Class_05 May10 Solving and Graphing Polynomials

Plan For Today:

1. Question about anything? Hand back Test 1 - please return to me in class.
2. Finish Chapter 3: Polynomial Functions
   - 3.1: Characteristics of Polynomial Functions (Review?)
   - 3.2: Dividing Polynomials (Finish)
   - 3.3: Factoring and Solving Polynomials
   - 3.4: Characteristics of Polynomials Graphs

5. Work on practice questions from Textbook

Page 124: #1-2, 3a, 4c, 5b, 6-8
Page 133: #1-4, 5ace, 7bd, 9, 11
Page 147: #1-2, 3ac, 4ac, 5, 9ae, 14, 16

Factor Completely Using Synthetic Division

$f(x) = 2x^4 + 5x^3 + 5x^2 + 20x - 12$ $\quad x = -3 \quad x = \frac{1}{2}$

-3	2	5	5	20	-12
	↓	-6	3	-24	12
½	2	-1	8	-4	0
	↓	1	0	4	
	2	0	8	0	

$$2x^4 + 5x^3 + 5x^2 + 20x - 12 = 2(x+3)\left(x - \frac{1}{2}\right)(x^2 + 4)$$

Plan Going Forward:

1. Finish going through practice question from 3.2-3.4 in textbook. Focus on completing the questions in the chapter 3 assignment.

   - **CHAPTER 3 ASSIGNMENT DUE MONDAY, MAY 15TH**

2. You will start Chapter 4 Trigonometry on Thursday (tomorrow). Have a look through these sections to prepare for tomorrow.

   - **TEST 2 ON MONDAY, MAY 15TH (ON CH3 & 4.1 FROM THURSDAY)**

Please let us know if you have any questions or concerns about your progress in this course. The notes from today will be posted at egolfmath.weebly.com after class.
Anurita Dhiman = adhiman@sd35.bc.ca
Susana Egolf = segolf@sd35.bc.ca

Review

$f(x) = x^4 - 4x^3 + 2x^2 + x + 4$	$f(x) = -x^5 + 6x^2 - 11x + 3$
1. Degree = 2. Leading coefficient = 3. End behavior = 4. Possible number of x-intercepts = 5. value of y-intercept = 6. domain = 7. range =	1. Degree = 2. Leading coefficient = 3. End behavior = 4. Possible number of x-intercepts = 5. value of y-intercept = 6. domain = 7. range =

Dividing Polynomials

Long Division

$$
\begin{array}{r|l}
 & 2x^2 \quad + x \quad - 5 \\
\hline
x-3 & 2x^3 - 5x^2 - 8x + 15 \\
 & \underline{2x^3 - 6x^2} \\
 & \qquad\quad x^2 - 8x \\
 & \qquad\quad \underline{x^2 - 3x} \\
 & \qquad\qquad\quad -5x + 15 \\
 & \qquad\qquad\quad \underline{-5x + 15} \\
\end{array}
$$

Remainder 0

Synthetic Division

$$
\begin{array}{c|cccc}
3 & 2 & -5 & -8 & 15 \\
 & & 6 & 3 & 15 \\
\hline
 & 2 & 1 & -5 & \big| \; 0
\end{array}
$$

Remainder

Section 3.2-3.3: Practice Dividing Polynomials

Do long division AND synthetic division for each of the following and write the division statement at the end.

1. $\left(x^3+7x^2+14x+3\right)\div\left(x+2\right)$

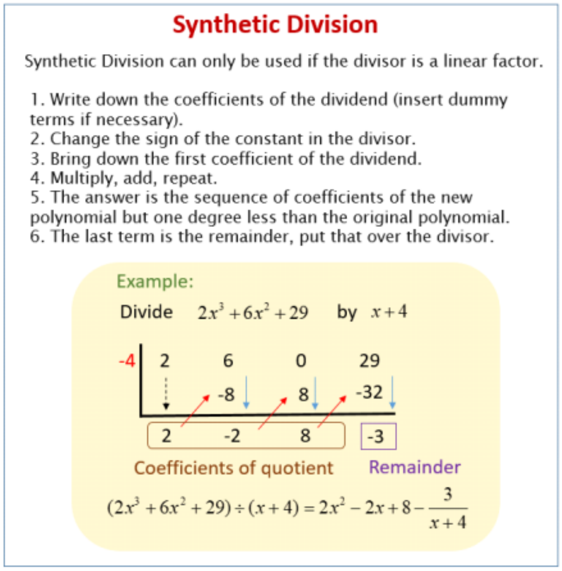
Long Division	Synthetic Division
$$\begin{array}{r} x^2 + 5x + 4 \\ x+2 \overline{\big) x^3+7x^2+14x+3} \\ -\ \ x^3+2x^2 \qquad\qquad \\ \hline 0 \quad 5x^2+14x \qquad \\ -\ \ 5x^2+10x \qquad \\ \hline 0 \quad 4x+3 \\ -\ \ 4x+8 \\ \hline 0 \quad -5 \end{array}$$ Remainder $= -5$ binomial $= x+2$ Remainder $= P(-2) = (-2)^3 + 7(-2)^2 + 14(-2) + 3$ verified ✓ $P(-2) = \boxed{-5}$	$$\begin{array}{c\|cccc} x+2 & x^3 & +7x^2 & +14x & +3 \\ 2 & 1 & 7 & 14 & 3 \\ - & & 2 & 10 & 8 \\ \hline x & 1 & 5 & 4 & -5 \end{array}$$ R $x^2 + 5x + 4 - \dfrac{5}{x+2}$

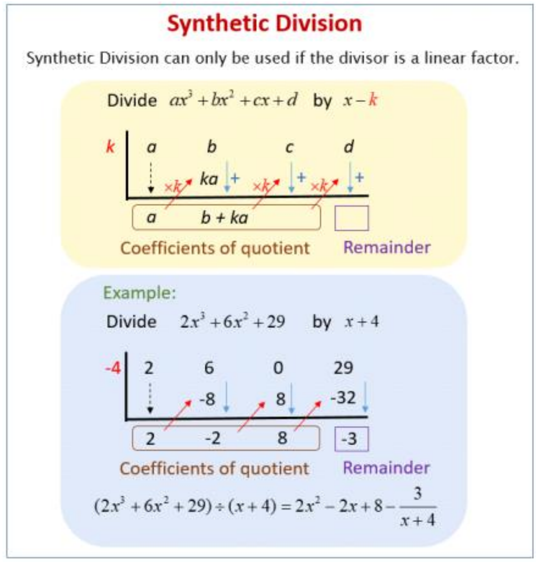
Division Statement:

$$\boxed{x^3+7x^2+14x+3 = x^2+5x+4-\frac{5}{x+2}}$$

OR

$$(x^3+7x^2+14x+3)(x+2) = (x^2+5x+4)(x+2)-5$$





Find the value of $P(-3)$, for $P(x)=2x^3+3x^2-5x+2$. $\rightarrow$ $2(-3)^3+3(-3)^2-5(-3)+2$

-3 -3 -3

$P(-3)=\boxed{-10}$

Remainder Theorem:
When a polynomial, *P(x)*, is divided by a binomial, $x-a$, the remainder is $P(a)$.

If $P(a)=0$, then the binomial $x-a$ ***is*** a factor of *P(x)*.
If $P(a)\neq 0$, then the binomial $x-a$ is ***not*** a factor of *P(x)*.

$x-2 \rightarrow P(2)$
$x+2 \rightarrow P(-2)$

Example
a) Use the Remainder Theorem to find the remainder when $P(x)=8x^3+4x^2-19$ is divided by $x+2$

0x

$P(-2)=8(-2)^3+4(-2)^2-19$
$=\boxed{-67}$

b) Check your answer by using synthetic division.

$$\begin{array}{c|cccc} 2 & 8 & 4 & 0 & -19 \\ - & & 16 & -24 & 48 \\ \hline \times & 8 & -12 & 24 & -67 \end{array}$$

$8x^2-12x+24 \quad -\frac{67}{x+2}$

c) Use the Remainder Theorem to find the remainder when $P(x)=8x^3+4x^2-19$ is divided by $x-1$.

$P(1)=8(1)^3+4(1)^2-19$
$=\boxed{-7}$

p.121

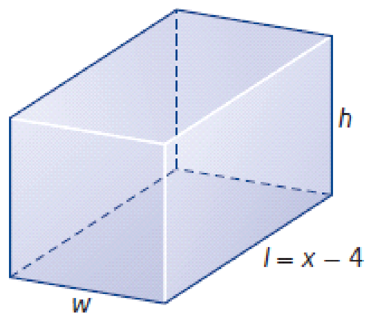
Example 2

Apply Polynomial Long Division to Solve a Problem

The volume, V, of the nested boxes in the introduction to this section, in cubic centimetres, is given by $V(x) = x^3 + 7x^2 + 14x + 8$. What are the possible dimensions of the boxes in terms of x if the height, h, in centimetres, is $x + 1$?

Your Turn

The volume of a rectangular prism is given by $V(x) = x^3 + 3x^2 - 36x + 32$. Determine possible measures for w and h in terms of x if the length, l, is $x - 4$.



p.121 $$\frac{x^3+3x^2-36x+32}{x-4} = wh$$

-4	1	3	-36	32
−		-4	-28	32
x	1	7	-8	0

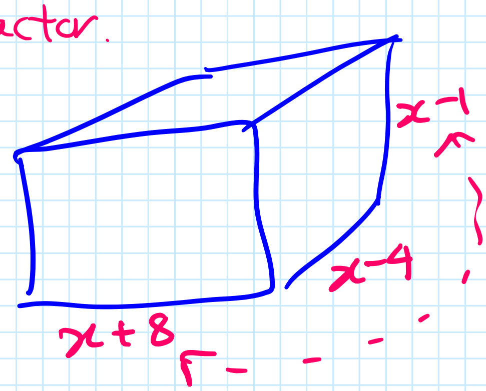
x4

$wh = x^2 + 7x - 8$ factor.

$wh = (x+8)(x-1)$

w h

h w



p.124

8. For each dividend, determine the value of k if the remainder is 3.

a) $(x^3 + 4x^2 - x + k) \div (x - 1)$

b) $(x^3 + x^2 + kx - 15) \div (x - 2)$

#5+6 in ch3 HW

a) $(x^3 + 4x^2 - x + k) \div (x - 1)$

b) $(x^3 + x^2 + kx - 15) \div (x - 2)$

c) $(x^3 + kx^2 + x + 5) \div (x + 2)$

d) $(kx^3 + 3x + 1) \div (x + 2)$

binomial $x - 1$

$$R = P(1) = 3$$

$$(1)^3 + 4(1)^2 - 1 + k = 3$$

$$\underbrace{1 + 4 - 1} + k = 3$$

$$4 + k = 3$$

$$-4 \qquad -4$$

$$\boxed{k = -1}$$

$x+2$

$$P(-2) = 3$$

$$(-2)^3 + k(-2)^2 + (-2) + 5 = 3$$

$$-8 + 4k - 2 + 5 = 3$$

$$4k - 5 = 3$$

$$+5 \qquad +5$$

$$4k = 8$$

$$/4 \qquad /4$$

$$\boxed{k = 2}$$

3.3 Factor Theorem

2. $(10x^3+37x^2+37x+6)\div(x+2)$

Synthetic Division

$x+2$ | $10x^3+37x^2+37x+6$

$$\begin{array}{c|cccc} 2 & 10 & 37 & 37 & 6 \\ - & & 20 & 34 & 6 \\ \hline \times & 10 & 17 & 3 & 0 \end{array}$$

← Remainder

$\underset{A}{10x^2}+\underset{B}{17x}+\underset{C}{3}$ AC = 30

$10x^2+15x+2x+3$ 15, 2 → 17 (B)

Notice you can factor the quadratic further:

$5x(2x+3)+1(2x+3)$

$(5x+1)(2x+3)$

In this case this polynomial can be fully factored to the following:

$(x+2)(5x+1)(2x+3) = 10x^3+37x^2+37x+6$

R = 0 b/c it was a factor

∴ you can factor this polynomial.

divide $2x^3 + 3x^2 - 4x + 15$ by $x + 3$

$$\begin{array}{c|cccc} & 2x^3 & 3x^2 & -4x & 15 \\ +3 & 2 & 3 & -4 & 15 \\ - & & & & \\ \hline \times & & & & \end{array}$$

$$\begin{array}{c|cccc} +3 & 2 & 3 & -4 & 15 \\ - & & 6 & -9 & 15 \\ \hline \times & 2 & -3 & 5 & 0 \end{array}$$

remainder

$(2x^3 + 3x^2 - 4x + 15) \div (x + 3) = 2x^2 - 3x + 5$

Restriction: $x + 3 \neq 0$ or $x \neq -3$

Remainder Theorem

If a polynomial f(x) is divided by (x – a), the remainder is f(a).

f(x) = (x – a) Q(x) + f(a)

Factor Theorem

A polynomial f(x) has a factor (x – a) if and only if f(a) = 0.

Synthetic Division

Synthetic Division can only be used if the divisor is a linear factor.

Divide ax^3+bx^2+cx+d by $x-k$

$$\begin{array}{c|cccc} k & a & b & c & d \\ & & ka & & \\ \hline & a & b+ka & & \boxed{} \end{array}$$

Coefficients of quotient — Remainder

Example:

Divide $2x^3+6x^2+29$ by $x+4$

$$\begin{array}{c|cccc} -4 & 2 & 6 & 0 & 29 \\ & & -8 & 8 & -32 \\ \hline & 2 & -2 & 8 & \boxed{-3} \end{array}$$

Coefficients of quotient — Remainder

$(2x^3+6x^2+29)\div(x+4)=2x^2-2x+8-\dfrac{3}{x+4}$

x − a

P(a) = 0

means

x − a IS a factor.

3.3 The Factor Theorem

For each binomial below, find the remainder when $P(x)=x^3-4x^2+x+6$ is divided by the binomial. Which of the following binomials are factors of $P(x)=x^3-4x^2+x+6$?

a) $x+1$ $P(-1)=(-1)^3-4(-1)^2+(-1)+6$ $= 0$ YES $x+1$ is a factor of the polynomial

d) $x-1$ $P(1)=(1)^3-4(1)^2+1+6$ $= 4$ NO $x-1$ is not a factor

b) $x+2$ $P(-2) = -20$ NO

e) $x-2$ $P(2) = 0$ YES.

c) $x+3$ $P(-3) = -60$ NO

f) $x-3$ $P(3) = 0$ YES.

Factor Theorem: $x-a$ is a factor of a polynomial, $P(x)$, if and only if $P(a)=0$

Example

Consider the polynomial: $P(x)=x^3+7x^2-28x+20$.

a) Given that $P(-10)=0$, what binomial must be a factor of $P(x)$? → factor is $(x+10)$

b) Factor $P(x)$ completely.

Integral Zero Theorem:
If $x - a$ is a factor of a polynomial with integral coefficients, $P(x)$, then a must divide evenly into the ***constant term*** of the polynomial $P(x)$.

The Zeros or Roots of a Polynomial Function

Graphically: the real zeros or real roots of a polynomial function are the x-intercepts of the graph.

Rational Zeros Theorem: If a polynomial function has rational zeros, they will be a ratio of the factors of the constant to the factors of the leading coefficient.

Example: $f(x) = 2x^3 - 9x^2 + 7x + 6$

$6 : \pm1, \pm2, \pm3, \pm6$
$2 : \pm1, \pm2$
$\pm1, \pm2, \pm3, \pm6$

The relationship between the factors of a polynomial and the constant term of the polynomial is stated in the **integral zero theorem**.

The integral zero theorem states that if $x - a$ is a factor of a polynomial function $P(x)$ with integral coefficients, then a is a factor of the constant term of $P(x)$.

integral zero theorem

- if $x = a$ is an integral zero of a polynomial, $P(x)$, with integral coefficients, then a is a factor of the constant term of $P(x)$

Example
Factor fully without using technology: $2x^3 - 5x^2 - 4x + 3$ $P(x)$

a) According to the integral zero theorem, which values could possibly give factors of this polynomial?

☆ look at constant term → 3, determine factors of 3
$\pm 1, \pm 3$ possible factors.

b) Use the remainder and factor theorems to find a factor.

TEST factors + determine which one gives a remainder of zero

NO $P(1) = 2(1)^3 - 5(1)^2 - 4(1) + 3 = \boxed{-4}$ $P(3)$ = skip

YES $P(-1) = \boxed{0}$ $P(-3)$ = skip.

c) Use either long division or synthetic division to divide $2x^3 - 5x^2 - 4x + 3$ by the factor found in part (b).

$x+1$

$$\begin{array}{r|rrrr} 1 & 2 & -5 & -4 & 3 \\ - & & 2 & -7 & 3 \\ \hline \times & 2 & -7 & 3 & 0 \end{array}$$

$2x^2 - 7x + 3$ AC = 6 → $-6, -1$ → -7
$2x^2 - 6x - x + 3$
$2x(x-3) - (x-3)$
$(x-3)(2x-1)$

d) What is the fully factored form of $2x^3 - 5x^2 - 4x + 3$?

$(x+1)(x-3)(2x-1)$

e) Consider the equation $2x^3 - 5x^2 - 4x + 3 = 0$, what are the solutions to this equation?

$(x+1)(x-3)(2x-1) = 0$

$x + 1 = 0$, $x = -1$ $x - 3 = 0$, $x = 3$ $2x - 1 = 0$, $2x = 1$, $x = \frac{1}{2}$

$x = -1, 3, \frac{1}{2}$

f) Graph $2x^3 - 5x^2 - 4x + 3$ on a graphing calculator. Find the values of its x-intercepts.

x-intercepts
$(-1,0), (3,0), (\frac{1}{2},0)$

#8 ch3 assign.

Your Turn

p.131 What is the fully factored form of $x^4 - 3x^3 - 7x^2 + 15x + 18$?

Your Turn

p.131 What is the fully factored form of $x^4 - 3x^3 - 7x^2 + 15x + 18$?

since x^4 find TWO factors by testing

$\pm1, \pm2, \pm3, \pm6, \pm9, \pm18$

#8 _ _ _ _ +45 $\quad \pm1, \pm3, \pm5, \pm9, \pm15, \pm45.$

p. 131 YourTurn

$$x^4 - 3x^3 - 7x^2 + 15x + 18$$

notice 4th degree = need synthetic division twice to get to x^2 ∴ find TWO binomial factors.

$\pm1, \pm2, \pm3, \pm6, \pm9, \pm18$

Test $P(1) = 24 \qquad (1)^4 - 3(1)^3 - 7(1)^2 + 15(1) + 18$

$P(-1) = 0$

$P(3) = 0$

$P(-3) =$ ~~~

$x+1 \qquad x-3$

① $x+1$

	$1x^4$	-3	-7	15	18
$-$		1	-4	-3	18
$\times$	1	-4	-3	18	0 ✓

$$x^3 - 4x^2 - 3x + 18$$

② $x-3$

	$1x^3$	-4	-3	18
$-$		-3	3	18
$\times$	1	-1	-6	0 ✓

$$x^2 - x - 6 \quad \text{factor}$$

$$(x-3)(x+2)$$

Fully Factored form = collect all binomials

$$\boxed{(x+1)(x-3)(x-3)(x+2) \quad \text{OR} \quad (x+1)(x-3)^2(x+2)}$$

Solve? $\boxed{x = -1, 3, 2}$

Example 4

Solve Problems Involving Polynomial Expressions

An intermodal container that has the shape of a rectangular prism has a volume, in cubic feet, represented by the polynomial function $V(x) = x^3 + 7x^2 - 28x + 20$, where x is a positive real number.

What are the factors that represent possible dimensions, in terms of x, of the container?

Your Turn

A form that is used to make large rectangular blocks of ice comes in different dimensions such that the volume, V, in cubic centimetres, of each block can be modelled by $V(x) = x^3 + 7x^2 + 16x + 12$, where x is in centimetres. Determine the possible dimensions, in terms of x, that result in this volume.

Graphing a cubic function with transformations: follow the steps for transformations you learned in Chapter 1.

Graphing Polynomial Functions using Transformations

The graph of a function of the form $y = a(b(x - h))^n + k$ is obtained by applying transformations to the graph of the general polynomial function $y = x^n$, where $n \in \mathrm{N}$. The effects of changing parameters in polynomial functions are the same as the effects of changing parameters in other types of functions.

Your Turn

Transform the graph of $y = x^3$ to sketch the graph of $y = -4(2(x + 2))^3 - 5$.

ch3 assign.

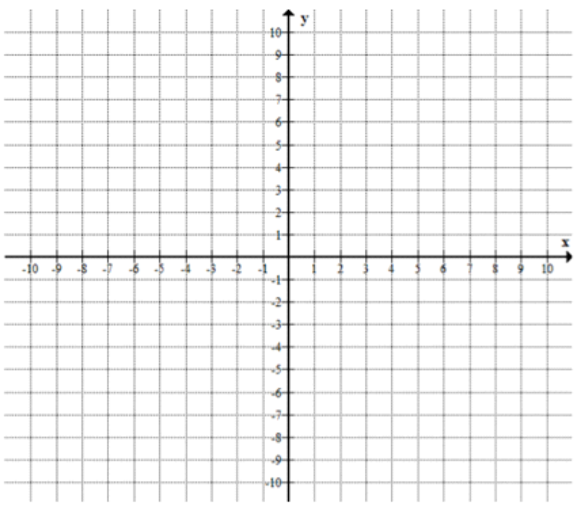
11a) Give the mapping that shows what happens to points on the base function, $y = x^3$, when the equation is changed to $y=\frac{1}{2}\left(-\frac{1}{2}x-1\right)^3-2$

Factor $y = \frac{1}{2}\left(-\frac{1}{2}(x+2)\right)^3 - 2$

b) Fill in the table of key points for the base function. Using the mapping, create the table of image points that result when the original points are transformed. Sketch the transformed graph on the grid.

$y = x^3$

x	y
-2	-8
-1	-1
0	0
1	1
2	8



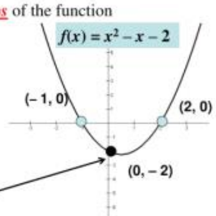
INTERCEPTS AND ZEROS

To find the x-intercepts of $y = f(x)$, set $y = 0$ and solve for x.
x-intercepts correspond to the *zeros* of the function

$x^2 - x - 2 = 0$

$(x + 1)(x - 2) = 0$

$x = -1 \qquad x = 2$



To find the y-intercepts of $y = f(x)$, set $x = 0$; the y-intercept is $f(0)$.

$f(0) = (0)^2 - 0 - 2$
$= -2$

1

Multiplicity is the number of times an x-intercept occurs in a graph:

Roots

Multiplicity of roots

Single root — Example: Factor (x+2)
root: -2
graph goes straight through root

double root — Example: Factor $(x-1)^2$
root: 1 (M2)
graph goes through the root like a quadratic

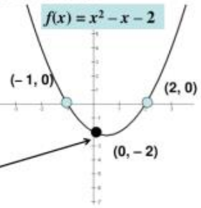
To find the x-intercepts of $y = f(x)$, set $y = 0$ and solve for x.

x-intercepts correspond to the *zeros* of the function

$x^2 - x - 2 = 0$

$(x + 1)(x - 2) = 0$

$x = -1 \qquad x = 2$



To find the y-intercepts of $y = f(x)$, set $x = 0$; the y-intercept is $f(0)$.

$f(0) = (0)^2 - 0 - 2$
$= -2$

1

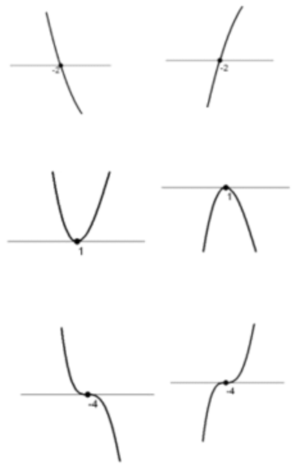
Roots

Multiplicity of roots

Single root — Example: Factor (x+2)
root: -2
graph goes straight through root

double root — Example: Factor $(x-1)^2$
root: 1 (M2)
graph goes through the root like a quadratic

triple root — Example: Factor $(x+4)^3$
root: -4 (M3)
graph goes through the root like a cubic



If you factor the polynomial, you can see the value of the x-intercepts (roots) and the corresponding multiplicity

Real roots are x-intercepts.

To find the roots, we let y = 0 and solve for x.

Example: Find the roots for the following.

$y = (x+5)(x-3)(2x+4)$ Roots: -5, 3, -2

$y = x(x-4)^2(3x+1)$ Roots: 0, 4 (M2), $-\frac{1}{3}$

$y = x^2(3-x)(x+4)^3$ Roots: 0 (M2), 3, -4 (M3)

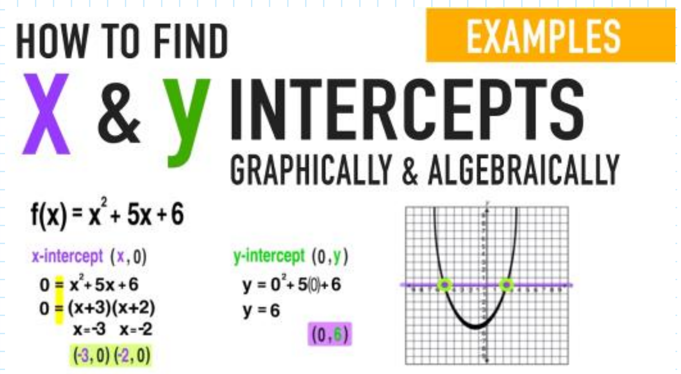
To find the y-intercept, make x=0 and solve for y:

Example: Find the y-intercept for each

$y = (x+5)(x-3)(2x+4)$ y-intercept: (0, -60)

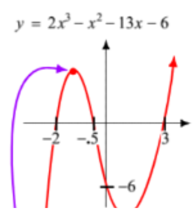
$y = -x^4 + 2x^3 - x^2 + 3x + 20$ y-intercept: (0, 20)

$y = x^2(3-x)(x+4)^3$ y-intercept: (0,0)

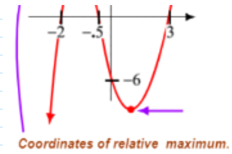


To graph a polynomial function:

1. Determine the y-intercept by making x=0
2. Determine the x-intercepts by solving the polynomial when y=0
(or you can use your graphing calculator to find these characteristics)
3. Use the degree and leading coefficient to determine behaviour.
4. Use the table of values to estimate the relative and/or



3. Use the degree and leading coefficient to determine behaviour.
4. Use the table of values to estimate the relative and/or absolute Max's and Min's and get other points.
5. Draw the curve with the correct behaviour.



3.4 Equations and Graphs of Polynomial Functions

As we just saw, the solutions of an equation match up with the x-intercepts of the graph.

Example

a) Graph the function $f(x) = x^4 + x^3 - 10x^2 - 4x + 24$ ~~using graphing technology.~~ What are its x-intercepts?

Integral zero theorem

b) Use the results from part (a) to help fully factor $f(x) = x^4 + x^3 - 10x^2 - 4x + 24$.

$\pm 1, \pm 2, \pm 3$
$\pm 4, \pm 6,$
$\pm 8, \pm 12$
± 24

$P(2) = 0$ $\quad$ $P(-2) = 0$

① $x-2$ $\quad$ ② $x+2$

① $x-2$ | $x^4 + x^3 - 10x^2 - 4x + 24$

−2	1	1	−10	−4	24
−		−2	−6	8	24
×	1	3	−4	−12	0 ✓

② $x+2$

	1	3	−4	−12
2				
−		2	2	−12
×	1	1	−6	0 ✓

$x^2 + x - 6$

$(x+3)(x-2)(x+2)(x-2)$

$f(x) = (x+3)(x+2)(x-2)^2$ multiplicity of 2

c) What are the solutions to the equation: $x^4 + x^3 - 10x^2 - 4x + 24 = 0$

$x = -3, -2, 2$

these are x-intercepts.

Example
Factor completely, then analyze and sketch the graph of this polynomial function ***without*** using technology: $f(x) = x^3 + 3x^2 - 6x - 8$

$\pm 1, \pm 2, \pm 4, \pm 8$
$P(1) = (1)^3 + 3(1)^2 - 6(1) - 8$
$= -10$
$P(-1) = 0$

$x+1$

$$\begin{array}{c|cccc} 1 & 1 & 3 & -6 & -8 \\ - & \downarrow & 1 & 2 & -8 \\ \hline x & 1 & 2 & -8 & 0 \end{array}$$

$x^2 + 2x - 8$
$(x-2)(x+4)$

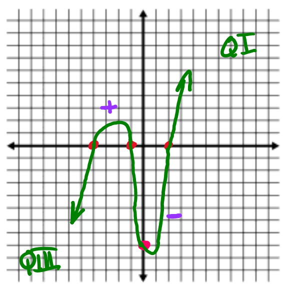
y-int = -8
(0, -8)

collect all factors
$(x+1)(x-2)(x+4) = 0$

solutions = x-intercepts
$x = -1, 2, -4$ ← Multiplicity = 1 ∴ draw line straight through x-intercepts

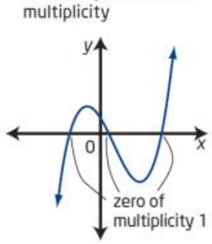
$x^3 \rightarrow$ behavior ↙ QIII ↗ QI

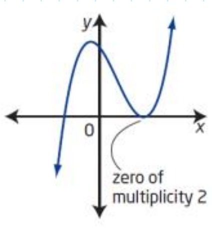
Degree	
Leading coefficient	
End behavior	
Zeros/x-intercepts	
y-intercept	
Interval(s) where function is positive or negative	$+ \ -4 < x < 1$ $- \ -1 < x < 2$

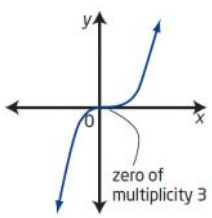


If a polynomial has a factor $x - a$ that is repeated n times, then $x = a$ is a zero of **multiplicity**, n. The function $f(x) = (x - 1)^2(x + 2)$ has a zero of multiplicity 2 at $x = 1$ and the equation $(x - 1)^2(x + 2) = 0$ has a root of multiplicity 2 at $x = 1$.

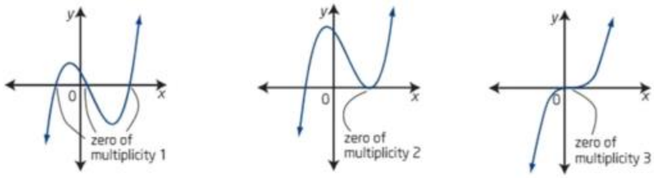
multiplicity (of a zero)
- the number of times a zero of a polynomial function occurs
- the shape of the graph of a function close to a zero depends on its multiplicity



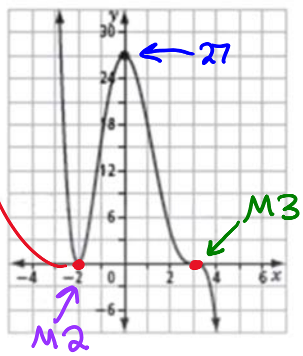




Multiplicity of a zero – the multiplicity of a zero is the number of times a zero of a polynomial occurs.



Example
Find the equation for the given graph.



① x-intercepts + multiplicity

$x=-2$, $x=3$

② write as binomials

M2 $(x+2)^2$ $(x-3)^3$ M3

③ use another point on graph to solve for 'a' by substitution

y-int = (0, 27)

$$y=a(x+2)^2(x-3)^3$$

$$27=a(0+2)^2(0-3)^3$$

$$27=a(4)(-27)$$

$$\frac{27}{-108}=\frac{-108a}{-108} \rightarrow a=-\frac{1}{4}$$

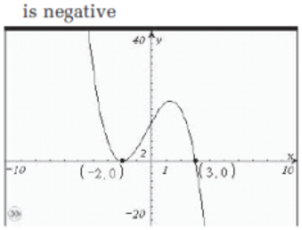
④ Final Equation

$$\boxed{y=-\frac{1}{4}(x+2)^2(x-3)^3}$$

Your Turn

For the graph of the polynomial function shown, determine

- the least possible degree
- the sign of the leading coefficient
- the x-intercepts and the factors of the function of least possible degree
- the intervals where the function is positive and the intervals where it is negative



Your Turn

Sketch a graph of each polynomial function by hand. State the characteristics of the polynomial functions that you used to sketch the graphs.

a) $g(x) = (x - 2)^3(x + 1)$

b) $f(x) = -x^3 + 13x + 12$

C_05 Key and Poly Graph Equations WS

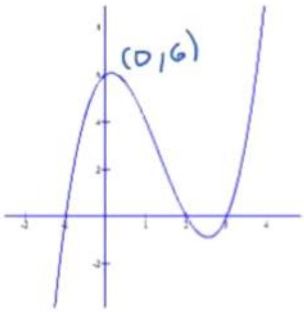
Wednesday, September 22, 2021 7:16 PM

For each polynomial graph, determine

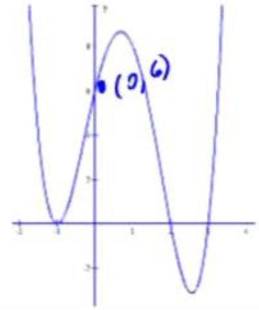
- **Coordinates of its x-intercepts**
- **Coordinates of its y-intercept**
- **Equation of the polynomial, in factored form**

Polynomial Graphs

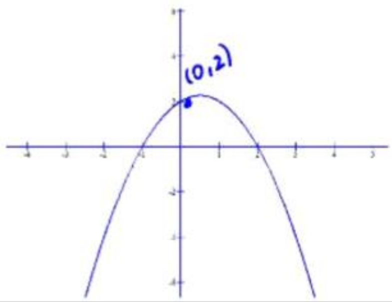
#1



#2



#3



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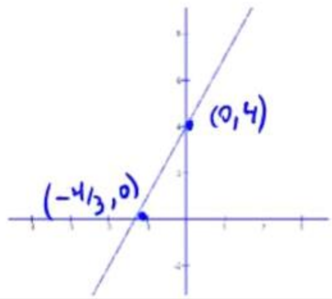
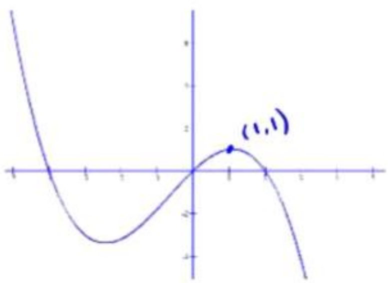


#5



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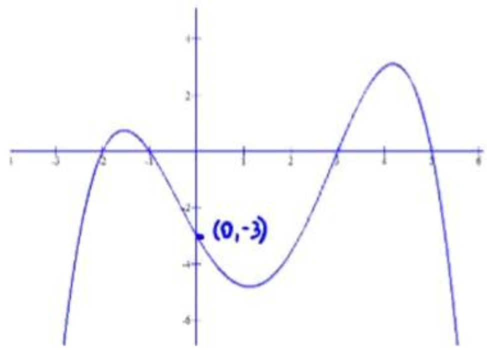

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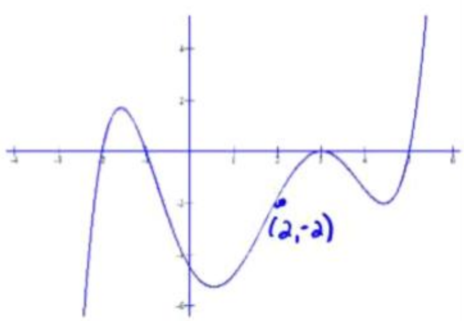
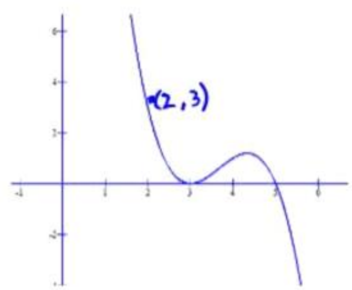
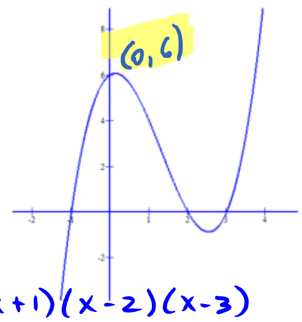


#8



Polynomial Graphs Key

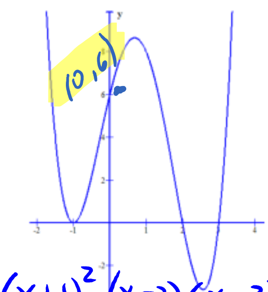
#1



odd
+
x-int
(-1,0)
(2,0)
(3,0)
y-int
(0,6)

$y=(x+1)(x-2)(x-3)$

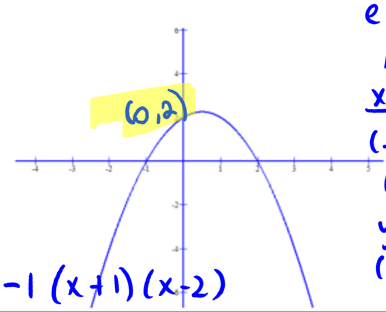
#2



even
+
x-int
(-1,0)
(2,0)
(3,0)
y-int
(0,6)

$y=(x+1)^2(x-2)(x-3)$

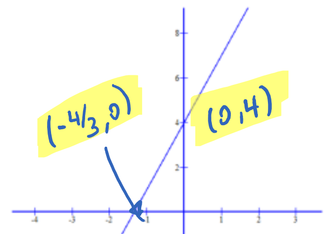
#3



even
NEG
x-int
(-1,0)
(2,0)
y-int
(0,2)

$y=-1(x+1)(x-2)$

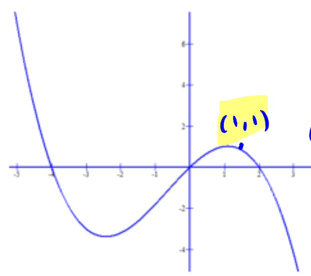
#4



ODD
+
x-int
$(-4/3,0)$
y-int
(0,4)

$y=3(x+4/3)$ OR $y=3x+4$

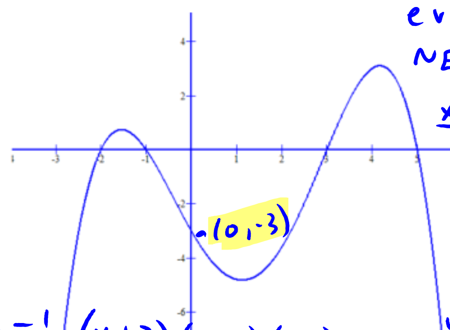
#5



ODD
NEG
x-int
(-4,0)
(0,0)
(2,0)
y-int
(0,0)

$y=-\frac{1}{5}x(x+4)(x-2)$

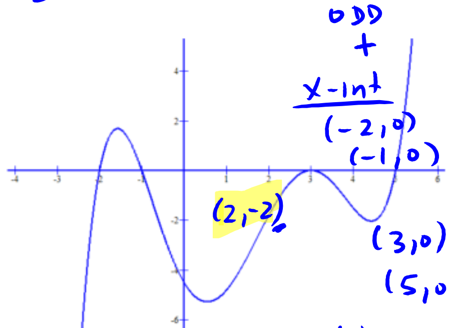
#6



even
NEG
x-int
(-2,0)
(-1,0)
(3,0)
(5,0)
y-int
(0,-3)

$y=-\frac{1}{10}(x+2)(x+1)(x-3)(x-5)$

#7

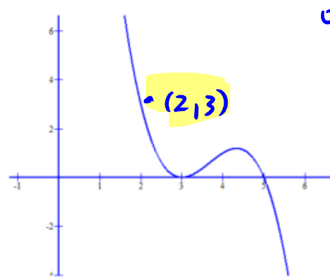


ODD
+
x-int
(-2,0)
(-1,0)
(3,0)
(5,0)
y-int
?

$y=\frac{1}{18}(x+2)(x+1)(x-3)^2(x-5)$

(see below for y-intercept)

#8



ODD
NEG
x-int
(3,0)
(5,0)
y-int
?

$y=-(x-3)^2(x-5)$

(see below for y-intercept)

$y = \frac{1}{18}(x+2)(x+1)(x-3)^2(x-5)$

Let $x = 0$, to find y-intercept:

$y = \frac{1}{18}(2)(1)(-3)^2(-5)$

$y = \frac{1}{18}(2)(9)(-5)$

$y = -5$

$(0, -5)$

Let $x = 0$

$y = -(-3)^2(-5)$

$= -(9)(-5)$

$= 45$

$(0, 45)$