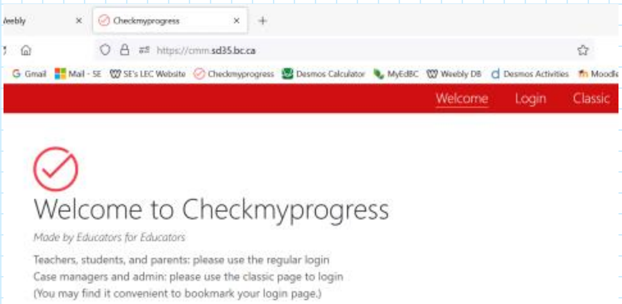
Class_06 Sep 27 Polynomials and Starting Trig

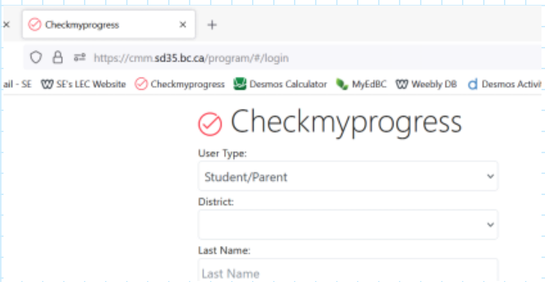
Tuesday, September 27, 2022 9:36 PM

Tonight's Class

- **Polynomial Graphs (3.4)**
- **Starting Trigonometry (4.0-4.1)**

You can check the progress for FName LName at cmm.sd35.bc.ca using the Last Name LName and password StPassword.





Desmos Test Mode

Pre-Calc 12 – Unit 1
Page 25

Example

Factor fully without using technology: $2x^3 - 5x^2 - 4x + 3$

a) According to the integral zero theorem, which values could possibly give factors of this polynomial?

$1, -1, 3, -3$ (only the numbers that are factors of the constant, 3)

b) Use the remainder and factor theorems to find a factor.

$P(-1) = 2(-1)^3 - 5(-1)^2 - 4(-1) + 3$

(constant, 3)

b) Use the remainder and factor theorems to find a factor.

$$P(-1) = 2(-1)^3 - 5(-1)^2 - 4(-1) + 3$$
$$= -2 - 5(1) + 4 + 3$$
$$= -2 - 5 + 7$$
$$= 0$$

$x+1$

c) Use either long division or synthetic division to divide $2x^3 - 5x^2 - 4x + 3$ by the factor found in part (b).

$$\begin{array}{r} 2x^2 - 7x + 3 \\ x+1 \overline{\big)\, 2x^3 - 5x^2 - 4x + 3} \\ -(2x^3 + 2x^2) \\ \hline -7x^2 - 4x \\ -(-7x^2 - 7x) \\ \hline 3x + 3 \\ -(3x+3) \\ \hline 0 \end{array}$$

$x+1$

$$\begin{array}{r|rrrr} -1 & 2 & -5 & -4 & 3 \\ & \downarrow & 2 & -7 & 3 \\ \hline & 2 & -7 & 3 & 0 \end{array}$$

$2x^2 - 7x + 3$

$$P(x) = (x+1)(2x^2 - 7x + 3)$$

A, B, C

Decomposition method:

$AC = 2(3) = 6$

sum $= -7$

mult $= 6$

add to $B = -7$

$-6, -1$

$$2x^2 - 7x + 3$$
$$= 2x^2 - 6x - 1x + 3$$
$$= 2x(x-3) - 1(x-3)$$
$$= (x-3)(2x-1)$$

d) What is the fully factored form of $2x^3 - 5x^2 - 4x + 3$?

$$P(x) = (x+1)(x-3)(2x-1)$$

e) Consider the equation $2x^3 - 5x^2 - 4x + 3 = 0$, what are the solutions to this equation?

$$(x+1)(x-3)(2x-1) = 0$$

$x + 1 = 0$

$x - 3 = 0$

$2x - 1 = 0$

$2x = 1$

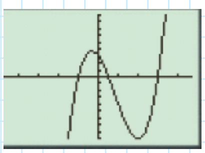
$x = 1/2$

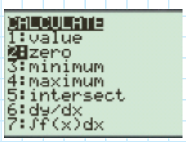
$x = -1$ $\quad x = 3$ $\quad x = 1/2$

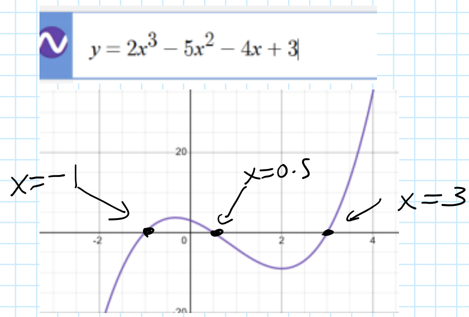
f) Graph $2x^3 - 5x^2 - 4x + 3$ on a graphing calculator. Find the values of its x-intercepts.

$x = -1$ $\quad x = 3$

$x = 0.5 = 1/2$







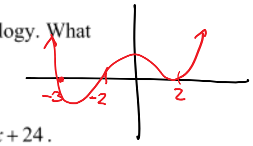
3.4 Equations and Graphs of Polynomial Functions

As we just saw, the solutions of an equation match up with the x-intercepts of the graph.

Example

a) Graph the function $f(x)=x^4+x^3-10x^2-4x+24$ using graphing technology. What are its x-intercepts?

$x=-3,\ x=-2,\ x=2$



b) Use the results from part (a) to help fully factor $f(x)=x^4+x^3-10x^2-4x+24$.

x-intercepts $\Rightarrow$ I know $(x+3)$, $(x+2)$ and $(x-2)$ are factors.

Use one of them to divide the original polynomial, & help us factor it.

Use either kind of division

$x+3$

$$\begin{array}{r|rrrrr} 3 & 1 & 1 & -10 & -4 & 24 \\ - & \downarrow & 3 & -6 & -12 & 24 \\ \hline \times & 1 & -2 & -4 & 8 & 0 \end{array}$$

$$\underbrace{1 \quad -2 \quad -4 \quad 8}_{x^3-2x^2-4x+8}$$

$$\begin{array}{r} x^3-2x^2-4x+8 \\ x+3\,\big)\overline{x^4+x^3-10x^2-4x+24} \\ -(x^4+3x^3) \\ \hline -2x^3-10x^2 \\ -(-2x^3-6x^2) \\ \hline -4x^2-4x \\ -(-4x^2-12x) \\ \hline 8x+24 \\ 8x+24 \\ \hline 0 \end{array}$$

$f(x)=(x+3)(x^3-2x^2-4x+8)$

$x+2$

$$\begin{array}{r|rrrr} 2 & 1 & -2 & -4 & 8 \\ - & \downarrow & 2 & -8 & 8 \\ \hline \times & 1 & -4 & 4 & 0 \end{array}$$

$$\begin{array}{r} x^2-4x+4 \\ x+2\,\big)\overline{x^3-2x^2-4x+8} \\ -(x^3+2x^2) \\ \hline -4x^2-4x \\ -(-4x^2-8x) \\ \hline 4x+8 \\ -(4x+8) \\ \hline 0 \end{array}$$

$f(x)=(x+3)(x+2)(x^2-4x+4)$

$f(x)=(x+3)(x+2)(x-2)(x-2)$

$\hookrightarrow f(x)=(x+3)(x+2)(x-2)^2$

c) What are the solutions to the equation: $x^4+x^3-10x^2-4x+24=0$

$(x+3)(x+2)(x-2)^2=0$

$$\boxed{x=-3 \qquad x=-2 \qquad x=2}$$

y-int. $(0, -8)$

Example
Factor completely, then analyze and sketch the graph of this polynomial function ***without*** using technology: $f(x) = x^3 + 3x^2 - 6x - 8$

1) List possible zeros (using the integral zero theorem) - $\pm 1, 2, 4, 8$

2) Show using the remainder theorem what is one of the factors.

$$f(1) = 1^3 + 3(1)^2 - 6(1) - 8 = 1 + 3 - 6 - 8 = -10$$

$$f(-1) = (-1)^3 + 3(-1)^2 - 6(-1) - 8 = 0$$

$\Rightarrow \boxed{x+1 \text{ is a factor}}$

3) Divide, whichever method you prefer.

$(x+1)$

$$\begin{array}{r|rrrr} -1 & 1 & 3 & -6 & -8 \\ & \downarrow & -1 & -2 & 8 \\ \hline & 1 & 2 & -8 & 0 \end{array}$$

$\underbrace{1 \quad 2 \quad -8}_{x^2 + 2x - 8}$

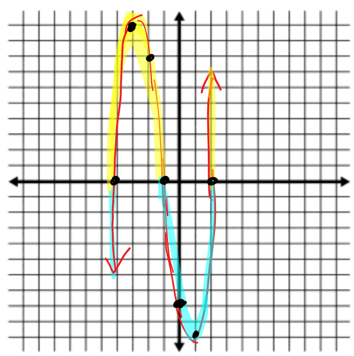
$$\begin{array}{r} x^2 + 2x - 8 \\ x+1 \overline{)\, x^3 + 3x^2 - 6x - 8} \\ -(x^3 + x^2) \\ \hline 2x^2 - 6x \\ -(2x^2 + 2x) \\ \hline -8x - 8 \\ -(-8x - 8) \\ \hline 0 \end{array}$$

4) $f(x) = (x+1)(x^2 + 2x - 8)$
$= (x+1)(x-2)(x+4)$

$x = -1 \qquad x = 2 \qquad x = -4$

Degree	3	(odd)
Leading coefficient	1	(+)
End behavior	↓Q3 ↑Q1	
Zeros/*x*-intercepts	$(-4,0)$ $(-1,0)$	$(2,0)$
y-intercept	$(0,-8)$	
Interval(s) where function is positive or negative		

positive $-4 < x < -1, \; x > 2$
negative $x < -4, \; -1 < x < 2$



1) plot x-intercepts
2) plot y-intercept

x	y
-3	10
-2	8
1	-10

Whiteboards

Factor and sketch: $f(x) = -x^4 - 2x^3 + 7x^2 + 8x - 12$

1) possibilities: $\pm 1, 12, 2, 6, 3, 4$

2) try to find one: $f(1) = -(1^4) - 2(1)^3 + 7(1)^2 + 8(1) - 12$
$= -1 - 2 + 7 + 8 - 12$
$= 0$ ✓ $\Rightarrow \boxed{x-1 \text{ is a factor}}$

3) divide: $(x-1)$

$$\begin{array}{r|rrrrr} -1 & -1 & -2 & 7 & 8 & -12 \\ & & 1 & 3 & -4 & -12 \end{array}$$

3) divide: $x-1$

$$\begin{array}{r|rrrrr} -1 & -1 & -2 & 7 & 8 & -12 \\ & & 1 & 3 & -4 & -12 \\ \hline & -1 & -3 & 4 & 12 & 0 \end{array}$$

$-x^3 - 3x^2 + 4x + 12$

what might work here? let's try $x = -1$

$$-(-1)^3 - 3(-1)^2 + 4(-1) + 12$$
$$= -(-1) + 3 - 4 + 12$$
$$= 1 + 3 - 4 + 12 \neq 0$$

try $x = 2$

$$-(2)^3 - 3(2)^2 + 4(2) + 12$$
$$= -8 - 12 + 8 + 12 = 0$$

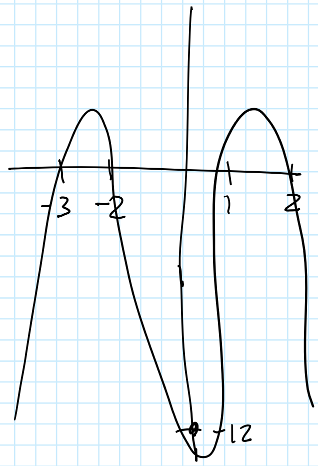
$\Rightarrow$ $x-2$ is a factor

$x-2$

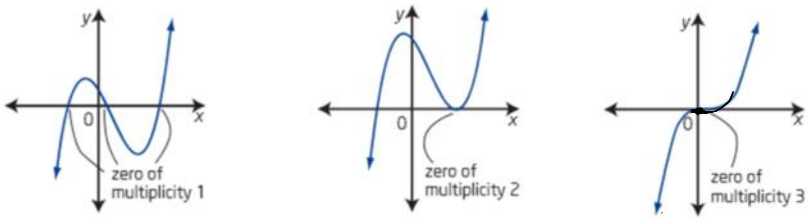
$$\begin{array}{r|rrrr} -2 & -1 & -3 & 4 & 12 \\ & & 2 & 10 & 12 \\ \hline & -1 & -5 & -6 & 0 \end{array}$$

$-x^2 - 5x - 6$

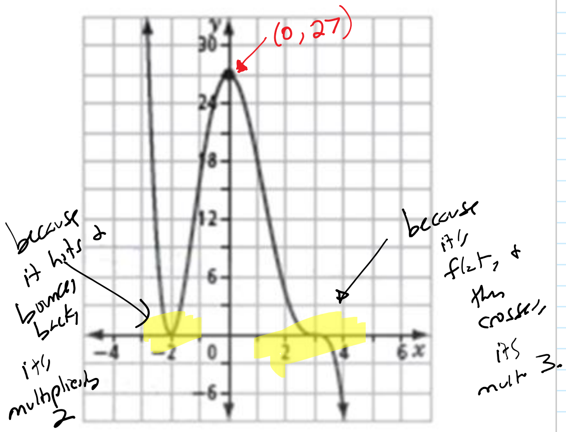
$$f(x) = (x-1)(x-2)(-1)(x^2 + 5x + 6)$$
$$= -1(x-1)(x-2)(x+2)(x+3)$$



Multiplicity of a zero – the multiplicity of a zero is the number of times a zero of a polynomial occurs.



Example
Find the equation for the given graph.



stretch factor

$$y = a(x+2)^2(x-3)^3$$

contains this point: $(0, 27)$
x   y

$$27 = a(0+2)^2(0-3)^3$$
$$27 = a(4)(-3)^3$$
$$27 = a(4)(-27)$$
$$\frac{27}{-108} = \frac{a(\cancel{-108})}{\cancel{-108}}$$

$$a = \frac{27}{-108} = \frac{1}{-4}$$

$$\boxed{y = -\frac{1}{4}(x+2)^2(x-3)^3}$$

x-intercepts are very important, but they aren't everything - a stretch factor can change the shape of the graph, even if the x-intercepts don't change at all. (It can make a vertical expansion or compression happen, as well as a reflection)

$$y = a(x-1)(x+1)(x-3)$$

Polynomial Graphs Worksheet

For each graph, find:

- Degree (odd or even)
- Sign of leading coefficient
- Coordinates of x-intercept(s)
- Equation of the polynomial
- y-intercept

Check your equation on a graphing calculator to see if it contains all the points that are on the pictured graph.

Solutions to this worksheet will be posted.

Unit 2 - Trigonometry

What do you think of when you hear the word "trigonometry"?

UNIT circle

triangles

SOHCAHTOA

radians

$a^2+b^2=c^2$ (Pythagorean Theorem)

sine, cosine, tangent

$\sin^{-1}$ $\cos^{-1}$ $\tan^{-1}$

π

Law of Sines

Law of Cosines

ambiguous case

$$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$$

trig·o·nom·e·try

/ˌtrigəˈnämətrē/

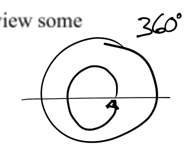
noun

the branch of mathematics dealing with the relations of the sides and angles of triangles and with the relevant functions of any angles.

Chapter 4: Trigonometry and the Unit Circle

4.0 Trigonometry Review

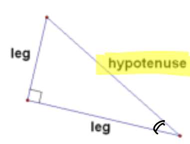
Trigonometry is the study of triangles and trigonometric functions. First, we review some trigonometry dealing with triangles.



Triangles

- have three angles, the measures add up to 180°
- longest side of triangle is across from the largest angle
- shortest side of a triangle is across from the smallest angle

- ***right triangles*** are triangles that have a ***right angle*** (90°)
  - ***hypotenuse*** is the longest side of a right triangle
  - other sides of the triangle are often called ***legs***
  - hypotenuse is always **across** from the right angle
  - in a right triangle, we can use the ***Pythagorean Theorem***



a RIGHT TRIANGLE

If the two legs of a right triangle are called a and b, and the hypotenuse is called c, then $a^2 + b^2 = c^2$.

$(\text{side})^2 + (\text{side})^2 = (\text{hyp})^2$

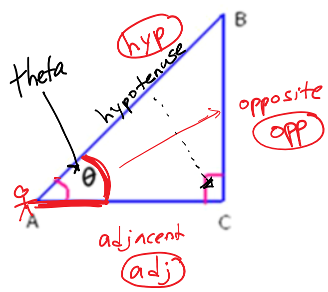
Look at the right triangle shown below. The angle by point A is labeled with the Greek letter θ, read "theta." Angles are very commonly labeled with the letter θ.

Which side is the hypotenuse?

(it's across from the right angle)

Which side is opposite θ?

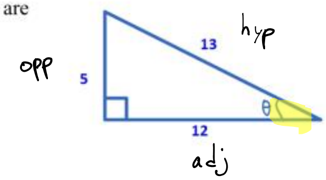
Which side is adjacent to θ?



When we know the lengths of the sides of a right triangle, we can calculate **ratios** that compare the lengths of two different sides

When we know the lengths of the sides of a right triangle, we can calculate **ratios** that compare the lengths of two different sides.

First, we label the hypotenuse, and the sides that are opposite and adjacent to angle θ .



There are six different ratios one can create. We'll leave the ratios in fractional form.

$$\frac{opposite}{hypotenuse} = \frac{5}{13} \qquad \frac{adjacent}{hypotenuse} = \frac{12}{13} \qquad \frac{opposite}{adjacent} = \frac{5}{12}$$

$$\frac{hypotenuse}{opposite} = \frac{13}{5} \qquad \frac{hypotenuse}{adjacent} = \frac{13}{12} \qquad \frac{adjacent}{opposite} = \frac{12}{5}$$

These ratios are called the ***trigonometric ratios***. Knowing them makes it possible to find the measure of each angle in the triangle.

$S \frac{O}{H}$ "soh"
$C \frac{A}{H}$ "cah"
$T \frac{O}{A}$ "toa"

Primary Trigonometric Ratios

$SINE = \sin\theta = \frac{opposite}{hypotenuse} = \frac{O}{H}$

$COSINE = \cos\theta = \frac{adjacent}{hypotenuse} = \frac{A}{H}$

$TANGENT = \tan\theta = \frac{opposite}{adjacent} = \frac{O}{A}$

Reciprocal Trigonometric Ratios

$COSECANT = \csc\theta = \frac{hypotenuse}{opposite} = \frac{H}{O}$

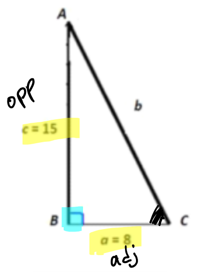
$SECANT = \sec\theta = \frac{hypotenuse}{adjacent} = \frac{H}{A}$

$COTANGENT = \cot\theta = \frac{adjacent}{opposite} = \frac{A}{O}$

★ **Remember:**
sin/csc are reciprocals
cos/sec are reciprocals
tan/cot are reciprocals

Example Find the measure of each side and angle (correct to nearest degree) in the right triangle shown below.

"Solve the triangle" (find the lengths and the angle measures of all of it)



Pythagoras:

$(side)^2 + (side)^2 = (hyp)^2$

$15^2 + 8^2 = (hyp)^2$

$225 + 64 = (hyp)^2$

$\sqrt{289} = \sqrt{(hyp)^2}$

$17 = hyp$

$B = 90^\circ$

$\tan C = \frac{opp}{\ }$

$$\tan C = \frac{15}{8}$$ (adj)

$$\tan^{-1}(\tan C) = \tan^{-1}\left(\frac{15}{8}\right)$$

$$C = \tan^{-1}\left(\frac{15}{8}\right) \doteq 61.9^\circ \doteq \boxed{62^\circ}$$

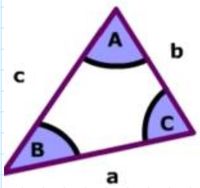
$$A = 180^\circ - 90^\circ - 62^\circ$$

$$\boxed{A = 28^\circ}$$

You may also remember using Law of Sines and Law of Cosines to help solve triangles that are not right triangles.

The Law of Sines	
$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
Use to find ANGLES	Use to find sides

Law of Cosines



$a^2 = b^2 + c^2 - 2bc \cdot \cos(A)$

$b^2 = a^2 + c^2 - 2ac \cdot \cos(B)$

$c^2 = a^2 + b^2 - 2ab \cdot \cos(c)$

However, there's a lot more to trigonometry than triangles.

trig·o·nom·e·try

/ˌtrigəˈnämətrē/

noun

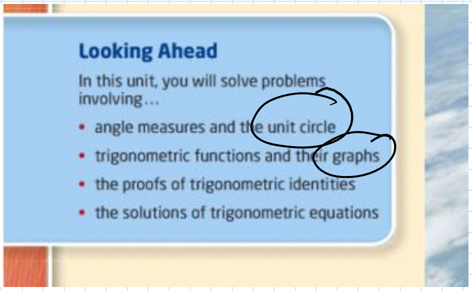
the branch of mathematics dealing with the relations of the sides and angles of triangles and with the relevant functions of any angles.

TB p 162


Unit 2

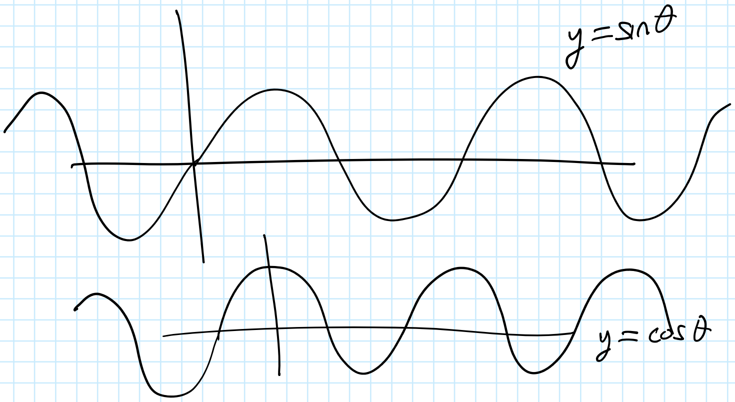
Trigonometry

Trigonometry is used extensively in our daily lives. For example,


Looking Ahead

In this unit, you will solve problems involving…

- angle measures and the unit circle
- trigonometric functions and their graphs
- the proofs of trigonometric identities
- the solutions of trigonometric equations

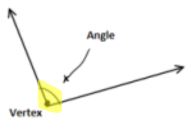


(Notes package, page 3)

4.1 Angles and Angle Measure

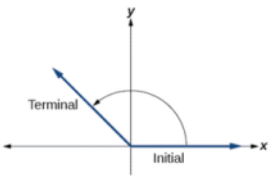
Trigonometry does a lot more than solve triangles. It can be used to analyze many ***repeating patterns*** – things like sound, light, ocean tides, and circular motion.

We start by looking carefully at **ANGLES**.
Remember, angles measure the space between two rays that meet at the ***vertex*** of the angle.



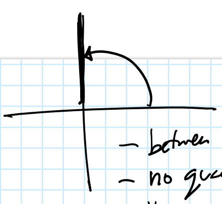
Angles in ***standard position***

- Have the vertex at the origin (0, 0)
- Have a specific direction of rotation, shown with an arrow.
- Have the ***initial arm*** on the positive x-axis
- Have the ***terminal arm*** either in one of the four quadrants, or on the x- or y-axis.



Wherever the terminal arm is, that's how we decide what to call an angle. Options are:

- First quadrant angle
- Second quadrant angle
- Third quadrant angle
- Fourth quadrant angle
- Quadrantal angle

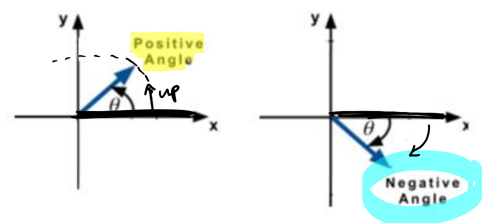


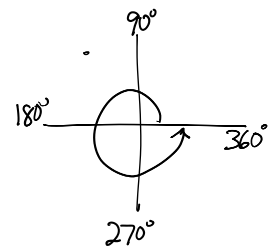
– between Q1 and Q2
– no quadrant
" quadrantal angle"

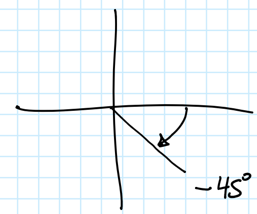
– "quadrantal angle"

positive angles start on the positive x-axis, and rotate counter-clockwise (anti-clockwise)

negative angles start on the positive x-axis, and rotate clockwise

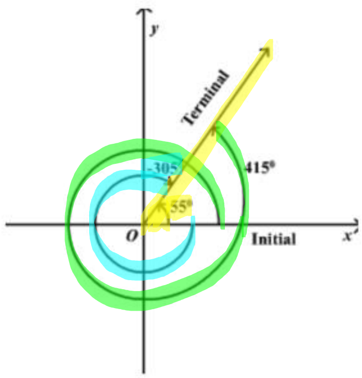






Coterminal Angles are

- different in size but
- terminate in the same place



Find another positive angle and another negative angle that are coterminal to the shown angles.

55° and −305° and 415° are coterminal

General form for coterminal angles – this is an expression that generates ALL the angles that are coterminal to a specific angle. Here's how we write the angles coterminal to 55° in general form:

integer

$$(\text{starter angle}) + 360^\circ \underline{n}, \quad n \in I$$

$$55^\circ + 360^\circ n, \quad n \in I$$

This means we are adding onto the original angle integer multiples of 360:
one 360,
two 360's,
three 360's,
four 360's . . .
OR
adding -360,
or two -360's,
three -360's,
and so on.

(Not adding on a fractional or decimal portion of a full rotation.)

For next class

- **Complete the Chapter 3 Hand-in Assignment**
- **Prepare for the Unit 1 test**

Optional worksheet:

- **Unit 1 Review with full solutions, posted on the class website**