Class_06 Jan 24 - Multiplying and Dividing Radicals

Saturday, January 14, 2023 7:43 PM

Tonight's Class:
- **Chapter 1 Test Return**
- **Any questions from 2.1-2.2?**
- **Working through sections 2.2 and 2.3**
  - **Adding and subtracting radicals (continued)**
  - **Multiplying and dividing radicals**

p 96, #8d

$$\sqrt[3]{\frac{-48^{\div 2}}{54^{\div 2}}} = \sqrt[3]{\frac{-24^{\div 3}}{27^{\div 3}}}$$

$$= \sqrt[3]{\frac{-8}{9}}$$

$$= \frac{\sqrt[3]{-8}}{\sqrt[3]{9}}$$

$$= \frac{-2}{\sqrt[3]{9}} = -2 \cdot \sqrt[3]{\frac{1}{9}}$$

$$= -2\,\frac{\sqrt[3]{1}}{\sqrt[3]{9}} = \frac{-2 \cdot 1}{\sqrt[3]{9}}$$

p98 # 11a) $\sqrt[3]{\frac{27x^2}{16}}$ $x \in \mathbb{R}$

odd index

$$= \sqrt[3]{\frac{3^3 \cdot x^2}{8 \cdot 2}} = \sqrt[3]{\frac{3^3 \cdot x^2}{2^3 \cdot 2}} = \frac{3}{2}\sqrt[3]{\frac{x^2}{2}}$$

e) $\sqrt[3]{\frac{-27m^8}{16}}$ odd index

$$\xrightarrow{m \in \mathbb{R}} \sqrt[3]{\frac{-27m^6 \cdot m^2}{8 \cdot 2}} = \frac{-3m^2}{2}\sqrt[3]{\frac{m^2}{2}}$$

p100, #15

Determine the radii of sphere C and sphere D, in terms of A.

B

SA = "A" square units

$\boxed{\text{Surface area of sphere } A = 4\pi r^2}$

SA = "A" square units

r_c

C: SA is twice that of sphere B = 2A

D: SA is 9 times the SA of sphere B = 9A

Surface area of sphere $A = 4\pi r^2$

Sphere C, its surface area = 2A

$$\frac{\cancel{4\pi}(r_c)^2}{\cancel{4\pi}} = \frac{2A}{4\pi}$$

$$\sqrt{(r_c)^2} = \sqrt{\frac{2A}{4\pi}}$$

$$r_c = \sqrt{\frac{2A}{4\pi}} \qquad \frac{2}{4} = \frac{1}{2}$$

$$r_c = \sqrt{\frac{A}{2\pi}}$$

Summary:

$\sqrt[odd]{7x^2y^3}$ — even, odd

$x \in \mathbb{R}$, $y \in \mathbb{R}$

$\sqrt[even]{5x^2y^3}$ — even, odd

$x \in \mathbb{R}$

$y \geq 0$

If the index is even, variables in the radicand that have an ODD exponent will be defined only in some places, not for all real numbers.

- **By "some places" I mean either for values ≥ 0, or, sometimes, for values ≤ 0**

Radicals

Radicals

Radicals

Radicals

You may add radicals that have the same index and same radicand. If this is the case, add the coefficients and keep the radicand the same.	$2\sqrt{3}+6\sqrt{2}+4\sqrt{3}$ $=6\sqrt{3}+6\sqrt{2}$ $\sqrt[3]{x^2}+\sqrt[3]{27x^2}$ $=1\sqrt[3]{x^2}+3\sqrt[3]{x^2}$ $=4\sqrt[3]{x^2}$	$12\sqrt[3]{10}-14\sqrt[3]{10}$ $=-2\sqrt[3]{10}$ $\sqrt{5x}-\sqrt{20x}+3\sqrt{x}$ $=\sqrt{5x}-\sqrt{4\cdot 5x}+3\sqrt{x}$ $=\sqrt{5x}-2\sqrt{5x}+3\sqrt{x}$ $=-\sqrt{5x}+3\sqrt{x}$	You may subtract radicals that have the same index and same radicand. If this is the case, subtract the coefficients and keep the radicand the same.

WT, page 104

Example 2 Simplifying Expressions with Variable Radicands

Identify the values of the variables for which each radical is defined, then simplify.

a) $5\sqrt{x}+3\sqrt{x}-4\sqrt{x}$

b) $\sqrt{25a^2b}+\sqrt{4a^2b}$

c) $\sqrt[4]{81p^3q^5}-2\sqrt[4]{p^3q^5}$

restrictions

a) $x\geq 0$ $\quad (5+3-4)\sqrt{x}=4\sqrt{x}$

b) $a\in\mathbb{R}$, $b\geq 0$

$\sqrt{25a^2b}+\sqrt{4a^2b}$

$=\sqrt{25a^2\cdot b}+\sqrt{4a^2\cdot b}$

$=5|a|\sqrt{b}+2|a|\sqrt{b}$

$=7|a|\sqrt{b}$

these are needed, so we can guarantee that whats taken outside of the $\sqrt{\ }$ is positive

even index

c) $\sqrt[4]{81p^3q^5}-2\sqrt[4]{p^3q^5}$

$p\geq 0$ $\quad q\geq 0$

OR $p<0$ and $q<0$

$=\sqrt[4]{81q^4\cdot p^3\cdot q}-2\sqrt[4]{q^4\cdot qp^3}$

$= 3q\sqrt[4]{p^3q} - 2\cdot q\sqrt[4]{p^3q}$

$= |q|\sqrt[4]{p^3q}$

$\boxed{\text{CYU, p 104}}$

WT, page 105

Example 3 **Simplifying with More than One Set of Like Terms**

Simplify.

a) $2\sqrt{x} - 3\sqrt{y} + 5\sqrt{x} + 2\sqrt{y}$, $x, y \geq 0$

b) $8\sqrt[3]{2x} + 7\sqrt{2x} - 5\sqrt[3]{2x} + \sqrt{2x}$, $x \geq 0$

c) $5\sqrt{8x^3} + 4y\sqrt{75y^3} - 2\sqrt{27y^5} - 3x\sqrt{50x}$, $x, y \geq 0$

a) $7\sqrt{x} - 3\sqrt{y} + 2\sqrt{y} = 7\sqrt{x} - \sqrt{y}$

b) $3\sqrt[3]{2x} + 8\sqrt{2x}$

c) $5\sqrt{8x^3} + 4y\sqrt{75y^3} - 2\sqrt{27y^5} - 3x\sqrt{50x}$

$= 5\sqrt{4\cdot 2\cdot x^2\cdot x} + 4y\sqrt{25\cdot 3\cdot y^2\cdot y} - 2\sqrt{9\cdot 3\cdot y^4\cdot y} - 3x\sqrt{25\cdot 2\cdot x}$

$= 5\cdot 2\cdot x\sqrt{2x} + 4y\cdot 5\cdot y\sqrt{3y} - 2\cdot 3\cdot y^2\sqrt{3y} - 3x\cdot 5\sqrt{2x}$

$= 10x\sqrt{2x} + 20y^2\sqrt{3y} - 6y^2\sqrt{3y} - 15x\sqrt{2x}$

$= (10x - 15x)\sqrt{2x} + (20y^2 - 6y^2)\sqrt{3y}$

$= -5x\sqrt{2x} + 14y^2\sqrt{3y}$

Recap 5 - try it now

Precap 6

2.3 Multiplying and Dividing Radical Expressions

Focus: simplify products and quotients of radical expressions

Multiplication Property of Radicals:

for $a, b \geq 0$

$\sqrt{ab} = \sqrt{a} \times \sqrt{b}$

Multiply Radicals

1. Multiply the coefficients.
2. Multiply the numbers inside the radicals.
3. Simplify.

Example:

for $a, b \geq 0$

$$\sqrt[n]{ab} = \sqrt[n]{a} \times \sqrt[n]{b}$$

3. Simplify.

Example:

$\sqrt{6} \times \sqrt{15}$
$= \sqrt{6 \times 15}$
$= \sqrt{90}$
$= \sqrt{9 \times 10}$
$= 3\sqrt{10}$

$3\sqrt{2} \times 5\sqrt{6}$
$= 3 \times 5\sqrt{2 \times 6}$
$= 15\sqrt{12}$
$= 15\sqrt{2 \times 2 \times 3}$
$= 15 \times 2\sqrt{3}$
$= 30\sqrt{3}$

You may multiply radicals that have the same index.

If this is the case, multiply the coefficients and multiply the radicands.

Simplify.

Examples

1) $\sqrt{6} \times \sqrt{2} = \sqrt{12}$
$= \sqrt{4 \cdot 3}$
$= 2\sqrt{3}$

2) $3\sqrt{6} \times 5\sqrt{3}$
$= 15\sqrt{18}$
$= 15\sqrt{9 \cdot 2}$
$= 15 \cdot 3\sqrt{2}$
$= 45\sqrt{2}$

3) $4\sqrt{3xz} \cdot 5\sqrt{6xy^2}$
$= 20\sqrt{3xz \cdot 6xy^2}$
$= 20\sqrt{18x^2y^2z}$
$= 20\sqrt{9 \cdot 2 \cdot x^2 \cdot y^2 \cdot z}$ simplify
$= 20 \cdot 3xy\sqrt{2z} = 60x|y|\sqrt{2z}$

restrictions?
$x \geq 0$
$y \in \mathbb{R}$
$z \geq 0$

1) $\sqrt{6}(\sqrt{8} + \sqrt{12})$
$= \sqrt{48} + \sqrt{72}$
$= \sqrt{16 \cdot 3} + \sqrt{36 \cdot 2}$
$= 4\sqrt{3} + 6\sqrt{2}$

4
9
16
25
36
⋮

$3(x + y)$
$= 3x + 3y$

$3(2x + 5x)$
$= 3(7x)$
$= 21x$

$3(2x + 5x)$
$= 6x + 15x$
$= 21x$

$-3\sqrt{12}(5\sqrt{3} - 4\sqrt{6})$

$(4 + 3\sqrt{2})(\sqrt{10} + \sqrt{5})$

$-3\sqrt{12}\,(5\sqrt{3} - 4\sqrt{6})$

$= -15\sqrt{36} + 12\sqrt{72}$

$= -15 \cdot 6 + 12\sqrt{36 \cdot 2}$

$= -90 + 12 \cdot 6\sqrt{2}$

$= -90 + 72\sqrt{2}$

$(4+3\sqrt{2})(\sqrt{10}+\sqrt{5})$

$= \underline{4\sqrt{10}} + 4\sqrt{5} + 3\sqrt{20} + \underline{3\sqrt{10}}$

$= 7\sqrt{10} + 4\sqrt{5} + 3\sqrt{4 \cdot 5}$

$= 7\sqrt{10} + 4\sqrt{5} + \underbrace{3 \cdot 2\sqrt{5}}$

$= 7\sqrt{10} + \underline{4\sqrt{5}} + \underline{6\sqrt{5}}$

$= 7\sqrt{10} + 10\sqrt{5}$

$(3\sqrt{x} - \sqrt{y})^2 = (3\sqrt{x} - \sqrt{y})(3\sqrt{x} - \sqrt{y})$

$= 9\sqrt{x^2} - 3\sqrt{xy} - 3\sqrt{xy} + \sqrt{y^2}$

$= 9x - 6\sqrt{xy} + y$

Division Property of Radicals:

for $a \geq 0$, $b > 0$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

$$\frac{18\sqrt{20}}{2\sqrt{4}} = 9\sqrt{5}$$

Next time, we'll look at

"rationalizing the denominator"

For next class

- **Finish worktext questions for 2.2 and the multiplication ones only, from 2.3**