrt[n]{a}$ $\quad a^{m/n} = \sqrt[n]{a^m}$ OR $\sqrt[n]{a^m}$	$49^{1/2} = \sqrt{49} = 7$ $\left(\frac{8}{27}\right)^{2/3} = \sqrt[3]{\frac{8}{27}}^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$

Simplifying Numerical Expressions

Work text examples, pp 64-65

Example 1 **Simplifying Numerical Expressions with Rational Bases**

Write each expression as a single power, then evaluate.
Write each answer as a fraction in lowest terms.

a) $1.2^{-4} \cdot 1.2^7$

b) $\left[\left(\frac{4}{5}\right)^3\right]^{-2} \cdot \left[\left(\frac{4}{5}\right)^2\right]^4$

c) $5\left(\frac{2}{3}\right)^{-4}$

a) $1.2^{-4} \cdot 1.2^7 = 1.2^{-4+7}$

$= 1.2^3$

$= \left(\frac{12}{10}\right)^3$ $\quad \frac{10}{10}(1.2) = \frac{12}{10}$

$= \left(\frac{6}{5}\right)^3$

$= \frac{216}{125}$

b) $\left[\left(\frac{4}{5}\right)^3\right]^{-2} \left[\left(\frac{4}{5}\right)^2\right]^4$

$= \left(\frac{4}{5}\right)^{-6} \left(\frac{4}{5}\right)^8$

$= \left(\frac{4}{5}\right)^{-6+8}$

c) $5\left(\frac{2}{3}\right)^{-4} = 5\left(\frac{3}{2}\right)^4$

$= \left(\frac{4}{5}\right)^{-6+8}$

$= \left(\frac{4}{5}\right)^{2} = \frac{16}{25}$

c) $5\left(\frac{3}{2}\right) = 5\left(\frac{3}{2}\right)$

$= 5\left(\frac{3^4}{2^4}\right)$

$= \frac{5}{1}\left(\frac{81}{16}\right)$

$= \frac{405}{16}$

"CYU" p. 64

→ 1(a) $2.5^3 \cdot 2.5^{-5} = 2.5^{-2}$

$= \left(\frac{5}{2}\right)^{-2}$

$= \left(\frac{2}{5}\right)^{+2}$

$= \frac{4}{25}$

$2.5 = \frac{25}{10} \div 5 \div 5$

$= \frac{5}{2}$

p65

Example 2 **Simplifying Numerical Expressions with Rational Exponents**

Write each expression as a single power, then evaluate.
Write each answer as a fraction or a whole number.

a) $\left(\frac{12^{\frac{7}{8}}}{12^{\frac{1}{8}} \cdot 12^{\frac{1}{4}}}\right)^4$ **b)** $\left(8^{-\frac{1}{3}} + 8^{\frac{2}{3}}\right)^2$ **c)** $\left(3^{-\frac{1}{2}} \cdot 4^{\frac{2}{3}}\right)^6$ **d)** $2^{\frac{1}{4}} \cdot 128^{\frac{1}{4}}$

a) $\left(\frac{12^{7/8}}{12^{1/8} \cdot 12^{1/4}}\right)^4$ = Simplifying inside brackets first $\left(\frac{12^{7/8}}{12^{1/8 + 1/4 \cdot \frac{2}{2}}}\right)^4$

$= \left(\frac{12^{7/8}}{12^{1/8 + 2/8}}\right)^4$

$= \left(12^{7/8 - 3/8}\right)^4$

$= \left(12^{4/8}\right)^4$

$= \left(12^{1/2}\right)^4$

$= 12^2$

$= \left(\frac{12^{7/8}}{12^{3/8}}\right)^4$

$= 12^2$

$= \boxed{144}$

b) $\left(8^{-1/3} + 8^{2/3}\right)^2 = \left(\frac{1}{8^{1/3}} + \sqrt[3]{8^2}\right)^2$

$= \left(\frac{1}{\sqrt[3]{8}} + 4\right)^2$

$= \left(\frac{1}{2} + \frac{4}{1}\cdot\frac{2}{2}\right)^2$

$= \left(\frac{1}{2} + \frac{8}{2}\right)^2$

$= \left(\frac{9}{2}\right)^2$

$= \boxed{\frac{81}{4}}$

$\left(\frac{1}{2} + 4\right)^2$

$= \left(\frac{1}{2} + 4\right)\left(\frac{1}{2} + 4\right)$

$= \frac{1}{4} + 2 + 2 + 16$

$= \frac{1}{4} + 20$

$= 20\frac{1}{4}$

$= \frac{81}{4}$

2c) $\left(3^{-1/2} \cdot 4^{2/3}\right)^6$ power to power law

$= \left(3^{-1/2}\right)^6 \cdot \left(4^{2/3}\right)^6$

$= 3^{-6/2} \cdot 4^{12/3}$

$= 3^{-3} \cdot 4^4$

$= \frac{1}{3^3} \cdot 256$

$= \frac{1}{27} \cdot \frac{256}{1} = \boxed{\frac{256}{27}}$

Try - page 65: #2, and page 69: 5abcd, 7ab, 14ace, 21aceg (no variables)

Simplifying Algebraic Expressions
Work text examples, p 67

Example 3 — Simplifying Algebraic Expressions with Rational Exponents

Simplify each expression and write with positive exponents.

a) $3(2x^3y^{-4})(5x^2y^8)$ **b)** $\dfrac{(4a^2b^{-3})^2}{3a^{\frac{1}{2}}b^{-5}}$ **c)** $(2x^2 + 3y)(5x^2 - 2y)$

a) $3(2x^3y^{-4})(5x^2y^8)$

$= 3 \cdot 2 \cdot 5 \; x^3 x^2 y^{-4} y^8$

$= 30x^5y^4$

b) $\dfrac{(4a^2b^{-3})^2}{3a^{1/2}b^{-5}} = \dfrac{16a^4b^{-6}}{3a^{1/2}b^{-5}}$

$= \dfrac{16a^{4-1/2}b^{-6-(-5)}}{3}$

$\frac{2}{2}(4) - \frac{1}{2}$

$= \frac{8}{2} - \frac{1}{2}$

$= 7/2$

$= \dfrac{16a^{7/2}b^{-1}}{3}$

$= \boxed{\dfrac{16a^{7/2}}{3b}}$ $\dfrac{16\sqrt{a^7}}{3b}$

c) $(2x^2+3y)(5x^2-2y)$

$= 10x^4 - 4x^2y + 15x^2y - 6y^2$

$= 10x^4 + 11x^2y - 6y^2$

* mistake in answer, for 3c

p67

Check Your Understanding

Answers:

3. a) $\frac{60y^8}{x}$ b) $\frac{1}{2y^3}$

c) $18x^4 + 3xy^2 - 10y^4$

should be $18x^2 + 3xy^2 - 10y^4$

Example 4 **Applying Rational Exponents**

The height, h metres, of a certain species of fir tree can be estimated from the formula $h = 35d^{\frac{2}{3}}$, where d metres is the diameter of the tree at its base.

a) The base of a fir tree has diameter 4.1 m. Determine the approximate height of the tree. Give the answer to the nearest metre.

$h = 35d^{2/3}$

$= 35(4.1)^{2/3}$ use calculator

$= 35 (4.1) \wedge (2/3)$

$\doteq 89.658 \longrightarrow$ 90 m

b) A lumberjack climbs to the top of a fir tree and estimates that the tree is 52.5 m high. Determine the diameter of the tree at its base. Give the answer to 1 decimal place.

$h = 35d^{2/3}$

$\frac{52.5}{35} = \frac{35d^{2/3}}{35}$

$\left(\frac{52.5}{35}\right)^{3/2} = \left(d^{2/3}\right)^{3/2}$

$\left(\frac{52.5}{35}\right)^{3/2} = d$

$d \doteq 1.837\ldots$

$d \doteq 1.8$ m

For next class

- **Complete the "Recap" from tonight!**
- **Finish worktext questions for all of Chapter 1**
- **Complete the Chapter 1 Hand-in, due next class**
- **Prepare for the Chapter 1 Test, next class**
  - **4 multiple-choice questions**
  - **9 written questions**
  - **Out of 20 marks total**
  - **You will be permitted to use the Exponent Rules Foldable during the test**