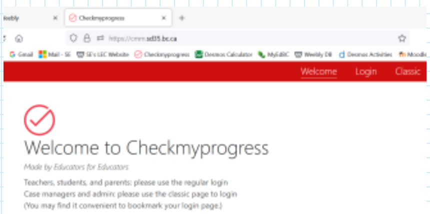
Class_06 May 11 Trig and Angles

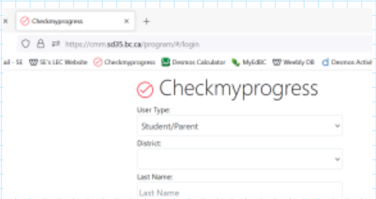
Thursday, May 11, 2023 8:44 AM

Tonight's Class

- **Earthquake Preparedness**
- **Accessing Checkmyprogress**
- **Chapter 3 questions?**
- **Starting Trigonometry (4.0-4.1)**

You can check the progress for FName LName at cmm.sd35.bc.ca using the Last Name LName and password StPassword.





Chapter 3 Questions?

Trigonometry - chapters 4, 5 and 6

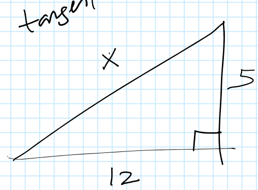
What do you think of when you hear the word "trigonometry"?

$S\frac{O}{H}$ $C\frac{A}{H}$ $T\frac{O}{A}$

Sine
cosine
tangent

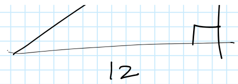
Sine Law

Cosine Law



Sine Law

Cosine Law



Pythagorean Thm.

CHAPTER 4

Trigonometry and the Unit Circle

Have you ever wondered about the repeating patterns that occur around us? Repeating patterns occur in sound, light, tides, time, and molecular motion. To analyse these repeating, cyclical patterns, you need to move from using ratios in triangles to using circular functions to approach trigonometry.

In this chapter, you will learn how to model and solve trigonometric problems using the unit circle and circular functions of radian measures.

trig·o·nom·e·try

/ˌtrigəˈnämətrē/

noun

the branch of mathematics dealing with the relations of the sides and angles of triangles and with the relevant functions of any angles.

Chapter 4: Trigonometry and the Unit Circle

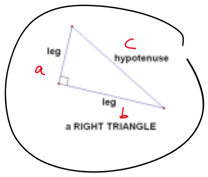
4.0 Trigonometry Review

Trigonometry is the study of triangles and trigonometric functions. First, we review some trigonometry dealing with triangles.

Triangles

- have three angles, the measures add up to 180°
- longest side of triangle is across from the largest angle
- shortest side of a triangle is across from the smallest angle

- ***right triangles*** are triangles that have a ***right angle*** (90°)
  - *hypotenuse* is the longest side of a right triangle
  - other sides of the triangle are often called ***legs***
  - hypotenuse is always **across** from the right angle
  - in a right triangle, we can use the ***Pythagorean Theorem***



If the two legs of a right triangle are called a and b, and the hypotenuse is called c, then $a^2 + b^2 = c^2$.

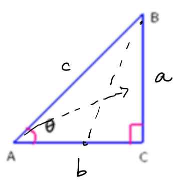
Look at the right triangle shown below. The angle by point A is labeled with the Greek letter θ, read "theta." Angles are very commonly labeled with the letter θ.

Which side is the hypotenuse? $= c$

Which side is opposite θ? $= a$

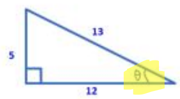
Which side is adjacent to θ? $= b$

next to



When we know the lengths of the sides of a right triangle, we can calculate **ratios** that compare the lengths of two different sides.

First, we label the hypotenuse, and the sides that are opposite and adjacent to angle θ .



There are six different ratios one can create. We'll leave the ratios in fractional form.

Sine $= \dfrac{opposite}{hypotenuse} = \dfrac{5}{13}$ Cosine $\dfrac{adjacent}{hypotenuse} = \dfrac{12}{13}$ Tangent $\dfrac{opposite}{adjacent} = \dfrac{5}{12}$

Cosecant $= \dfrac{hypotenuse}{opposite} = \dfrac{13}{5}$ csc secant sec $\dfrac{hypotenuse}{adjacent} = \dfrac{13}{12}$ $\dfrac{adjacent}{opposite} = \dfrac{12}{5}$ cotangent cot

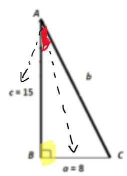
These ratios are called the ***trigonometric ratios***. Knowing them makes it possible to find the measure of each angle in the triangle.

Primary Trigonometric Ratios	***Reciprocal Trigonometric Ratios***
$SINE = \sin\theta = \dfrac{opposite}{hypotenuse} = \dfrac{O}{H}$	$COSECANT = \csc\theta = \dfrac{hypotenuse}{opposite} = \dfrac{H}{O}$
$COSINE = \cos\theta = \dfrac{adjacent}{hypotenuse} = \dfrac{A}{H}$	$SECANT = \sec\theta = \dfrac{hypotenuse}{adjacent} = \dfrac{H}{A}$
$TANGENT = \tan\theta = \dfrac{opposite}{adjacent} = \dfrac{O}{A}$	$COTANGENT = \cot\theta = \dfrac{adjacent}{opposite} = \dfrac{A}{O}$

Remember:
sin/csc are reciprocals
cos/sec are reciprocals
tan/cot are reciprocals

Example Find the measure of each side and angle (correct to nearest degree) in the right triangle shown below.

0



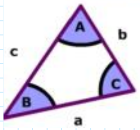
$(side)^2 + (side)^2 = (hyp)^2$
$8^2 + 15^2 = b^2$
$64 + 225 = b^2$
$289 = b^2$
$\sqrt{289} = b$
$\boxed{b = 17}$

$\tan\theta = \dfrac{opp}{adj}$
$\tan^{-1}(\tan A) = \tan^{-1}\left(\dfrac{8}{15}\right)$
$A = \tan^{-1}\left(\dfrac{8}{15}\right)$
$\boxed{A \doteq 28^\circ}$
$\boxed{C \doteq 62^\circ}$

You may also remember using Law of Sines and Law of Cosines to help solve triangles that are not right triangles.

The Law of Sines	
$\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$	$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$
Use to find ANGLES	Use to find sides

Law of Cosines



$$a^2 = b^2 + c^2 - 2bc \cdot \cos(A)$$
$$b^2 = a^2 + c^2 - 2ac \cdot \cos(B)$$
$$c^2 = a^2 + b^2 - 2ab \cdot \cos(c)$$

There's much more to trigonometry than triangles.

Looking Ahead

In this unit, you will solve problems involving…

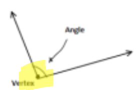
- angle measures and the unit circle
- trigonometric functions and their graphs
- the proofs of trigonometric identities
- the solutions of trigonometric equations

(Notes package, page 3)

4.1 Angles and Angle Measure

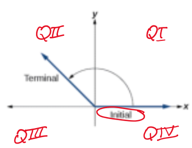
Trigonometry does a lot more than solve triangles. It can be used to analyze many ***repeating patterns*** – things like sound, light, ocean tides, and circular motion.

We start by looking carefully at **ANGLES**.
Remember, angles measure the space between two rays that meet at the ***vertex*** of the angle.



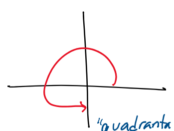
Angles in ***standard position***

- Have the vertex at the origin (0, 0)
- Have a specific direction of rotation, shown with an arrow.
- Have the ***initial arm*** on the positive *x*-axis
- Have the ***terminal arm*** either in one of the four quadrants, or on the *x*- or *y*-axis.



Wherever the terminal arm is, that's how we decide what to call an angle. Options are:

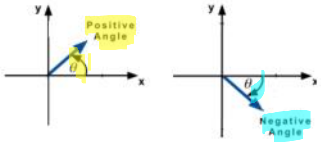
- First quadrant angle
- Second quadrant angle
- Third quadrant angle
- Fourth quadrant angle
- Quadrantal angle → an angle terminates on either the x-axis or the y-axis



"quadrantal angle"
"no quadrant"

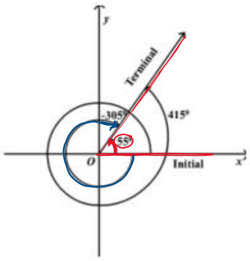
positive angles start on the positive *x*-axis, and rotate counter-clockwise

negative angles start on the positive *x*-axis, and rotate clockwise



Coterminal Angles are

- different in size but
- terminate in the same place



Find another positive angle and another negative angle that are coterminal to the shown angles.

Find two more angles that are coterminal to 55°.

$775° = 55° + (360°)(2)$

$-665° = 55° - (360°)(2)$

General form for coterminal angles – this is an expression that generates ALL the angles that are coterminal to a specific angle. Here's how we write the angles coterminal to 55° in general form:

$$\boxed{55° + 360°n, \quad n \in I}$$

(n is an element of the integers)

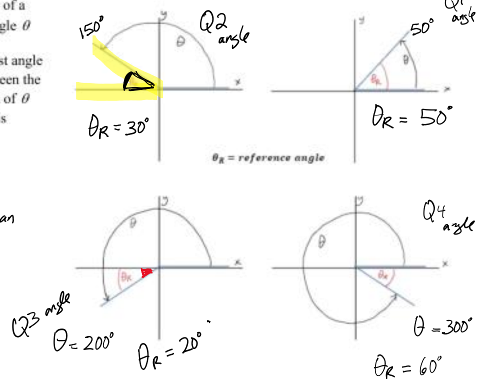
This means we are adding onto the original angle integer multiples of 360:
one 360,
two 360's,
three 360's,
four 360's . . .
OR
adding -360,
or two -360's,
three -360's,
and so on.

(Not adding on a fractional or decimal portion of a full rotation.)

θ_R

reference angle θ_R of a standard position angle θ

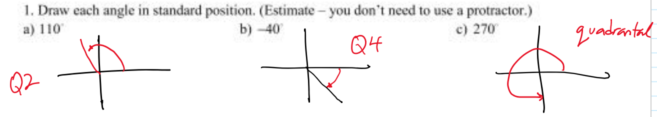
- is the smallest angle formed between the terminal arm of θ and the x-axis
- is positive
- is "acute", smaller than $90°$.



Try this

1. Draw each angle in standard position. (Estimate – you don't need to use a protractor.)

a) $110°$ b) $-40°$ c) $270°$



2. Find a positive and a negative coterminal angle to the angle $160°$.

$160° + 360° = 520°$
$160° - 360° = -200°$

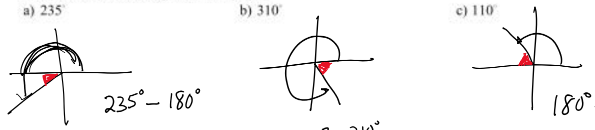
3. Give the general expression for ALL angles coterminal to the angle $25°$.

$25° + 360°n, \; n \in I$

$-4, -3, -2, -1, 0, 1, 2, 3, \ldots$

4. Find the reference angle for each angle.

a) $235°$ b) $310°$ c) $110°$



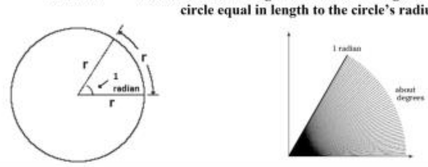
$235° - 180° = 55°$

$360° - 310° = 50°$

$180° - 110° = 70°$

Another unit used to measure angles (besides degrees) is ***radians***. We need to know how to work with radians, as they make some calculus questions much easier (and this is PreCalculus, after all!)

1 *radian* = **the measure of an angle that cuts off a length on the circle equal in length to the circle's radius.**

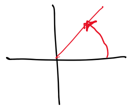


1 radian $\approx 57^\circ$

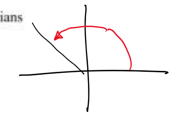
Try this

Sketch a standard position angle measuring:

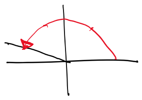
a) 1 radian



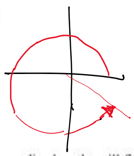
b) 2 radians



c) 3 radians



b) 6 radians



How many radians are in a full rotation? (Think about how many radius lengths will fit onto the full circumference of a circle.)

$A = \pi r^2$

$C = 2\pi r$

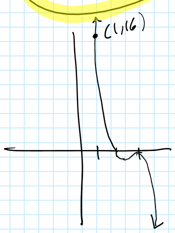
circumference $= (2\pi) r$

$\approx 6.28\ldots$

2π *radians = a complete rotation* = 360°

π *radians = a straight angle* = 180°

Ch 3 flashback



A graph has these roots:

$x = 2$, multiplicity of 1

$x = 3$, multiplicity of 2

and it goes through this point: (1, 16)

What is its equation?

$$y = a(x-2)(x-3)^2$$

substitute in $x=1$, and $y=16$, to figure out "a"

$$16 = a(1-2)(1-3)^2$$

$$16 = a(-1)(-2)^2$$

$$16 = a(-1)(4)$$

$$\frac{16}{-4} = \frac{\cancel{-4}a}{\cancel{-4}}$$

$$a = -4$$

$$\boxed{y = -4(x-2)(x-3)^2}$$

Common Angles

Some angles are used so frequently that it is very helpful to simply KNOW their measurement in both radians and degrees.

40"

★ Make sure you include degree symbol if that's what you mean.

$\pi =$ $180°$ $2\pi =$ $360°$ $\frac{\pi}{2} =$ $90°$ $\frac{\pi}{3} =$ $60°$

$\frac{\pi}{4} =$ $45°$ $\frac{\pi}{6} =$ $30°$ $0 =$ $0°$ $\frac{3\pi}{2} =$ $270°$

Converting Units

We know that 30 minutes is the same thing as ½ an hour. But what about 7452 minutes. What is that, in hours? Here's one way to change units – multiply by a factor of "1".

$$7452 \text{ } \cancel{\text{minutes}} \times \left(\frac{1 \text{ hour}}{60 \text{ } \cancel{\text{minutes}}}\right) = \frac{7452}{60} = 124.2 \text{ hours}$$

Converting Angle Measure

For angles that are not the common ones listed above, we convert angle measurements between degrees and radians by multiplying by the appropriate conversion unit.

(degrees) x $\left(\frac{\pi}{180°}\right)$	=	radians
(radians) x $\left(\frac{180°}{\pi}\right)$	=	degrees

$22° \times \frac{\pi}{180°} =$ "exact form" leave π in the answer $\frac{22\pi}{180} =$ reduces to $\frac{11\pi}{90}$ radians / $22° \times \frac{\pi}{180°} = \frac{22\pi}{180} \doteq$ 0.38 radians rounded answer

Try these

Convert from degrees to radians. Express answer correct to 2 decimal places.

425°

$425° \times \frac{\pi}{180°} \doteq \boxed{7.42 \text{ radians}}$

Convert from degrees to radians. ***Leave answer as a simplified fraction, in terms of π.***

−330°

$-330° \times \frac{\pi}{180°} = \frac{-330\pi}{180} = \frac{-33\pi}{18} = \boxed{\frac{-11\pi}{6}}$

Convert from radians to degrees. Express correct to 2 decimal places.

a) $\frac{3\pi}{8}$

$\frac{3\cancel{\pi}}{8} \cdot \frac{180°}{\cancel{\pi}} = \boxed{67.5°}$

b) 2 radians

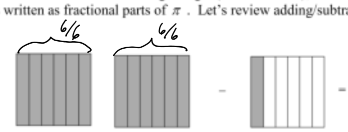
$2 \times \frac{180°}{\pi} = \frac{360°}{\pi}$

$= \boxed{114.59°}$

Working with Radians in Fraction Form

Because π radians is the size of a straight angle (half a rotation), we end up working a lot with angles written as fractional parts of π . Let's review adding/subtracting fractions.

$2 - \frac{1}{6} =$



$\frac{6}{6} \cdot \frac{2}{1} - \frac{1}{6}$

$\frac{12}{6} - \frac{1}{6} = \frac{11}{6}$

Try

a) $1 + \frac{1}{4} =$ $\frac{5}{4}$

b) $2 - \frac{1}{3} =$ $\frac{6}{3} - \frac{1}{3} = \frac{5}{3}$

c) $1 - \frac{1}{6} =$ $\frac{6}{6} - \frac{1}{6} = \frac{5}{6}$

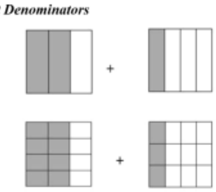
d) $2 - \frac{1}{4} =$ $\frac{8}{4} - \frac{1}{4} = \frac{7}{4}$

e) $1 - \frac{1}{4} =$ $\frac{4}{4} - \frac{1}{4} = \frac{3}{4}$

f) $1 + \frac{1}{3} =$ $\frac{3}{3} + \frac{1}{3} = \frac{4}{3}$

Different Denominators

$\frac{2}{3} + \frac{1}{4} =$



$\frac{2}{3} \cdot \frac{4}{4} + \frac{1}{4} \cdot \frac{3}{3}$

$= \frac{8}{12} + \frac{3}{12} = \frac{11}{12}$

Try

a) $\frac{1}{2} + \frac{1}{3} =$ $\frac{1}{2} \cdot \frac{3}{3} + \frac{1}{3} \cdot \frac{2}{2}$
$= \frac{3}{6} + \frac{2}{6} = \boxed{\frac{5}{6}}$

b) $\frac{7}{6} + \frac{1}{4} =$ $\frac{7}{6} \cdot \frac{2}{2} + \frac{1}{4} \cdot \frac{3}{3}$
$= \frac{14}{12} + \frac{3}{12} = \boxed{\frac{17}{12}}$

c) $\frac{3}{5} + \frac{1}{7} =$ $\frac{3}{5} \cdot \frac{7}{7} + \frac{1}{7} \cdot \frac{5}{5}$
$= \frac{21}{35} + \frac{5}{35}$
$= \boxed{\frac{26}{35}}$

Is π in the fraction? It still works the same way!

Try

★ a) $\pi + \frac{\pi}{6} =$ $\boxed{\frac{7\pi}{6}}$

b) $2\pi - \frac{\pi}{2} =$ $\frac{4\pi}{2} - \frac{\pi}{2}$
$= \boxed{\frac{3\pi}{2}}$

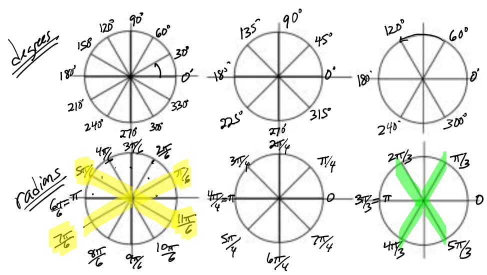
c) $\frac{3\pi}{4} + 2\pi \cdot \frac{4}{4}$
$\frac{3\pi}{4} + \frac{8\pi}{4}$
$= \boxed{\frac{11\pi}{4}}$

d) $\frac{4}{4} \cdot \frac{2\pi}{3} + \frac{\pi}{4} \cdot \frac{3}{3} = \frac{8\pi}{12} + \frac{3\pi}{12} = \boxed{\frac{11\pi}{12}}$

d) $\frac{4}{4}\cdot\frac{2\pi}{3} + \frac{\pi}{4}\cdot\frac{3}{3} = \frac{8\pi}{12} + \frac{3\pi}{12} = \boxed{\frac{11\pi}{12}}$ $\boxed{4}$

Standard-Position Angles in Radian Measure

A straight angle measures π radians. This helps us when sketching angles that are fractions involving π.



For each angle below:

- Draw the angle in standard position.
- List two angles that are coterminal to the given angle. Use radians, not degrees!

a) $\frac{5\pi}{6}$

$\frac{5\pi}{6} + 2\pi\cdot\frac{6}{6}$

$= \frac{5\pi}{6} + \frac{12\pi}{6}$

$\boxed{\frac{17\pi}{6}}$

$\frac{5\pi}{6} - \frac{12\pi}{6}$

$= \boxed{\frac{-7\pi}{6}}$

b) $-\frac{\pi}{4}$

$-\frac{\pi}{4} + 2\pi\cdot\frac{4}{4}$

$= -\pi/4 + \frac{8\pi}{4}$

$= \boxed{\frac{7\pi}{4}}$

$-\pi/4 - \frac{8\pi}{4}$

$= \boxed{-\frac{9\pi}{4}}$

c) $\frac{5\pi}{3}$

$\frac{5\pi}{3} + 2\pi\cdot\frac{3}{3}$

$= \frac{5\pi}{3} + \frac{6\pi}{3}$

$= \boxed{11\pi/3}$

$\frac{5\pi}{3} - 6\pi/3$

$= \boxed{-\pi/3}$

d) $\frac{7\pi}{6}$

$\frac{7\pi}{6} + 2\pi\cdot\frac{6}{6}$

$= \frac{7\pi}{6} + \frac{12\pi}{6}$

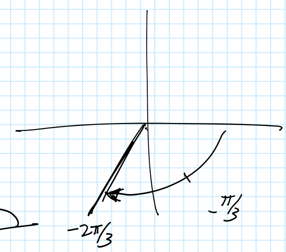
$= \boxed{\frac{19\pi}{6}}$

$\frac{7\pi}{6} - \frac{12\pi}{6}$

$= \boxed{-\frac{5\pi}{6}}$

Example

$-\frac{2\pi}{3}$



remember: $\pi =$

coterminal

$-\frac{2\pi}{3} + 2\pi$

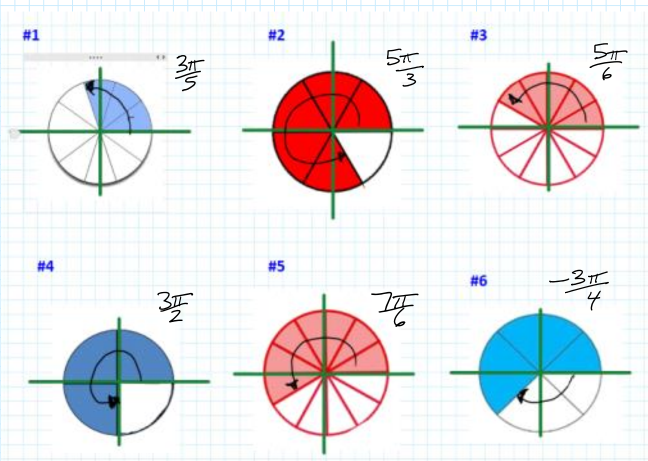
$= -\frac{2\pi}{3} + \frac{2\pi}{1}\cdot\frac{3}{3}$

$= -\frac{2\pi}{3} + \frac{6\pi}{3} = \boxed{\frac{4\pi}{3}}$

another one: $\frac{4\pi}{3} + \frac{6\pi}{3} = \boxed{\frac{10\pi}{3}}$

Quick Check-in

What is the size of each angle, in radian measure? π

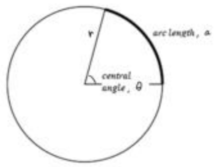


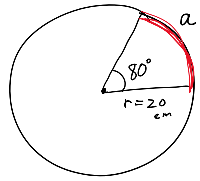
I don't understand why some people use fractions instead of decimals.

It's pointless.

Arc Length

Suppose we have a circle with radius = 20 cm. If we mark off a central angle measuring 80°, what arc length (length along the circle's circumference) does the angle cut off?





$\boxed{a = r\theta,\ \text{for } \theta \text{ in radian measure}}$

$2\pi r$

$a = (20)(80°)\left(\frac{\pi}{180°}\right)$

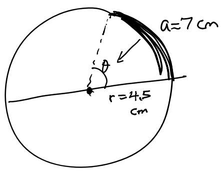
$\boxed{a \doteq 27.9 \text{ cm}}$

$\frac{80°}{360°} = \frac{a}{\text{Circumference}}$ $C = 2\pi r$

$(2\pi)(20)\left[\frac{80}{360}\right] = \left[\frac{a}{\cancel{(2\pi)(20)}}\right]\cancel{(2\pi)(20)}$

$\Rightarrow a = 27.9 \text{ cm}$

Suppose we have another circle, this one with diameter 9 cm. If we know that an arc measuring 7 cm is subtended by a central angle, what is the measure of that central angle, in radians? What is the measure of the central angle, in degrees?



$a = r\theta$

$\frac{7}{4.5} = \frac{\cancel{4.5}\theta}{\cancel{4.5}}$

$\theta = \frac{7}{4.5} \doteq \boxed{1.55 \text{ radians}}$

In degrees, $\theta = \frac{7}{4.5} \cdot \frac{180°}{\pi}$

$\doteq \boxed{89°}$

OR

$\cancel{2\pi} \cdot \left[\frac{\theta}{\cancel{2\pi}}\right] = \left[\frac{7}{(2\pi)(4.5)}\right] \cdot 2\pi$

$\theta = \frac{(7)(\cancel{2\pi})}{(\cancel{2\pi})(4.5)}$

$\theta = \frac{7}{4.5} \doteq 1.55 \text{ radians}$

Then, converting, we get $\theta = \left(\frac{7}{4.5}\right) \cdot \left(\frac{180°}{\pi}\right)$

$\doteq \boxed{89°}$

For next class:

- **Complete the Chapter 3 Hand-in Assignment - handing it in on Monday!**
- **Complete questions from 4.0-4.1 on the Chapter 4 Hand-in Assignment**
- **Prepare for the Week 2 Test (Chapter 3 and 4.0-4.1)**