Class_04 Sep 20 Polynomials

Thursday, September 15, 2022 1:50 PM

Due today

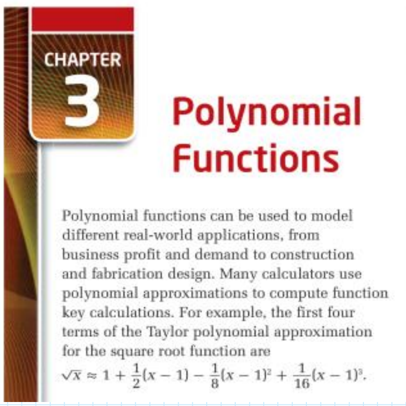
Hand-in Assignment: Chapter 1 Hand-in. Any questions?

Tonight's Class:

- **Chapter 1 Test warm-up; Test**
- **3.1 Polynomial Characteristics**
- **3.2 Remainder Theorem**

Please:

1. **Make sure your name is on your Chapter 1 Hand-in, and turn it in.**
2. **Put away your phone and all materials except for a calculator & something to write with.**
3. **On your test, write clearly and show all necessary steps.**
   When you are finished, please look over your test before handing it in.
4. **While other people are still finishing, respect them by being quiet. You can leave the classroom if you wish, but be back in time for the rest of class. Try the "Factoring Practice" worksheet.**



CHAPTER 3

Polynomial Functions

Polynomial functions can be used to model different real-world applications, from business profit and demand to construction and fabrication design. Many calculators use polynomial approximations to compute function key calculations. For example, the first four terms of the Taylor polynomial approximation for the square root function are $\sqrt{x} \approx 1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{1}{16}(x-1)^3$.

Chapter 3: Polynomial Functions

3.1 Characteristics of Polynomial Functions

- ***term*** - a single number (called a constant), a variable, or numbers and variables multiplied together

13
$5x^2$
$-6x$

- a polynomial can just one term, or it can be made up of several terms added/subtracted together

- ***exponents*** of polynomial terms must be positive integers (no negative, fractional or decimal exponents)

degree 2

x^{12} $5x^2$ $2x^{-1}$ } not a polynomial

- ***coefficients*** of polynomial terms must be real numbers (no square-roots of negative numbers). In this chapter, we will only use coefficients that are integers.

coefficient

- the ***degree of a constant*** is zero, ***degree*** of other terms is the exponent of x term

degree 12

$8x^0$ is a polynomial constant, degree 0

- the ***degree of a polynomial*** is found by looking at the degree of each of term ~~and choosing~~ the largest one

$5x^4 - 7x^2 + 11x - 3$ degree of this polynomial?

- the ***leading coefficient*** is the coefficient of the term with highest degree.

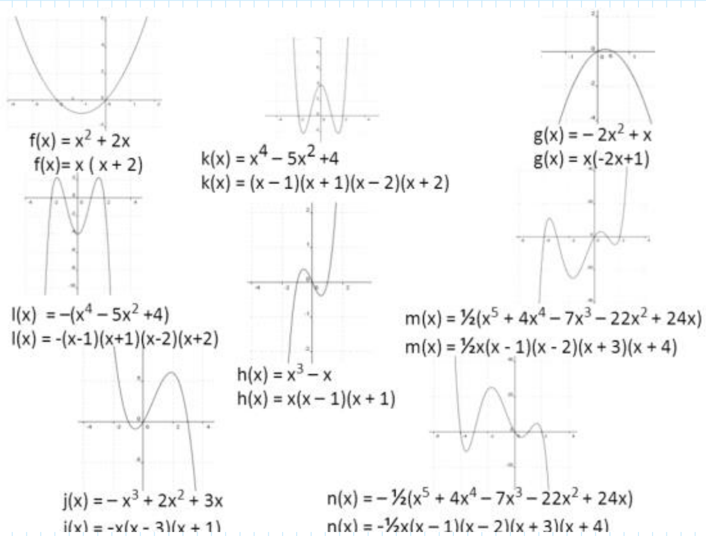
Any polynomial can be written in the form below:

Leading coefficient — Highest degree term first to lowest degree term last — Constant Term

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_2 x^2 + a_1 x^1 + a_0$$

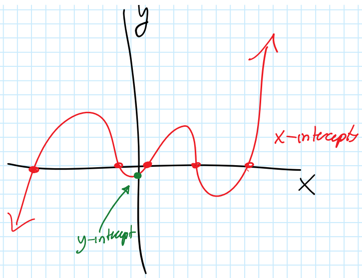
Degree of Polynomials		examples	Number of Terms		examples
Linear	degree 1	$5x^1+4$			
Quadratic	degree 2	$3x^2-17x+4$	***Monomial***	only 1 term	$8x$
Cubic	degree 3	$-2x^3+10$	***Binomial***	2 terms	$3x+7$
Quartic	degree 4	$2x^4-10x^2+17$	***Trinomial***	3 terms	x^2-4x+9
Quintic	degree 5	$-10x^3+25x+11x^5$			

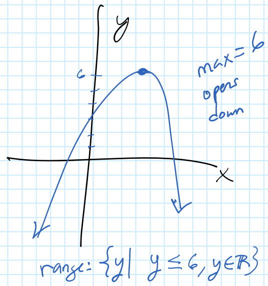
Polynomial graphs don't all look the same. Here are some examples:

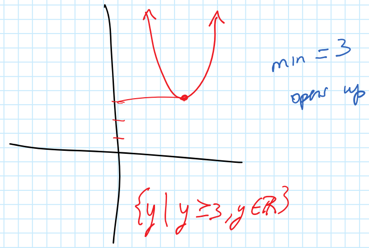


Important Features of Graphs

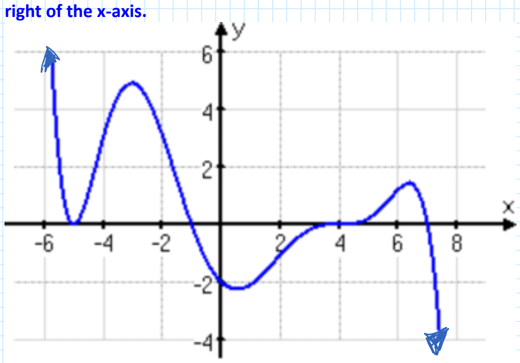
- x-intercept(s)
- y-intercept
- Domain $\{x \mid x \in \mathbb{R}\}$
- Range
- Maximum } highest y-value
- Minimum } lowest y-value
- Direction of opening
- End behavior

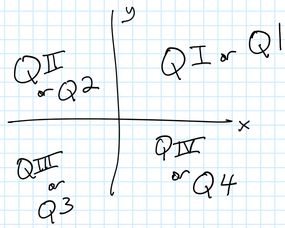






End behavior - Report what the graph does at the extreme left and extreme right of the x-axis.





end behavior: ↑Q2 ↓Q4

Activity: Graphs of Polynomial Functions

From this, we want to find some connections between key features of a polynomial's equation and its graph.

Section 3.1 Graphs of Polynomial Functions

Complete the tables to help you compare the graphs of different polynomial functions.

II I

III IV

Set	Function	Sketch	End Behavior Q# to Q#	Degree	Leading Coefficient
A (odd degree)	Linear $y=x$				
	Cubic $y=x^3$				
	Quintic $y=x^5$				
	Linear $y=-x$				
	Cubic $y=-2x^3$				
	Quintic $y=-2x^5$				
B (even degree)	Quadratic $y=x^2$				
	Quartic $y=x^4$				
	Quadratic $y=-3x^2$				
	Quartic $y=-2x^4$				

Section 3.1 Graphs of Polynomial Functions

Complete the tables to help you compare the graphs of different polynomial functions.

II I

III IV

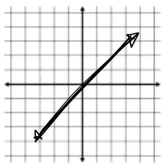
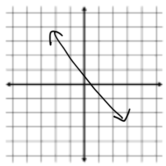
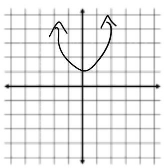
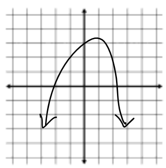
ODD degree

EVEN degree

Set	Function	Sketch	End Behavior Q# to Q#	Degree	Leading Coefficient
A (odd degree)	Linear $y=x$		↓Q3 ↑Q1	1	1
	Cubic $y=x^3$		↓Q3 ↑Q1	3	1
	Quintic $y=x^5$		↓Q3 ↑Q1	5	1
	Linear $y=-x$		↑Q2 ↓Q4	1	-1
	Cubic $y=-2x^3$		↑Q2 ↓Q4	3	-2
	Quintic $y=-2x^5$		↑Q2 ↓Q4	5	-2
B (even degree)	Quadratic $y=x^2$		↑Q2 ↑Q1	2	1
	Quartic $y=x^4$		↑Q2 ↑Q1	4	1
	Quadratic $y=-3x^2$		↓Q3 ↓Q4	2	-3
	Quartic $y=-2x^4$		↓Q3 ↓Q4	4	-2

Set		Function	Degree	Constant Term	Value of y-intercept	Number of x-Intercepts
C (odd degree)	Linear	$y=x+1$				
	Cubic	$y=x^3+4x^2+x-6$				
	Cubic	$y=x^3-2$				
	Quintic	$y=x^5+3x^4-5x^3-15x^2+4x+12$				
	Quintic	$y=x^5-3$				
D (even degree)	Quadratic	$y=x^2+5x+6$				
	Quadratic	$y=x^2+4$				
	Quartic	$y=x^4+2x^3-7x^2-8x+7$				
	Quartic	$y=x^4+2$				

Set		Function	Degree	Constant Term	Value of y-intercept	Number of x-Intercepts
C (odd degree)	Linear	$y=x+1$	1	1	1	1
	Cubic	$y=x^3+4x^2+x-6$	3	-6	-6	3
	Cubic	$y=x^3-2$	3	-2	-2	1
	Quintic	$y=x^5+3x^4-5x^3-15x^2+4x+12$	5	12	12	5
	Quintic	$y=x^5-3$	5	-3	-3	1
D (even degree)	Quadratic	$y=x^2+5x+6$	2	6	6	2
	Quadratic	$y=x^2+4$	2	4	4	0
	Quartic	$y=x^4+2x^3-7x^2-8x+7$	4	7	7	4
	Quartic	$y=x^4+2$	4	2	2	0

ODD degree	
Leading coefficient positive	Leading coefficient negative
 y = x	
• End behavior ↓Q3 ↑Q1 • *y*-intercept constant term • Number of *x-intercepts* 1 up to at most the degree • Domain $x \in \mathbb{R}$ • Range $y \in \mathbb{R}$ • Maximum/minimum none	• End behavior ↑Q2 ↓Q4 • *y*-intercept constant term • Number of *x-intercepts* • Domain • Range • Maximum/minimum
EVEN degree	
Leading coefficient positive	Leading coefficient negative
	
• End behavior ↑Q2 ↑Q1 • *y*-intercept constant term • Number of *x-intercepts* 0 up to the degree • Domain $x \in \mathbb{R}$ • Range $y \geq \text{min}$ • Maximum/minimum	• End behavior ↓Q3 ↓Q4 • *y*-intercept constant term • Number of *x-intercepts* • Domain • Range $y \leq \text{max}$ • Maximum/minimum

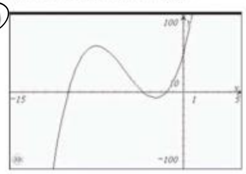
Try together, TB p 114, #3 and 4

Check Your Understanding

Practise

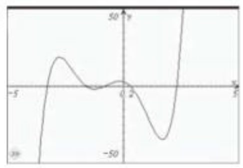
1. Identify whether each of the following is a polynomial function. Justify your answers.
   a) $h(x) = 2 - \sqrt{x}$
   b) $y = 3x + 1$
   c) $f(x) = 3^x$
   d) $g(x) = 3x^4 - 7$
   e) $p(x) = x^{-3} + x^2 + 3x$
   f) $y = -4x^3 + 2x + 5$
2. What are the degree, type, leading coefficient, and constant term of each polynomial function?
   a) $f(x) = -x + 3$
   b) $y = 9x^2$
   c) $g(x) = 3x^4 + 3x^2 - 2x + 1$
   d) $k(x) = 4 - 3x^3$
   e) $y = -2x^5 - 2x^3 + 9$
   f) $h(x) = -6$
3. For each of the following:
   - determine whether the graph represents an odd-degree or an even-degree polynomial function
   - determine whether the leading coefficient of the corresponding function is positive or negative
   - state the number of x-intercepts
   - state the domain and range

   a)

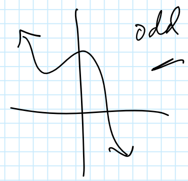


odd
+
3

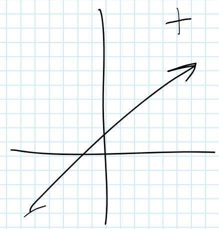
   b)



odd
+

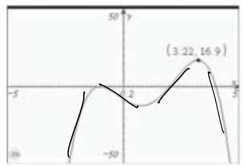


odd
−



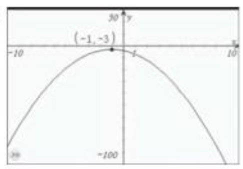
+

   c)



even
−

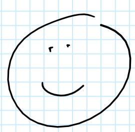
   d)



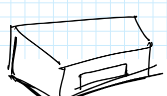
even
−

4. Use the degree and the sign of the leading coefficient of each function to describe the end behaviour of the corresponding graph. State the possible number of x-intercepts and the value of the y-intercept.
   a) $f(x) = x^2 + 3x - 1$ 0 up to 2 y-int: $(0, -1)$
   b) $g(x) = -4x^3 + 2x^2 - x + 5$
   c) $h(x) = -7x^4 + 2x^3 - 3x^2 + 6x + 4$
   d) $q(x) = x^5 - 3x^2 + 9x$
   e) $p(x) = 4 - 2x$
   f) $v(x) = -x^3 + 2x^4 - 4x^2$

You know what seems odd to me? Numbers that aren't divisible by two."



TB, page 112



Volume = l x w x h

Your Turn

A toaster oven is built in the shape of a rectangular prism. Its volume, V, in cubic inches, is related to the height, h, in inches, of the oven door by the function $V(h) = h^3 + 10h^2 + 31h + 30$.

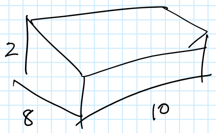
a) What is the volume, in cubic inches, of the toaster oven if the oven door height is 8 in.? let $h = 8$ $V(8) = 8^3 + 10(8)^2 + 31(8) + 30$ $= 1430 \text{ in}^3$

b) What is the height of the oven door for the least toaster oven volume? Explain.

$V(1) = 1^3 + 10(1)^2 + 31(1) + 30$
$= 1 + 10 + 31 + 30$
$= 72 \text{ in}^3$

$V(0) = 0^3 + 10(0)^2 + 31(0) + 30$
$V(0) = 30 \text{ in}^3$

Here's another volume question. Say we know a box is 8 cm wide, 10 cm long, and 2 cm high.
What is the volume of the box?



$V = l \times w \times h$

$V = 2 \times 8 \times 10$

$= 160\ cm^3$

What if we know the volume of a box? Can we find the dimensions?
In order to do that, we have to factor.......which in a way is like dividing.

Suppose Volume $= 30 cm^3$, and let's use dimensions that are whole numbers.

$1 \times 15 \times 2$
$1 \times 3 \times 10$ $\quad 2 \times 3 \times 5 \quad 1 \times 1 \times 30$
$1 \times 5 \times 6$

Given that the volume of a box is 30 cubic cm, what might the dimensions of the box be? Let's assume that each side length is a whole number.

What might the dimensions of a box be, if we know the volume is given by this polynomial?

$$V = x^3 + 2x^2 - 5x - 6$$

. . . we will learn!

To figure this out, we need to know how to factor a CUBIC polynomial.

Textbook, page 118

3.2

The Remainder Theorem

Focus on...

- describing the relationship between polynomial long division and synthetic division
- dividing polynomials by binomials of the form $x - a$ using long division or synthetic division
- explaining the relationship between the remainder when a polynomial is divided by a binomial of the form $x - a$ and the value of the polynomial at $x = a$

Whiteboard Activity: Long Division - remember that??

If possible, find a long-division "expert" to remind you how to do a long division question. There are a few shown on the screen.

How can you check whether your answer is correct?

$$\begin{array}{r} 119 \\ 7\overline{)833} \\ -7 \\ \hline 13 \\ -7 \\ \hline 63 \\ -63 \\ \hline 0 \end{array}$$

7×119

$= 833$

$6\overline{)5574}$

$$\begin{array}{r} 524 \\ 3\overline{)1574} \\ -15 \\ \hline 07 \\ -6 \\ \hline 14 \\ -12 \\ \hline 2 \end{array}$$

3×524

$+$ remainder 2

$= 1574$

$3\overline{)468}$ $\qquad$ $9\overline{)8272}$ $\qquad$ $2\overline{)1969}$

The basic process is shown in the first 50 seconds of this video:

https://www.youtube.com/watch?v=OK0ks0w8Kns

For next class:

- Complete: Factoring Practice Worksheet
- Practice ideas from tonight's class
    - TB (3.1) p 114: 1-3, 4ace, 6, 7, 9