Class_07 Jan 26 - Solving Radical Equations

Sunday, January 22, 2023 1:57 PM

Tonight's Class:

- **About the Learning Center**
- **Warm-up - small whiteboards**
- **Any questions from 2.2-2.3?**
- **Working through sections 2.3 and 2.5 (omitting 2.4, since we're not requiring the use of graphing technology)**
  - **Dividing radicals**
  - **Solving radical equations**

Warm-up

1. **List some perfect squares**

1, 4, 9, 16, 25, 36

2. **List some perfect cubes**

1, 8, 27, 64, 125, 216 ...

3. **Evaluate, without using a calculator:**

$$\left(\frac{9}{4}\right)^{-3/2} = \left(\frac{4}{9}\right)^{+3/2} = \sqrt[2]{\frac{4}{9}}^{\,3} = \left(\frac{2}{3}\right)^3 = \boxed{\frac{8}{27}}$$

4. Which one of the following problem and answer pairs is incorrect?

A) Problem: $\sqrt{16}\cdot\sqrt{25}$ Answer: 20

B) Problem: $\sqrt{16}\cdot\sqrt{x^2}$ Answer: $4|x|$

C) Problem: $\sqrt[3]{x}\cdot\sqrt[3]{y^2}$ Answer: $\sqrt[3]{xy^2}$

D) Problem: $\sqrt{20}\cdot\sqrt[3]{y}$ Answer: $\sqrt[3]{20y}$

54 → 2, 27; 27 → 3, 9; 9 → 3, 3

p108, 8c)

$$5e\sqrt{24e^3} - 7\sqrt{54e^5} + e^2\sqrt{6e} + 6e, \quad e \ge 0$$

$$= 5e\sqrt{4\cdot 6\cdot e^2\cdot e} - 7\sqrt{2\cdot 3\cdot 3\cdot 3\cdot e^4\cdot e} + e^2\sqrt{6e} + 6e$$

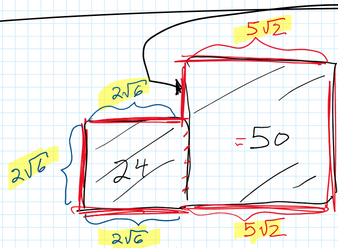
$$= 5e\cdot 2\cdot e\sqrt{6e} - 7\cdot 3\cdot e^2\sqrt{6e} + e^2\sqrt{6e} + 6e$$

$$= 10e^2\sqrt{6e} - 21e^2\sqrt{6e} + e^2\sqrt{6e} + 6e$$

$$= (10e^2 - 21e^2 + e^2)\sqrt{6e} + 6e$$

$$= -10e^2\sqrt{6e} + 6e$$

p109, #9



Perimeter = distance around the outside of a shape

Area = (y)(y)

$A = y^2 = 24$

$y = \sqrt{24}$

$= \sqrt{4\cdot 6}$

$= 2\sqrt{6}$

Area = (x)(x)

$A = x^2 = 50$

$x = \sqrt{50}$

$\sqrt{25\cdot 2} = 5\sqrt{2}$

Perimeter:

$2\sqrt{6} + 2\sqrt{6} + 2\sqrt{6}$

$+ 5\sqrt{2} + 5\sqrt{2} + 5\sqrt{2}$

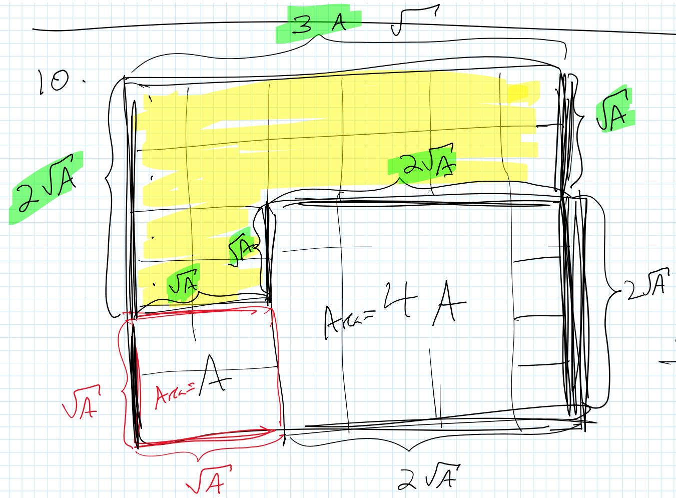
$+ (5\sqrt{2} - 2\sqrt{6})$

$= 6\sqrt{6} + 15\sqrt{2}$

$+ 5\sqrt{2} - 2\sqrt{6}$

$= 4\sqrt{6} + 20\sqrt{2}$

10.



$$\text{Perimeter} = 2\sqrt{A} + 3\sqrt{A} + \sqrt{A} + 2\sqrt{A} + \sqrt{A} + \sqrt{A}$$
$$= 10\sqrt{A}$$

$$\text{Area}_{\text{square}} = 4A$$

$$\text{side of square} = \sqrt{4A}$$
$$= 2\sqrt{A}$$

Area $= 4A$ (counting boxes works!)

p122, #7b)

$$-1(3\sqrt{2} - \sqrt{5})(\sqrt{2} + 7) - (2\sqrt{2} - \sqrt{5})^2$$
$$= (-3\sqrt{2} + \sqrt{5})(\sqrt{2} + 7) - (2\sqrt{2} - \sqrt{5})(2\sqrt{2} - \sqrt{5})$$
$$= -3\sqrt{4} - 21\sqrt{2} + \sqrt{10} + 7\sqrt{5} - [4\sqrt{4} - 2\sqrt{10} - 2\sqrt{10} + \sqrt{25}]$$
$$= -6 - 21\sqrt{2} + \sqrt{10} + 7\sqrt{5} - [8 - 4\sqrt{10} + 5]$$
$$= -6 - 21\sqrt{2} + \sqrt{10} + 7\sqrt{5} - [13 - 4\sqrt{10}]$$
$$= -6 - 21\sqrt{2} + \sqrt{10} + 7\sqrt{5} - 13 + 4\sqrt{10}$$
$$= -19 - 21\sqrt{2} + 5\sqrt{10} + 7\sqrt{5}$$

To multiply radical expressions:

To divide radical expressions:

You may multiply radicals that have the same index. If this is the case, multiply the coefficients and multiply the radicands. Simplify.	$2\sqrt{5}(3\sqrt{3} - 3\sqrt{5})$ $= 6\sqrt{45} - 6\sqrt{75}$ $= 6\sqrt{9\cdot 5} - 6\sqrt{25\cdot 3}$ $= 6\cdot 3\sqrt{5} - 6\cdot 5\sqrt{3}$ $= 18\sqrt{5} - 30\sqrt{3}$ $(5+4\sqrt{3})(3+\sqrt{3})$ $= 15 + 5\sqrt{3} + 12\sqrt{3} + 4\sqrt{9}$ $= 15 + 17\sqrt{3} + 4\cdot 3$ $= 15 + 17\sqrt{3} + 12$ $= 27 + 17\sqrt{3}$		You may divide radicals that have the same index. If this is the case, simplify the coefficients and simplify the radicands. Rationalize the denominator, if necessary.

Rationalizing the denominator

Ex $\quad \dfrac{3\sqrt{14}}{15\sqrt{2}} = \dfrac{1\sqrt{14}}{5\sqrt{2}} = \dfrac{1}{5}\sqrt{\dfrac{14}{2}} = \dfrac{1}{5}\sqrt{7}$

also written $\dfrac{\sqrt{7}}{5}$

Ex $\quad \dfrac{1}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}}$

Multiply by "1", in a specific format.

Ex $\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

Multiply by ... in a specific format.

$= \frac{\sqrt{3}}{3}$

Ex $\frac{3\sqrt{5}+6\sqrt{3}}{\sqrt{20}}$

if the radical in the denominator can be simplified, we should do that first!

$= \frac{(3\sqrt{5}+6\sqrt{3})}{2\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{3\sqrt{25}+6\sqrt{15}}{10}$

$= \frac{15+6\sqrt{15}}{10}$

$\frac{(3\sqrt{5}+6\sqrt{3})}{2\sqrt{5}} \cdot \frac{(2\sqrt{5})}{(2\sqrt{5})}$

$= \frac{6\sqrt{25}+12\sqrt{15}}{4\sqrt{25}}$

$= \frac{30+12\sqrt{15}}{20}$

$= \frac{\cancel{2}(15+6\sqrt{15})}{\cancel{2}(10)}$

$= \frac{15+6\sqrt{15}}{10}$

$\frac{4}{1+\sqrt{7}}$

denominator is a binomial. We tackle this differently!

Multiply (top + bottom) by the CONJUGATE.

= the same binomial except change the operation in the middle of the 2 terms.

$= \frac{4}{(1+\sqrt{7})} \cdot \frac{(1-\sqrt{7})}{(1-\sqrt{7})}$

$= \frac{4-4\sqrt{7}}{1-\sqrt{7}+\sqrt{7}-7}$ $= \frac{4-4\sqrt{7}}{1-7}$

$= \frac{4-4\sqrt{7}}{-6}$

$= \boxed{\frac{2-2\sqrt{7}}{-3}}$

WT p120, #4b

$\frac{(5\sqrt{3}+\sqrt{2})}{(2\sqrt{6}+4\sqrt{3})} \cdot \frac{(2\sqrt{6}-4\sqrt{3})}{(2\sqrt{6}-4\sqrt{3})}$

$= \frac{10\sqrt{18}-20\cdot 3+2\sqrt{12}-4\sqrt{6}}{4\cdot 6-\cancel{8\sqrt{18}}+\cancel{8\sqrt{18}}-16\cdot 3}$

$= \frac{10\sqrt{9\cdot 2}-60+2\sqrt{4\cdot 3}-4\sqrt{6}}{24-48}$

$= \frac{10\cdot 3\sqrt{2}-60+2\cdot 2\sqrt{3}-4\sqrt{6}}{-24}$

$= \frac{30\sqrt{2}-60+4\sqrt{3}-4\sqrt{6}}{-24}$

$= \frac{15\sqrt{2}-30+2\sqrt{3}-2\sqrt{6}}{-12}$

divided each constant & coefficient by 2

NOT the radicands!

also equal to: $\frac{\cancel{-1}(-15\sqrt{2}+30-2\sqrt{3}+2\sqrt{6})}{\cancel{-1}(-12)}$

$-15\sqrt{2}+30-2\sqrt{3}+2\sqrt{6}$

also equal to: $\frac{-\cancel{1}(-15\sqrt{2} + 30 \ldots}{-\cancel{1}(-12)}$ ... the radicands!

$$\frac{-15\sqrt{2} + 30 - 2\sqrt{3} + 2\sqrt{6}}{12}$$

$$\frac{5}{\sqrt[3]{32}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}} = \frac{5\sqrt[3]{2}}{\sqrt[3]{64}}$$

$$= \frac{5\sqrt[3]{2}}{4}$$

Think ... what number would we need to multiply by, to get a perfect CUBE in the radicand?

$$\frac{\sqrt[3]{7}}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{4}}{\sqrt[3]{4}} = \frac{\sqrt[3]{28}}{\sqrt[3]{8}} = \boxed{\frac{\sqrt[3]{28}}{2}}$$

You may multiply radicals that have the same index. If this is the case, multiply the coefficients and multiply the radicands. Simplify.		$\frac{18\sqrt{24}}{6\sqrt{3}} = 3\sqrt{8}$ $= 3\sqrt{4 \cdot 2}$ $= 3 \cdot 2\sqrt{2}$ $= 6\sqrt{2}$ $\frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$ $\frac{4\sqrt{2}}{(5 - 4\sqrt{3})} \cdot \frac{(5 + 4\sqrt{3})}{(5 + 4\sqrt{3})}$ $= \frac{20\sqrt{2} + 16\sqrt{6}}{25 + \cancel{20\sqrt{3}} - \cancel{20\sqrt{3}} - \underset{48}{16 \cdot 3}}$ $= \frac{20\sqrt{2} + 16\sqrt{6}}{-23}$ (OR: $\frac{-20\sqrt{2} - 16\sqrt{6}}{23}$)	You may divide radicals that have the same index. If this is the case, simplify the coefficients and simplify the radicands. Rationalize the denominator, if necessary.

2.5 Solving Radical Equations Algebraically

Focus: solve equations involving radicals

simpler type of equation:

$$4x + 5 = x + 17$$
$$-5 \qquad\qquad -5$$
$$4x = x + 12$$
$$-x \qquad -x$$
$$\frac{3x}{3} = \frac{12}{3}$$
$$x = 4$$

1) collect all our constants
2) collect our "x" terms
3) isolate "x"
4) check if it's right

Preview 7

Need practice with solving equations? Here's a hand-out with answers (not solutions)

WT, page 151

Example 1 **Solving an Equation with One Radical**

Solve each equation. Verify the solution. And, list the restrictions.

a) $3\sqrt{x} = 5$ **b)** $4\sqrt{x + 1} - 5 = 3$

radicand ≥ 0 and simplify

a) $3\sqrt{x} = 5$

restrictions
radicand ≥ 0
$\boxed{x \geq 0}$

$$\frac{3\sqrt{x}}{3} = \frac{5}{3}$$
$$(\sqrt{x})^2 = \left(\frac{5}{3}\right)^2$$
$$x = \frac{25}{9}$$

0) restrictions
1) collect constants
2) isolate radical
3) square both sides of the equation
4) CHECK it! in the original equation

(left side) LS	(right side) RS
	5

$x = \frac{25}{9}$

4) CHECK it! in the original equation

(left side) LS	(right side) RS
$3\sqrt{\frac{25}{9}}$	5 ✓
$3\cdot\left(\frac{5}{3}\right)$	
$\frac{15}{3}$	
5 ✓	

b) $4\sqrt{x+1} - 5 = 3$

radicand ≥ 0
$x+1 \geq 0$
$x \geq -1$

$+5 \quad +5$

$\frac{4\sqrt{x+1}}{4} = \frac{8}{4}$

$\left(\sqrt{x+1}\right)^2 = (2)^2$

$x+1 = 4$
$-1 \quad -1$

$\boxed{x = 3}$

LS	RS
$4\sqrt{3+1} - 5$	3 ✓
$4\sqrt{4} - 5$	
$4(2) - 5$	
$8 - 5$	
3 ✓	

WT p158 c) $\sqrt{5x+3} - 2 = -1$

restrictions: radicand ≥ 0
$5x+3 \geq 0$
$\frac{5x}{5} \geq \frac{-3}{5}$
$\boxed{x \geq -\frac{3}{5}}$

$\sqrt{5x+3} - 2 = -1$
$+2 \quad +2$

$\left(\sqrt{5x+3}\right)^2 = (1)^2$

$5x+3 = 1$
$-3 \quad -3$

$\frac{5x}{5} = \frac{-2}{5}$

$x = \frac{-2}{5}$ leave in fraction form

Check $\sqrt{5x+3} - 2 = -1$

LS	RS
$\sqrt{5\left(\frac{-2}{5}\right)+3} - 2$	-1 ✓
$\sqrt{-2+3} - 2$	
$\sqrt{1} - 2$	
$1 - 2$	
-1 ✓	

p157 b) $\sqrt{3x-1} + 5 = 2$
$-5 \quad -5$

restrictions
$3x-1 \geq 0$
$+1 \quad +1$
$\frac{3x}{3} \geq \frac{1}{3}$
$x \geq \frac{1}{3}$

$\left(\sqrt{3x-1}\right)^2 = (-3)^2$

$3x - 1 = 9$
$+1 \quad +1$

$\frac{3x}{3} = \frac{10}{3}$

$x = \frac{10}{3}$

check $\sqrt{3x-1} + 5 = 2$

LS	RS
$\sqrt{3\left(\frac{10}{3}\right)-1} + 5$	2
$\sqrt{10-1} + 5$	
$\sqrt{9} + 5$	
$3 + 5$	
8	

no solution!

$\sqrt[3]{3x+1} + 6 = 4$
$-6 \quad -6$

index = 3 no restrictions!

$\left(\sqrt[3]{3x+1}\right)^3 = (-2)^3$

$\sqrt[3]{3x+1} + 6 = 4$

$\qquad\quad -6 \quad -6$

no restrictions!

$\left(\sqrt[3]{3x+1}\right)^3 = (-2)^3$

$3x+1 = -8$

$\quad -1 \quad -1$

$\frac{3x}{3} = \frac{-9}{3}$

$x = -3$

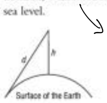
$\sqrt[3]{3x+1} + 6 = 4$

LS	RS
$\sqrt[3]{3(-3)+1} + 6$	4 ✓
$\sqrt[3]{-9+1} + 6$	
$\sqrt[3]{-8} + 6$	
$-2 + 6$	
4 ✓	

WT, page 153

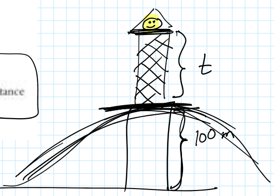
Example 2 **Using a Radical Equation to Solve a Problem**

The formula $d = \sqrt{13h}$ can be used to estimate the distance of the horizon, d kilometres, from an observer at a height of h metres above sea level.



An observation platform is to be built on a fire tower. The base of the tower is 100 m above sea level. How high must the observation platform be so an observer's distance from the horizon is 50 km?

$d = \sqrt{13h}$



$d = \sqrt{13h}$

$(50)^2 = \left(\sqrt{13(100+t)}\right)^2$

$\frac{2500}{13} = \frac{\cancel{13}(100+t)}{\cancel{13}}$

$\frac{2500}{13} = 100 + t$

$-100 \qquad -100$

$\frac{2500}{13} - 100 = t$

$t \doteq 92 \text{ m}$

For next class

- Complete the two "Recaps" from tonight!
- Finish worktext questions for all of Chapter 2
- Complete the Chapter 2 Hand-in, due next class
- Prepare for the Chapter 2 Test, next class
  - 4 multiple-choice questions
  - 10 written questions
  - Out of 20 marks total
  - You will be permitted to use both the foldables (Exponent Rules and Rationals) during the test
- Prepare for the Unit 1 Test, next Thursday
  - Includes concepts from Chapter 1 and Chapter 2
  - Out of 30-35 marks, something like that
  - You will be permitted to use both the foldables (Exponent Rules and Rationals) during the test