Class_04 May 9 Polynomials

Tuesday, May 9, 2023 10:42 AM

Due today

Hand-in Assignment: Chapter 1 Hand-in. Any questions?

Tonight's Class:

- **Chapter 1 Test warm-up; Test**
- **3.1 Polynomial Characteristics**
- **3.2 Remainder Theorem**

$$\boxed{y = af(b(x-c)) + d}$$

and 4 units down

6. The key point $(-12, 18)$ is on the graph of $y = f(x)$. What is its image point under each transformation of the graph of $f(x)$?

a) $y + 6 = f(x - 4)$

b) $y = 4f(3x)$

c) $y = -2f(x - 6) + 4$

d) $y = -2f\left(-\frac{2}{3}x - 6\right) + 4$

e) $y + 3 = -\frac{1}{3}f(2(x + 6))$

3 down

HC $\frac{1}{2}$

6 left

VC $\frac{1}{3}$, reflect across the x-axis

$(x, y) \rightarrow \left(\frac{1}{2}x - 6, -\frac{1}{3}y - 3\right)$

$y = -2f\left(-\frac{2}{3}x - 6\right) + 4$

9 left

4 up

$$\boxed{y = -2f\left(-\frac{2}{3}(x + 9)\right) + 4}$$

VE, 2 reflects across x-axis

HE $\frac{3}{2}$, reflect across y-axis

$-\frac{2}{3}(\text{something}) = -6$

$= -6 \div -\frac{2}{3}$

$= -\frac{6}{1} \times \frac{3}{-2}$

$= \frac{-18}{-2} = +9$

$(x, y) \rightarrow \left(-\frac{3}{2}x - 9, -2y + 4\right)$

$(-12, 18) \rightarrow \left[\left(-\frac{3}{2}\right)\left(\frac{-12}{1}\right) - 9, -2(18) + 4\right]$

$= (9, -32)$

Please:

1. **Make sure your name is on your Chapter 1 Hand-in, and turn it in.**
2. **Put away your phone and all materials except for a calculator & something to write with.**
3. **On your test, write clearly and show all necessary steps.**
   When you are finished, please look over your test before handing it in.
4. **While other people are still finishing, respect them by being quiet. You can leave the classroom if you wish, but be back in time for the rest of class. Try the "Factoring Practice" worksheet.**

4. Factor.

a) $7x - 21$ $= 7(x-3)$

Always look for common factors!

b) $x^2 - 2x - 15$

c) $6x^2 + 19x + 10$

A B C

1) AC =
$6(10)$
$= 60$

2) think of two numbers that multiply to AC (60) and add to B (19)

3) Split up the middle term into 2 pieces, using as coefficients

1, 60
2, 30
3, 20
4, 15

$6x^2 + 19x + 10$

$6x^2 + 4x + 15x + 10$

$2x(3x+2) + 5(3x+2)$

$(3x+2)(2x+5)$

4) factor by grouping the first 2 terms and last 2 terms

5) Pull out common binomial, + put the other terms in a bracket next to it

d) $4x^2 - 9$

square root of 9

square root of $4x^2$

$(2x-3)(2x+3)$

$= 4x^2 + \cancel{6x} - \cancel{6x} - 9$

$= 4x^2 - 9$

CHAPTER 3

Polynomial Functions

Polynomial functions can be used to model different real-world applications, from business profit and demand to construction and fabrication design. Many calculators use polynomial approximations to compute function key calculations. For example, the first four terms of the Taylor polynomial approximation for the square root function are
$\sqrt{x} \approx 1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{1}{16}(x-1)^3$.

Chapter 3: Polynomial Functions

3.1 Characteristics of Polynomial Functions

- ***term*** - a single number (called a constant), a variable, or numbers and variables multiplied together
- a polynomial can just one term, or it can be made up of several terms added/subtracted together
- ***exponents*** of polynomial terms must be positive integers (no negative, fractional or decimal exponents)

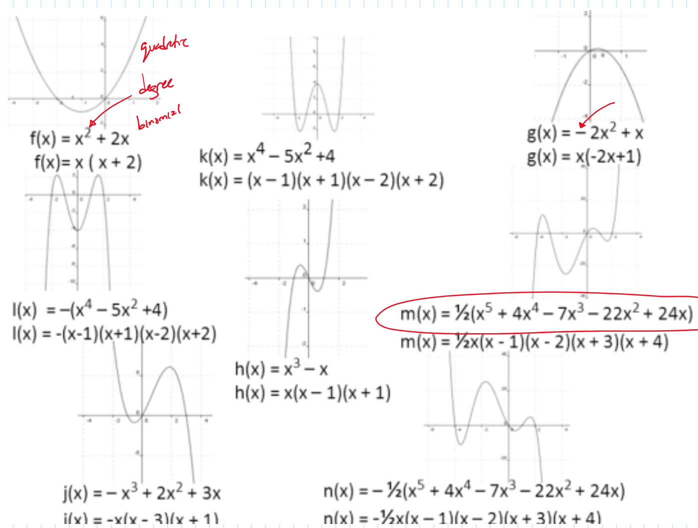
  x^{-2} ← not polynomial
  $\sqrt{x} = x^{1/2}$ $x^{3/4}$ ←
- ***coefficients*** of polynomial terms must be real numbers (no square-roots of negative numbers). In this chapter, we will only use coefficients that are integers.
- the ***degree of a constant*** is zero, ***degree*** of other terms is the exponent of *x* term
- the ***degree of a polynomial*** is found by looking at the degree of each of term and choosing the largest one
- the ***leading coefficient*** is the coefficient of the term with highest degree.

> Any polynomial can be written in the form below:
>
> Leading coefficient — Highest degree term first to lowest degree term last — Constant Term
>
> $$f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_2 x^2 + a_1 x^1 + a_0$$

Degree of Polynomials		examples
Linear	degree 1	$3x^1+5$
Quadratic	degree 2	$2x^2-9x+5$
Cubic	degree 3	$6x^3-4x^2+7$
Quartic	degree 4	$-5x^4+11$
Quintic	degree 5	$9x^5+2$

Number of Terms		examples
Monomial	1 term	$-5x$
Binomial	2 terms	$3x-6$
Trinomial	3 term	$x^2+7x+12$

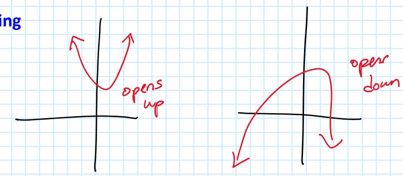
Polynomial graphs don't all look the same. Here are some examples:



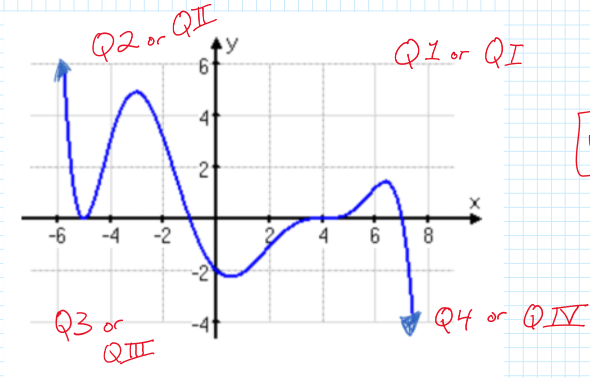
Important Features of Graphs
- **x-intercept(s)**
- **y-intercept**
- **Domain**
- **Range**
- **Maximum** } → highest or lowest y-value
- **Minimum**
- **Direction of opening**
- **End behavior**

open

- Minimum
- Direction of opening
- End behavior



End behavior - Report what the graph does at the extreme left and extreme right of the x-axis.



$\boxed{\text{up Q2 down Q4}}$

↑Q2 ↓Q4

Activity: Graphs of Polynomial Functions

Let's find some **CONNECTIONS** between a polynomial's equation and its graph.

odd degree end behavior is opposite direction

even degree, both ends point same direction

Section 3.1 Graphs of Polynomial Functions

Complete the tables to help you compare the graphs of different polynomial functions.

II I
III IV

Set	Function	Sketch	End Behavior Q# to Q#	Degree	Leading Coefficient
A (odd degree)	Linear $y=x^1$	1 3	↓Q3 ↑Q1	1	1
	Cubic $y=x^3$		↓Q3 ↑Q1	3	1
	Quintic $y=x^5$		↓Q3 ↑Q1	5	1
	Linear $y=-x$		↑Q2 ↓Q4	1	−1
	Cubic $y=-2x^3$		↑Q2 ↓Q4	3	−2
	Quintic $y=-2x^5$		↑Q2 ↓Q4	5	−2
B (even degree)	Quadratic $y=x^2$		↑Q2 ↑Q1	2	1
	Quartic $y=x^4$		↑Q2 ↑Q1	4	1
	Quadratic $y=-3x^2$		↓Q3 ↓Q4	2	−3
	Quartic $y=-2x^4$		↓Q3 ↓Q4	4	−2

Section 3.1 Graphs of Polynomial Functions

Complete the tables to help you compare the graphs of different polynomial functions.

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III IV

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A (odd degree)	Linear $y=x$		↓Q3 ↑Q1	1	1
	Cubic $y=x^3$		↓Q3 ↑Q1	3	1
	Quintic $y=x^5$		↓Q3 ↑Q1	5	1
	Linear $y=-x$		↑Q2 ↓Q4	1	−1
	Cubic $y=-2x^3$		↑Q2 ↓Q4	3	−2
	Quintic $y=-2x^5$		↑Q2 ↓Q4	5	−2
B (even degree)	Quadratic $y=x^2$		↑Q2 ↑Q1	2	1
	Quartic $y=x^4$		↑Q2 ↑Q1	4	1
	Quadratic $y=-3x^2$		↓Q3 ↓Q4	2	−3
	Quartic $y=-2x^4$		↓Q3 ↓Q4	4	−2

Equation Sketch

Set		Function	Degree	Constant Term	Value of y-intercept	# of x-intercepts	Sketch
C (odd degree)	Linear	$y = x + 1$	1	1	(0,1) = 1	1	
	Cubic	$y = x^3 + 4x^2 + x - 6$	3	−6	−6	3	
	Cubic	$y = x^3 - 2$	3	−2	−2	1	
	Quintic	$y = x^5 + 3x^4 - 5x^3 - 15x^2 + 4x + 12$	5	12	12	5	
	Quintic	$y = x^5 - 3$	5	−3	−3	1	
D (even degree)	Quadratic	$y = x^2 + 5x + 6$	2	6	6	2	
	Quadratic	$y = x^2 + 4$	2	4	4	0	
	Quartic	$y = x^4 + 2x^3 - 7x^2 - 8x + 7$	4	7	7	4	
	Quartic	$y = x^4 + 2$	4	2	2	0	

Set		Function	Degree	Constant Term	Value of y-intercept	# of x-intercepts	Sketch
C (odd degree)	Linear	$y = x + 1$	1	1	1	1	
	Cubic	$y = x^3 + 4x^2 + x - 6$	3	−6	−6	3	
	Cubic	$y = x^3 - 2$	3	−2	−2	1	
	Quintic	$y = x^5 + 3x^4 - 5x^3 - 15x^2 + 4x + 12$	5	12	12	5	
	Quintic	$y = x^5 - 3$	5	−3	−3	1	
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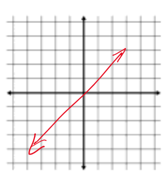
	Quadratic	$y = x^2 + 4$	2	4	4	0	
	Quartic	$y = x^4 + 2x^3 - 7x^2 - 8x + 7$	4	7	7	4	
	Quartic	$y = x^4 + 2$	4	2	2	0	

ODD degree

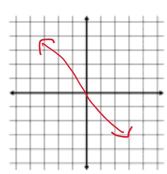
Leading coefficient positive



$y = x$

- End behavior ↓Q3 ↑Q1
- y-intercept constant
- Number of x-intercepts 1 up to "n", if n is the degree
- Domain $x \in \mathbb{R}$
- Range $y \in \mathbb{R}$
- Maximum/minimum none

Leading coefficient negative

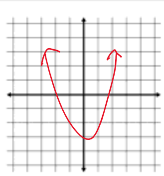


$y = -x$

- End behavior ↑Q2 ↓Q4
- y-intercept
- Number of x-intercepts
- Domain $x \in \mathbb{R}$
- Range $y \in \mathbb{R}$
- Maximum/minimum none

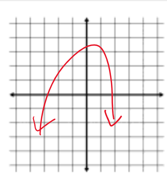
EVEN degree

Leading coefficient positive



- End behavior ↑Q2 ↑Q1
- y-intercept constant
- Number of x-intercepts 0 up to "n"
- Domain $x \in \mathbb{R}$
- Range $y \geq$ minimum
- Maximum/minimum

Leading coefficient negative



- End behavior ↓Q3 ↓Q4
- y-intercept constant
- Number of x-intercepts
- Domain $x \in \mathbb{R}$
- Range $y \leq$ maximum
- Maximum/minimum

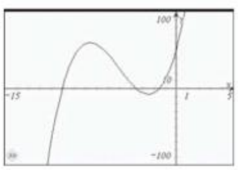
MC Characteristics of Graphs Practice - small whiteboards

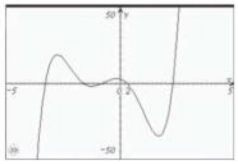
Try together, TB p 114, #3 and 4

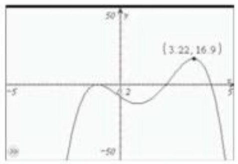
Check Your Understanding

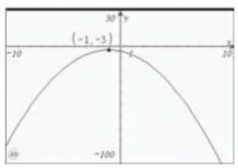
Practise

1. Identify whether each of the following is a polynomial function. Justify your answers.
   a) $h(x) = 2 - \sqrt{x}$
   b) $y = 3x + 1$
   c) $f(x) = 3^x$
   d) $g(x) = 3x^4 - 7$
   e) $p(x) = x^{-3} + x^2 + 3x$
   f) $y = -4x^3 + 2x + 5$
2. What are the degree, type, leading coefficient, and constant term of each polynomial function?
   a) $f(x) = -x + 3$
   b) $y = 9x^2$
   c) $g(x) = 3x^4 + 3x^2 - 2x + 1$ quartic
   d) $k(x) = 4 - 3x^3$
   e) $y = -2x^5 - 2x^3 + 9$
   f) $h(x) = -6$
3. For each of the following:
   - determine whether the graph represents an odd-degree or an even-degree polynomial function
   - determine whether the leading coefficient of the corresponding function is positive or negative
   - state the number of x-intercepts
   - state the domain and range

a) 

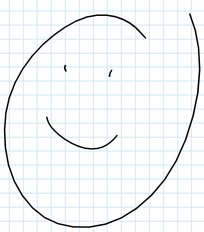
b) 

c) 

d) 

4. Use the degree and the sign of the leading coefficient of each function to describe the end behaviour of the corresponding graph. State the possible number of x-intercepts and the value of the y-intercept.
   a) $f(x) = x^2 + 3x - 1$
   b) $g(x) = -4x^3 + 2x^2 - x + 5$
   c) $h(x) = -7x^4 + 2x^3 - 3x^2 + 6x + 4$
   d) $q(x) = x^5 - 3x^2 + 9x$
   e) $p(x) = 4 - 2x$
   f) $v(x) = -x^3 + 2x^4 - 4x^2$

You know what seems odd to me? Numbers that aren't divisible by two."



TB, page 112

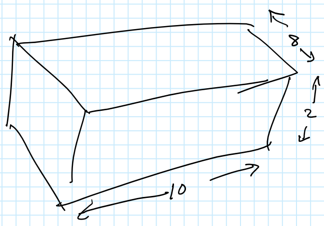
Your Turn

A toaster oven is built in the shape of a rectangular prism. Its volume, V, in cubic inches, is related to the height, h, in inches, of the oven door by the function $V(h) = h^3 + 10h^2 + 31h + 30$.

a) What is the volume, in cubic inches, of the toaster oven if the oven door height is 8 in.?

b) What is the height of the oven door for the least toaster oven volume? Explain.

Here's another volume question. Say we know a box is 8 cm wide, 10 cm long, and 2 cm high. What is the volume of the box?



$$\text{Volume} = lwh$$
$$= (10)(8)(2)$$
$$= 160 \text{ cm}^3$$

What if we know the volume of a box? Can we find the dimensions?
In order to do that, we have to factor.......which in a way is like dividing.

Given that the volume of a box is 30 cubic cm, what might the dimensions of the box be? Let's assume that each side length is a whole number.

$30 \times 1 \times 1$

$3 \times 10 \times 1$

$3 \times 5 \times 2$

What might the dimensions of a box be, if we know the volume is given by this polynomial?

$$V = x^3 + 2x^2 - 5x - 6$$

need to factor it!

To figure this out, we need to know how to FACTOR a CUBIC polynomial.

Textbook, page 118

3.2

The Remainder Theorem

Focus on…

- describing the relationship between polynomial long division and synthetic division
- dividing polynomials by binomials of the form $x - a$ using long division or synthetic division
- explaining the relationship between the remainder when a polynomial is divided by a binomial of the form $x - a$ and the value of the polynomial at $x = a$

Long Division - remember that??

The basic process is shown in the first 50 seconds of this video:

https://www.youtube.com/watch?v=OK0ks0w8Kns

3.2 The Remainder Theorem

Long division

Check : (divisor)(quotient) + remainder = dividend

divisor

quotient

$$6\overline{)253} = 42$$

$- 24$
13
$- 12$
1

dividend

remainder

Father / Find
Mother / Multiply
Sister / Subtract
Brother / Bring down

$(6)(42) + 1 = 253$

Ways to write the result:	Check:
$253 \div 6 = 42$, remainder 1 $\frac{253}{6} = 42$, remainder 1 $\frac{253}{6} = 42 + \left(\frac{1}{6}\right)$	(Divisor)(Quotient) + Remainder = Dividend (6)(42) + 1 = 253

Using long division to divide a polynomial by a binomial

Divide $x^3 - 12x^2 - 42$ by $x - 3$

1) Write dividend, P(x), in descending order (highest degree term first, then going on down)

2) If any degree is missing, include it, with a coefficient of 0.

$$x - 3\overline{)x^3 - 12x^2 + 0x - 42} = x^2 - 9x - 27$$

$-(x^3 - 3x^2)$
$-9x^2 + 0x$
$-(-9x^2 + 27x)$
$-27x - 42$
$-(-27x + 81)$
-123

remainder

Express the result in the form $\boxed{\frac{P(x)}{x-a} = Q(x) + \frac{R}{x-a}}$

What restrictions are there on the variable?

Verify (check) your answer.

Division Statement:

$$\frac{x^3 - 12x^2 - 42}{x - 3} = x^2 - 9x - 27 + \frac{-123}{x - 3}$$

restriction: $x \neq 3$

$P(x) \stackrel{?}{=} (x-3)(x^2 - 9x - 27) + -123$

$= x^3 - 9x^2 - 27x - 3x^2 + 27x + 81 + -123$

$= x^3 - 12x^2 - 42$ ✓

Division Statement
The result of dividing a polynomial $P(x)$ by a binomial of the form $x-a$ is:

$$\frac{P(x)}{x-a} = Q(x) + \frac{R}{x-a}$$, where $Q(x)$ is the quotient and R is the remainder.

Check: $P(x) = (x-a)Q(x) + R$
original polynomial = (divisor)(quotient) + remainder

1a) Divide the polynomial $5x^3 + 3x^2 - 12$ by $x+2$ using ***long division.***
Express the result in the form $\frac{P(x)}{x-a} = Q(x) + \frac{R}{x-a}$

$$
\begin{array}{r}
5x^2 - 7x + 14 \\
x+2 \overline{) 5x^3 + 3x^2 + 0x - 12} \\
-(5x^3 + 10x^2) \\
-7x^2 + 0x \\
-(-7x^2 - 14x) \\
14x - 12 \\
-(14x + 28) \\
-40
\end{array}
$$

$$\boxed{\frac{5x^3+3x^2-12}{x+2} = 5x^2 - 7x + 14 + \frac{-40}{x+2}}$$

b) What restrictions are there on the variable?

$x \neq -2$

c) Write the statement that can be used to check the division.

$P(x) = (x+2)(5x^2 - 7x + 14) + -40$

d) Verify your answer.

$(x+2)(5x^2 - 7x + 14) + -40$
$= 5x^3 - 7x^2 + \cancel{14x} + 10x^2 - \cancel{14x} + 28 + -40$
$= 5x^3 + 3x^2 - 12$ ✓

2. Divide the polynomial $5x^3 + 3x^2 - 12$ by $x+2$ using ***synthetic division.***
Express the result in the form $\frac{P(x)}{x-a} = Q(x) + \frac{R}{x-a}$

0x

$x+2$

$$
\begin{array}{r|rrrr}
-2 & 5 & 3 & 0 & -12 \\
 & \downarrow & 10 & -14 & 28 \\
\hline
 & 5 & -7 & 14 & -40
\end{array}
$$

$\underbrace{5 \quad -7 \quad 14}_{\text{coefficients of the quotient}}$; -40 remainder

$$\frac{P(x)}{x+2} = 5x^2 - 7x + 14 + \frac{-40}{x+2}$$

Divide, using synthetic division:
$(2x^3 + 3x^2 - 5x + 2) \div (x+3)$

$x+3$

$$
\begin{array}{r|rrrr}
-3 & 2 & 3 & -5 & 2 \\
 & \downarrow & -6 & 9 & -12 \\
\hline
 & 2 & -3 & 4 & -10
\end{array}
$$

Remainder

$2x^2 - 3x + 4 + \frac{-10}{x+3}$

$(x+3)(2x^2 - 3x + 4) - 10$
$= 2x^3 - 3x^2 + 4x + 6x^2 - 9x + 12 - 10$
$= 2x^3 + 3x^2 - 5x + 2$

Find the value of $P(-3)$, for $P(x) = 2x^3 + 3x^2 - 5x + 2$.

Remainder Theorem:
When a polynomial, *P(x)*, is divided by a binomial, $x-a$, the remainder is $P(a)$.

If $P(a) = 0$, then the binomial $x-a$ ***is*** a factor of *P(x).*
If $P(a) \neq 0$, then the binomial $x-a$ is ***not*** a factor of *P(x).*

Example
a) Use the Remainder Theorem to find the remainder when $P(x) = 8x^3 + 4x^2 - 19$ is divided by $x+2$

b) Check your answer by using synthetic division.

c) Use the Remainder Theorem to find the remainder when $P(x) = 8x^3 + 4x^2 - 19$ is divided by $x-1$.

TB, p 124

8. For each dividend, determine the value of k if the remainder is 3.

a) $(x^3 + 4x^2 - x + k) \div (x - 1)$

b) $(x^3 + x^2 + kx - 15) \div (x - 2)$

c) $(x^3 + kx^2 + x + 5) \div (x + 2)$

d) $(kx^3 + 3x + 1) \div (x + 2)$