Class_05 Jan 19 - Simplifying, Adding/Subtracting Radicals

Saturday, January 14, 2023 7:42 PM

Tonight's Class:

- **Recap #4**
- **Any questions from Chapter 1?**
- **Chapter 1 Test - closed book, but can use exponent rules foldable**
- **Working through sections 2.1 and 2.2**
  - **Simplifying radicals with variables**
  - **Adding and subtracting radicals**

$$\frac{x^m}{x^n} = x^{m-n}$$

p70, #7b)

$$\frac{(-4.5)^{3/4}}{(-4.5)^{-1/4}} = (-4.5)^{3/4-(-1/4)} = (-4.5)^{3/4+1/4} = (-4.5)^{4/4} = (-4.5)^1 = -4.5$$

$$-4.5 \times \frac{10}{10} = \frac{-45 \div 5}{10 \div 5} = \frac{-9}{2}$$

$-9 \div 2$

p75, 21 i)

$$\left(\frac{3}{4}\right)^{-1} \div \left(\frac{16}{9}\right)^{3/2} \cdot \left(\frac{4}{3}\right)^2$$

$$= \frac{4}{3}^{+1} \div \sqrt[2]{\frac{16}{9}}^{\,3} \cdot \frac{4^2}{3^2}$$

$$= \frac{4}{3} \div \left(\frac{4}{3}\right)^3 \cdot \frac{4^2}{3^2}$$

$$= \left(\frac{4}{3}\right)^1 \div \left(\frac{4}{3}\right)^3 \cdot \left(\frac{4}{3}\right)^2$$

$$= \left(\frac{4}{3}\right)^{1-3} \cdot \left(\frac{4}{3}\right)^2$$

$$= \left(\frac{4}{3}\right)^{-2} \cdot \left(\frac{4}{3}\right)^2 = \left(\frac{4}{3}\right)^{-2+2} = \left(\frac{4}{3}\right)^0 = 1$$

p76 22b)

$$\left(\frac{6^{\div 2} x^{3/4} y^{-2}}{4^{\div 2} x^{-1/4} y^{-3}}\right)^{-1} \cdot \left(\frac{2x^3 y}{3xy^{1/2}}\right)^2$$

$y^{1-1/2}$

$-2-(-3)$
$= -2+3$

$\frac{3}{4} - \left(-\frac{1}{4}\right)$
$= \frac{3}{4} + \frac{1}{4}$
$= \frac{4}{4}$

$$= \left(\frac{3x^{3/4-(-1/4)} y^{-2-(-3)}}{2}\right)^{-1} \cdot \left(\frac{2x^2 y^{1/2}}{3}\right)^2$$

$$= \left(\frac{3x^1 y^1}{2}\right)^{-1} \cdot \left(\frac{2x^2 y^{1/2}}{3}\right)^2$$

$= \frac{3}{4} + \frac{1}{4}$

$= \frac{4}{4}$

$= 1$

$$= \left(\frac{3x^{1}y^{1}}{2}\right)^{-1} \cdot \left(\frac{2x^{2}y^{1/2}}{3}\right)^{2}$$

$$= \left(\frac{2}{3xy}\right) \cdot \left(\frac{2^{2}x^{4}y^{\frac{1}{2}\cdot 2}}{3^{2}}\right)$$

$$= \left(\frac{2}{3xy}\right)\left(\frac{4x^{4}y}{9}\right) = \frac{8x^{4}\cancel{y}}{27x\cancel{y}} = \frac{8x^{3}}{27}$$

p70, #7d

$$(-0.125)^{2/3} \cdot (-0.125)^{-4/3}$$

$$= (-0.125)^{2/3 + -4/3}$$

$$= (-0.125)^{-2/3} = \frac{1}{(-0.125)^{2/3}} = \frac{1}{\sqrt[3]{-0.125}^{\,2}}$$

$$= \frac{1}{\sqrt[3]{-\frac{125}{1000}}^{\,2}}$$

$$= \frac{1}{\left(\frac{-5}{10}\right)^{2}}$$

$$= \frac{1}{(-\frac{1}{2})^{2}}$$

$$= \frac{1}{(\frac{1}{4})}$$

$$= 1 \div \frac{1}{4}$$

$$= 1 \times \frac{4}{1}$$

$$= 4$$

$$(-8)^{4/3} = \sqrt[3]{-8}^{\,4}$$

$$= (-2)^{4}$$

$$= 16$$

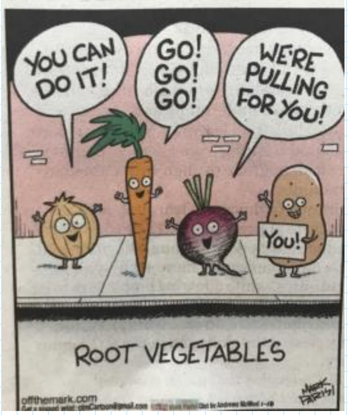
Please:

Make sure your name is on your <u>Chapter 1 Hand-in</u>, and turn it in.

Put away your phone and all materials except for the "foldable," a calculator, and something to write with.

On your test, write clearly and show all necessary steps - including on multiple-choice questions!
When you are finished, please look over your test before handing it in.

While other people are still finishing, respect them by being quiet. You can leave the classroom if you wish, but be back in time for the rest of class.



Preview 5

2.1 Simplifying Radical Expressions

Focus: simplify radical expressions with numerical or variable radicands

Arranging Radicals in Size Order

EX: ARRANGE THE FOLLOWING FROM LEAST TO GREATEST:

$6\sqrt{2}$,	$3\sqrt{7}$,	$2\sqrt{17}$,	$4\sqrt{5}$,	$2\sqrt{21}$
$=\sqrt{6^2\cdot 2}$	$\sqrt{3^2\cdot 7}$	$\sqrt{2^2\cdot 17}$	$\sqrt{4^2\cdot 5}$	$\sqrt{2^2\cdot 21}$
$=\sqrt{72}$	$=\sqrt{63}$	$=\sqrt{68}$	$=\sqrt{80}$	$=\sqrt{84}$
3	1	2	4	5

change each radical to entire form, it's MUCH easier to compare.

$3\sqrt{7}$, $2\sqrt{17}$, $6\sqrt{2}$, $4\sqrt{5}$, $2\sqrt{21}$

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Example 1 **Comparing and Ordering Radicals**

Arrange in order from least to greatest.

a) $9\sqrt{2}, 2\sqrt{6}, 8\sqrt{3}$

b) $7\sqrt[3]{3}, 3\sqrt[3]{3}, 8\sqrt[3]{3}$ $=$ all $\sqrt[3]{3}$ $3\sqrt[3]{3}, 7\sqrt[3]{3}, 8\sqrt[3]{3}$

c) $7\sqrt[3]{2}, 6\sqrt[4]{5}, 4\sqrt{5}$

if indices are not the same, use your calculator!

1 index
2 "indices"

$\sqrt[4]{5} = 5^{1/4}$

$6 \times (5)^{1/4} = 8.97209...$

Remember this:

Division Property

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

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Example 2 **Expressing Radicals in Different Forms**

a) Write $\sqrt[3]{-\frac{40}{81}}$ as a mixed radical.

b) Write $-2\sqrt[4]{\frac{3}{4}}$ as an entire radical.

a) $\sqrt[3]{\frac{-40}{81}} = \frac{\sqrt[3]{-40}}{\sqrt[3]{81}}$

$= \frac{\sqrt[3]{-8 \cdot 5}}{\sqrt[3]{27 \cdot 3}}$

$= \frac{\sqrt[3]{-2 \cdot -2 \cdot -2 \cdot 5}}{\sqrt[3]{3 \cdot 3 \cdot 3 \cdot 3}}$

$= \frac{-2\sqrt[3]{5}}{3\sqrt[3]{3}} = -\frac{2}{3}\sqrt[3]{\frac{5}{3}}$

1) factor each radicand

81 → 3, 27

40 → 4, 10 → 2, 2, 2, 5

40 → 8, 5

2) change to mixed form

Try CYU on p92

b) $-2\sqrt[4]{\frac{3}{4}}$ → to entire form

Index is even. Leave the negative OUT in front!

$= -\sqrt[4]{2^4 \cdot \frac{3}{4}}$

$= -\sqrt[4]{\frac{16}{1} \cdot \frac{3}{4}} \longrightarrow = -\sqrt[4]{\frac{48}{4}} = \boxed{-\sqrt[4]{12}}$

CYU b) $-3\sqrt[4]{\frac{2}{27}} = -\sqrt[4]{3^4 \cdot \frac{2}{27}}$

$= -\sqrt[4]{\frac{81}{1} \cdot \frac{2}{27}}$

$= -\sqrt[4]{\frac{162}{27}}$

$= -\sqrt[4]{\frac{54}{9}}$

$= -\sqrt[4]{6}$

When Are Radicals Defined?

- **Odd index**

  no restrictions, $x \in \mathbb{R}$

  x is an element of the real numbers

- **Even index**

  radicand must be ≥ 0

  $\sqrt[4]{x^3}$ where is this defined? $x^3 \geq 0$, $\sqrt[3]{x^3} \geq \sqrt[3]{0}$, $x \geq 0$

$\sqrt{x^2}$ where is this defined? $x \in \mathbb{R}$, because x^2 is always ≥ 0

$\sqrt{5x}$

radicand ≥ 0 (greater than or equal to)

$\frac{\cancel{5}x}{5} \geq \frac{0}{5}$

$x \geq 0$

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Example 3 Determining when a Radical Is Defined

For which values of the variable is each radical defined?

a) $\sqrt{54x^3}$ **b)** $\sqrt[3]{12x^5}$

We will also simplify each of these

index is even

radicand ≥ 0

$\frac{\cancel{54}x^3}{\cancel{54}} \geq \frac{0}{54}$

$\sqrt[3]{x^3} \geq \sqrt[3]{0}$

$x \geq 0$

index is odd

$x \in \mathbb{R}$

Simplify into mixed radical, if possible.

a) $\sqrt{54x^3}$, $x \geq 0$

$= \sqrt{9 \cdot 6 \cdot x^2 \cdot x}$

$= 3x\sqrt{6x}$

b) $\sqrt[3]{12x^5}$, $x \in \mathbb{R}$

$= \sqrt[3]{12 \cdot x^3 x^2}$

$= x\sqrt[3]{12x^2}$

$\boxed{\text{CYU p93}}$

a) $\sqrt{27x^2}$ $x \in \mathbb{R}$ (works for anything!!)

b) $\sqrt[4]{-12x^3}$ $x \leq 0$

radicand ≥ 0

$\frac{\cancel{-12}x^3}{\cancel{-12}} \geq \frac{0}{-12}$

$x^3 \leq 0$

$x \leq 0$

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Example 4 Simplifying Radicals with Variable Radicands

For which values of the variable is each radical defined?

Write as a mixed radical, if possible.

a) $\sqrt{75a^2}$ **b)** $\sqrt{18b^5}$ **c)** $\sqrt[3]{-15x}$ **d)** $\sqrt[4]{80e^7}$

a) $\sqrt{75a^2}$ $a \in \mathbb{R}$

$\sqrt{25 \cdot 3 \cdot a^2}$

b) $\sqrt{18b^5}$, $b \geq 0$

$= \sqrt{9 \cdot 2 \cdot (b^4) \cdot b}$

a) $\sqrt{75a^2} \quad a \in \mathbb{R}$

$= \sqrt{25 \cdot 3 \cdot a^2}$

$= 5a\sqrt{3}$

$= 5|a|\sqrt{3}$ or $5\sqrt{3}\,|a|$

b) $\sqrt{18b^5}, \quad b \geq 0$

$= \sqrt{9 \cdot 2 \cdot b^4 \cdot b}$

$= 3b^2\sqrt{2b}$

c) $\sqrt[3]{-15x} \quad x \in \mathbb{R}$

Can't reduce any further, because there aren't any perfect cube factors

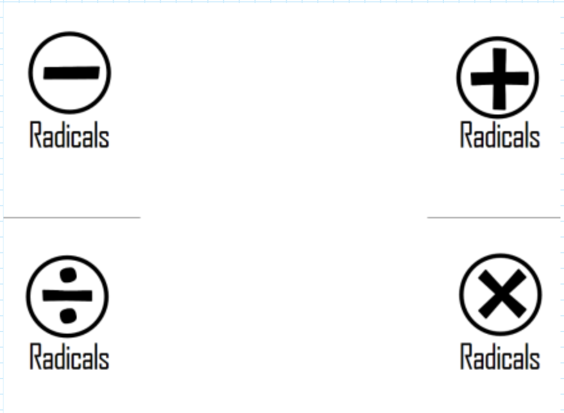
d) $\sqrt[4]{80e^7} \quad e \geq 0$

$= \sqrt[4]{16 \cdot 5 \cdot e^4 \cdot e^3}$

$= 2e\sqrt[4]{5e^3}$

2.2 Adding and Subtracting Expressions

Focus: simplify sums and differences of radical expressions



You may add radicals that have the same index and same radicand.

If this is the case, add the coefficients and keep the radicand the same.

You may subtract radicals that have the same index and same radicand.

If this is the case, subtract the coefficients and keep the radicand the same.

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Example 1 Simplifying Radical Expressions

Simplify.

a) $5\sqrt{6} - 2\sqrt{6}$

b) $\sqrt[3]{128} - \sqrt[3]{16} - \sqrt[3]{54}$

c) $\sqrt{20} + \sqrt{18} + \sqrt{45} - \sqrt{50}$

* Check to be sure that the index and radicand MATCH

a) $5\sqrt{6} - 2\sqrt{6} = 3\sqrt{6}$

b) $\sqrt[3]{128} - \sqrt[3]{16} - \sqrt[3]{54}$

Try to simplify fully, as this ... the

$= \sqrt[3]{64\cdot 2} - \sqrt[3]{8\cdot 2} - \sqrt[3]{27\cdot 2}$

possibly the radicand will match.

$= 4\sqrt[3]{2} - 2\sqrt[3]{2} - 3\sqrt[3]{2}$

$= -1\sqrt[3]{2}$ usually written as $-\sqrt[3]{2}$

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c) $\sqrt{20} + \sqrt{18} + \sqrt{45} - \sqrt{50}$

$= \sqrt{4\cdot 5} + \sqrt{9\cdot 2} + \sqrt{9\cdot 5} - \sqrt{25\cdot 2}$

$= 2\sqrt{5} + 3\sqrt{2} + 3\sqrt{5} - 5\sqrt{2}$

$= 5\sqrt{5} - 2\sqrt{2}$

if you need to, use a factor tree to help you out.

20

2 10

2 5

$\sqrt{2\cdot 2\cdot 5}$

$= 2\sqrt{5}$

For next class

- **Finish worktext questions for 2.1, and the ones in section 2.2 that relate to what we've already looked at tonight.**