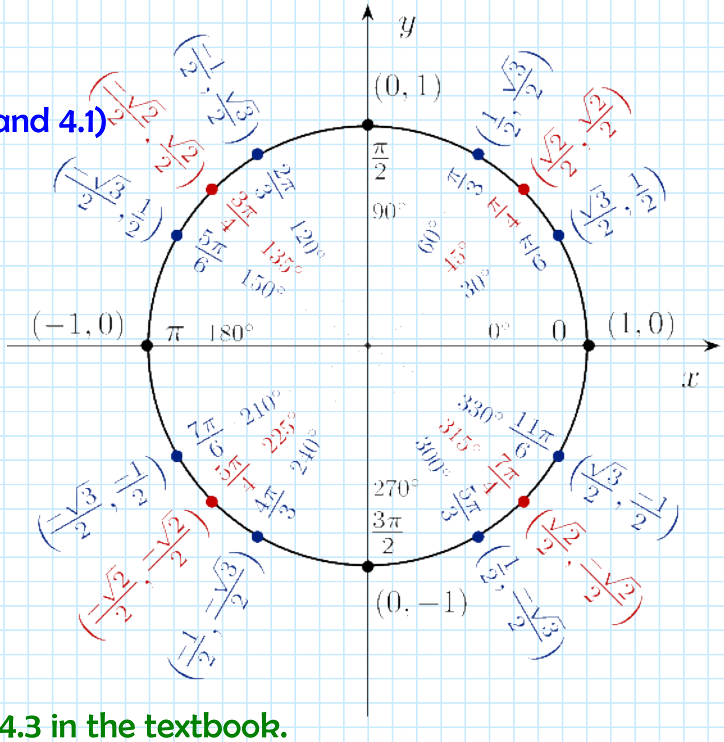


Plan For Today:

1. Question about anything from last week? (Ch3 and 4.1)
    - ▶ Do Test 2
    - ▶ Formula sheet provided
2. Continue Chapter 4:
    - ✓ 4.1: Angles and Angle Measure
    - ▶ **4.2: The Unit Circle**
    - ▶ **4.3: Trig Ratios**
    - ▶ 4.4: Intro to Trig Equations
3. Work on practice questions from Textbook

Page 186: #1c, 2ace, 3ac, 4, 5
Page 201: 1-2 (acegik), 3ace, 6ace, 9ace, 10 all, 11 all, 12ac

Plan Going Forward:

1. Finish going through practice question from 4.2-4.3 in the textbook.

- **CHAPTER 4 ASSIGNMENT DUE WEDNESDAY, MAY 17TH**

2. You will finish Chapter 4 Trigonometry on Tuesday (tomorrow). Have a look through the last sections in ch4 to prepare for tomorrow.

SCHOOL CLOSED ON MONDAY, MAY 22ND FOR VICTORIA DAY LONG WEEKEND

- **CHAPTER 5 ASSIGNMENT DUE TUESDAY, MAY 23RD**
- **TEST 3 ON TUESDAY, MAY 23RD (ON 4.2-5.4 OMIT 5.3, 6.1)**

Please let us know if you have any questions or concerns about your progress in this course.
The notes from today will be posted at egolfmath.weebly.com after class.
Anurita Dhiman = adhiman@sd35.bc.ca
Susana Egolf = segolf@sd35.bc.ca

C_09 Key and Trig Pr 3

Wednesday, October 10, 2018 11:22 AM

C_09 Trig Practice 3

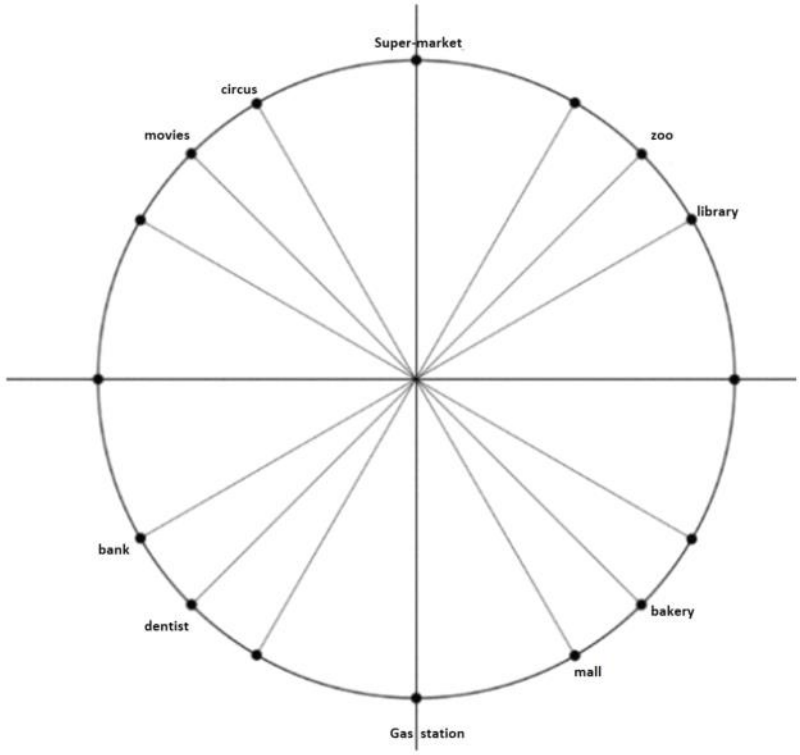
See solutions below

Trigonometry Practice - #3

1. Evil math teachers have replaced the steering wheel on your car with an app that requires you to enter the standard position angle you want your car to rotate through, before it drives to a location. Additionally, this app doesn't work in degrees, but **only in radians.**

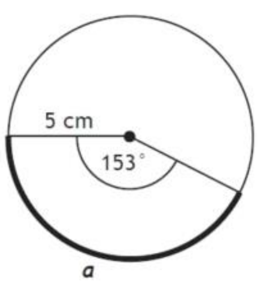
So……… What angle would you need to enter, if you want to go to:

1. The supermarket
2. The zoo
3. The gas station
4. The bank
5. The movies
6. The bakery
7. The library
8. The circus
9. The dentist
10. The mall

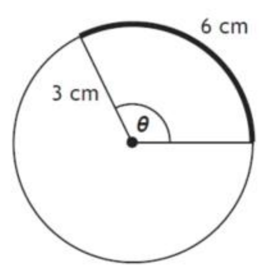


2. Find the requested information. Include units.

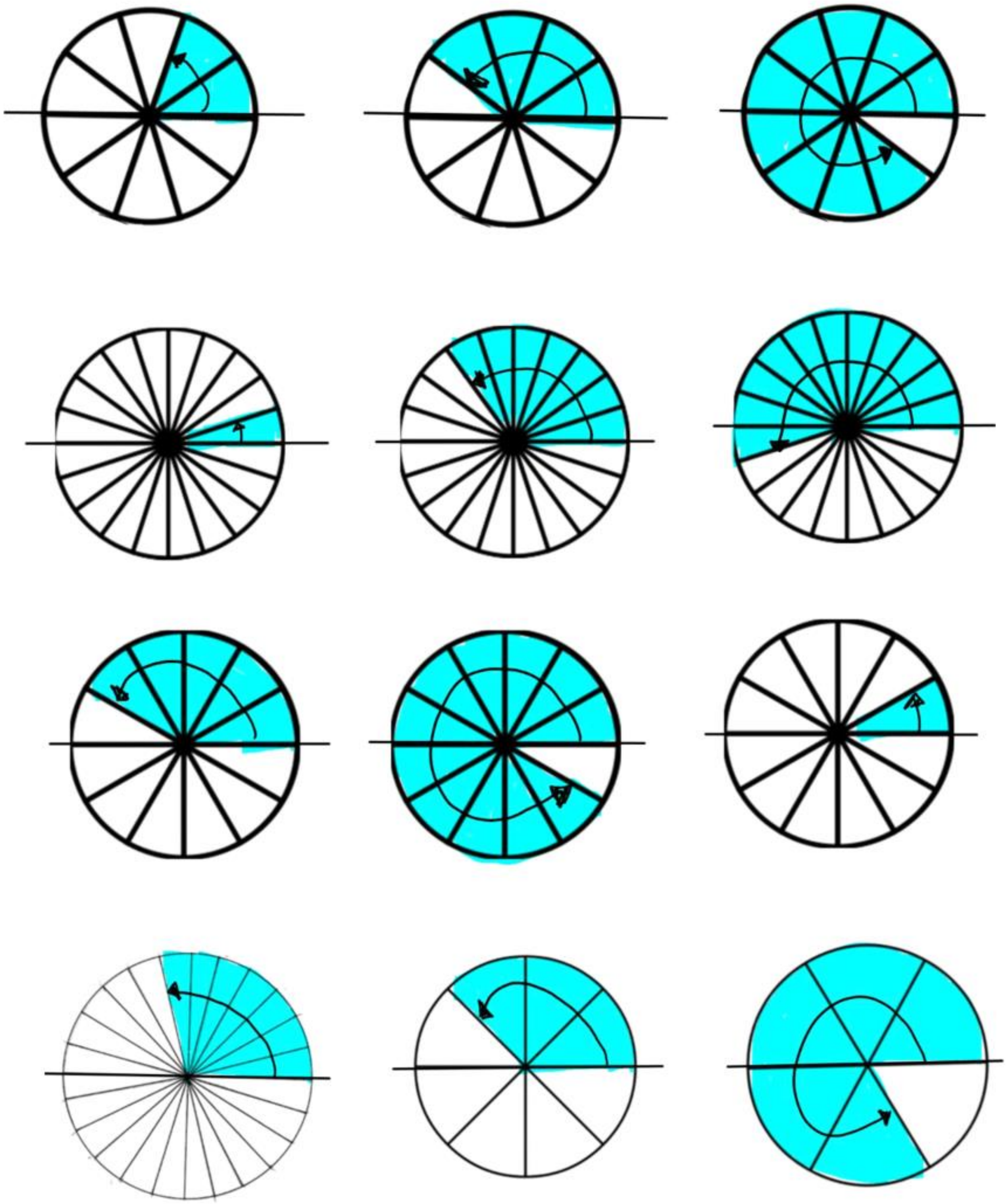
a) Solve for a, to the nearest hundredth.



b) Solve for θ. *(express your answer as a degree, to the nearest hundredth.)*



3 For each diagram, find the size of the smallest positive angle between the terminal arm of the given angle and the X-axis. (Answers should be in RADIANS.)



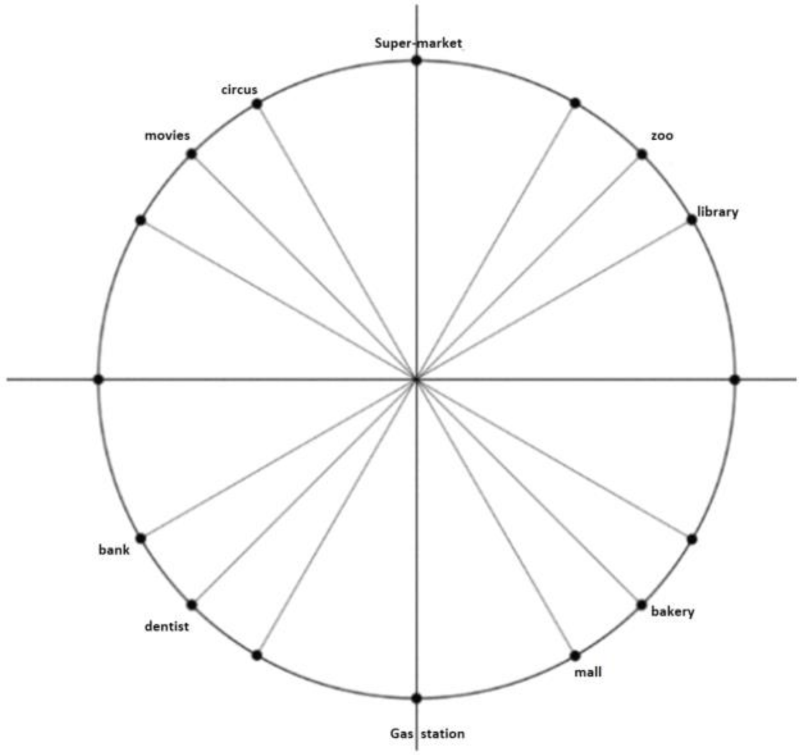
Key

Trigonometry Practice - #3

1. Evil math teachers have replaced the steering wheel on your car with an app that requires you to enter the standard position angle you want your car to rotate through, before it drives to a location. Additionally, this app doesn't work in degrees, but **only in radians.**

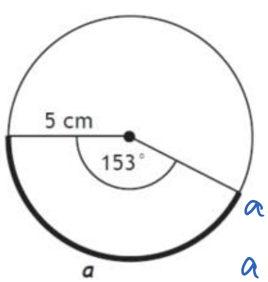
So……… What angle would you need to enter, if you want to go to:

1. The supermarket $\pi/2$
2. The zoo $\pi/4$
3. The gas station $\frac{3\pi}{2}$
4. The bank $\frac{7\pi}{6}$
5. The movies $\frac{3\pi}{4}$
6. The bakery $\frac{7\pi}{4}$
7. The library $\frac{\pi}{6}$
8. The circus $\frac{2\pi}{3}$
9. The dentist $\frac{5\pi}{4}$
10. The mall $\frac{5\pi}{3}$



2. Find the requested information. Include units.

a) Solve for a, to the nearest hundredth.

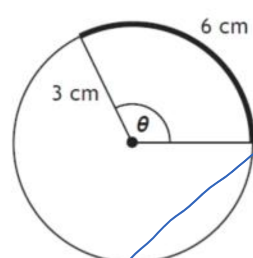


$a = r\,\theta$

$a = 5 \times 153^\circ \left(\frac{\pi}{180^\circ}\right)$

$a \doteq \boxed{13.35\text{ cm}}$

b) Solve for θ. *(express your answer as a degree, to the nearest hundredth.)*



$a = r\,\theta$

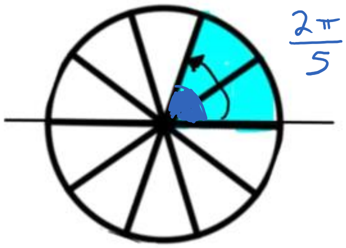
$\frac{6}{3} = \frac{3}{3}\theta$

$2 = \theta$

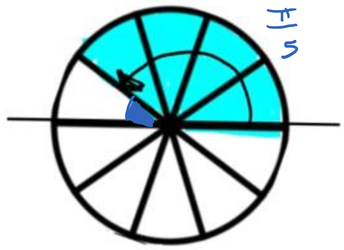
$\theta = 2$ radians

Convert: $\frac{2}{1} \times \left(\frac{180^\circ}{\pi}\right) \doteq \boxed{114.59^\circ}$

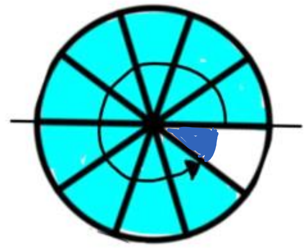
3 For each diagram, find the size of the smallest positive angle between the terminal arm of the given angle and the X-axis. (Answers should be in RADIANS.)



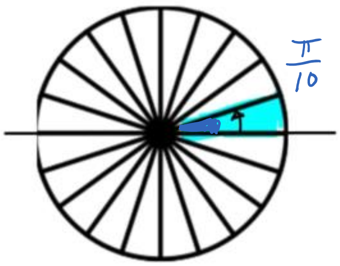
$\frac{2\pi}{5}$



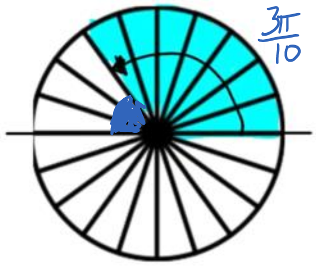
$\frac{\pi}{5}$



$\frac{\pi}{5}$

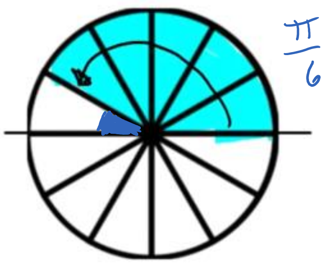


$\frac{\pi}{10}$

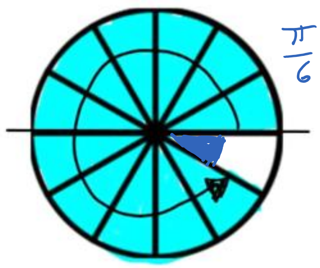


$\frac{3\pi}{10}$

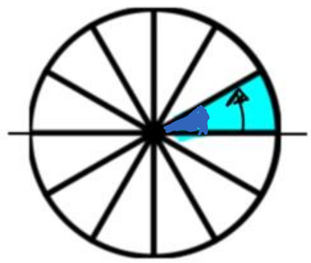
$\frac{\pi}{10}$



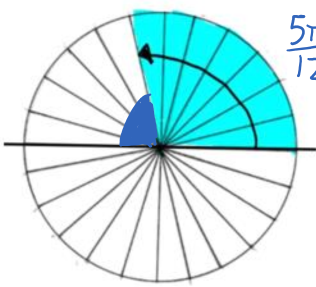
$\frac{\pi}{6}$



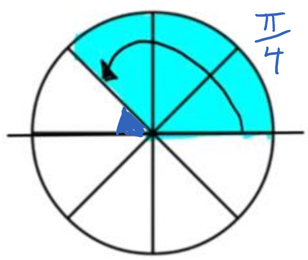
$\frac{\pi}{6}$



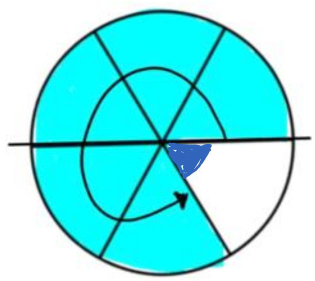
$\frac{\pi}{6}$



$\frac{5\pi}{12}$



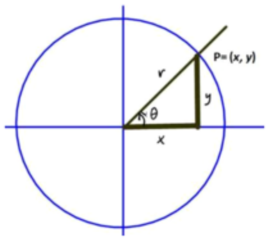
$\frac{\pi}{4}$



$\frac{\pi}{3}$

4.2 The Unit Circle

A circle is the set of all points that are a certain distance, ***radius***, from a given point, the center. Using the Pythagorean Theorem, we can get an equation for a circle.



The equation for a circle with center $(0,0)$ and radius ***r*** is:

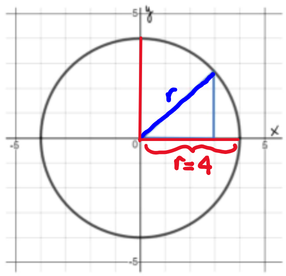
$$x^2 + y^2 = r^2$$

Try

a) Find the equation of this circle.

4^2

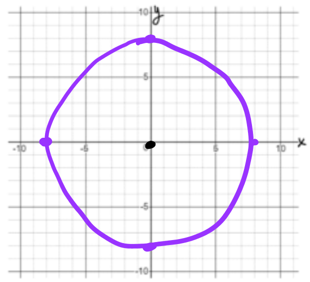
$x^2+y^2=16$



b) Sketch the graph of $x^2+y^2=64$

$\sqrt{r^2}=\sqrt{64}$

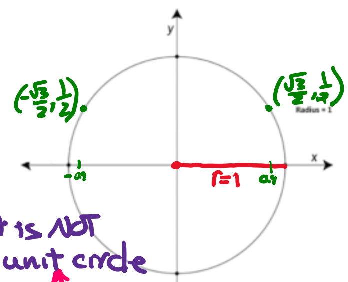
$r=8$



Unit Circle

If we choose $r=1$, we get a circle with radius 1 unit in length. This is called the ***unit circle***, and its equation is $x^2+y^2=1$.

(x, y)



a) Is the point $(0.6, 0.4)$ on the unit circle?

$\underbrace{(0.6)^2 + (0.4)^2}_{0.52} = 1$

$0.52 \neq 1$ ∴ this point is NOT on the unit circle

practice p.186 #2b

#7 on ch4 Assist

b) The point below is on the unit circle. Use the unit circle equation, $x^2+y^2=1$, to find the value of the unknown coordinate.

$\left(x, \frac{1}{2}\right)$

$x^2 + \left(\frac{1}{2}\right)^2 = 1$

$x^2 + \frac{1}{4} = 1$

$-\frac{1}{4}$

$x^2 = \frac{4}{4} - \frac{1}{4}$

$\sqrt{x^2} = \sqrt{\frac{3}{4}}$

$x = \pm\frac{\sqrt{3}}{2}$

$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

Two coordinates

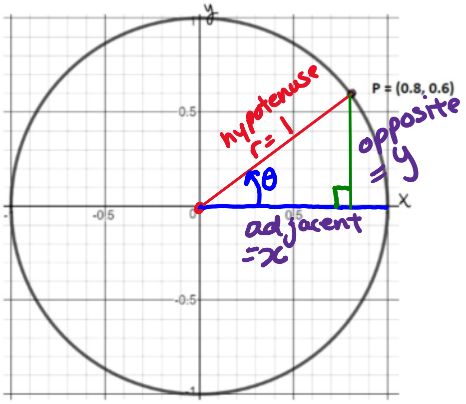
$\left(\frac{\sqrt{5}}{8}, \frac{7}{8}\right)$ on unit circle?

$\left(\frac{\sqrt{5}}{8}\right)^2 + \left(\frac{7}{8}\right)^2 = 1$

$\underbrace{\frac{5}{64} + \frac{49}{64}}_{\frac{54}{64}}$

$\frac{54}{64} \neq 1$ ∴ NOT on unit circle.

On the unit circle below, we have a point P, with coordinates (0.8, 0.6). We draw a line segment connecting P to the origin, (0, 0). This radius and the x-axis form a standard position angle, which we call θ. Because this is an accurate drawing, we could use a protractor and get the size of angle θ – it is about 36.87°. By drawing in a line segment that connects P to the x-axis, we create a right-triangle, with the right-angle on the x-axis.



From the diagram, we get:

SOH ↓ $\sin\theta = \frac{opp}{hyp.}$ CAH ↓ $\cos\theta = \frac{adj}{hyp}$ TOA ↓ $\tan\theta = \frac{opp}{adj}$

$\cos\theta = \frac{adj}{hyp} = \frac{x}{1} = x$

$\sin\theta = \frac{opp}{hyp} = \frac{y}{1} = y$ $\tan\theta = \frac{opp}{adj} = \frac{y}{x}$

Using the calculator, we get:

$\cos 36.87^\circ = 0.8$ ← x

$\sin 36.87^\circ = 0.6$ ← y

36.87° ← degrees ∴ DEGREE MODE

> Let $P(\theta) = (x, y)$ be the point where the terminal arm of a standard-position angle θ intersects the **unit circle**. Then we know:
> - the x-coordinate's value is equal to the cosine of the angle $x = \cos\theta$
> - the y-coordinate's value is equal to the sine of the angle $y = \sin\theta$

We now have a way to find sine and cosine values for ANY angle, including:

- negative angles
- 0°
- angles larger than 90°

Using the triangle definitions (SOHCAHTOA) for those types of angles doesn't really make sense. For example, what would be the adjacent, opposite, and hypotenuse lengths for an angle measuring 0°?

Try – NO calculator. *(You don't need it! You can figure them out yourself!)* = quadrant angles.

$P(0^\circ) = (1, 0)$ $\cos(0^\circ) = 1$ (x-coordinate) $\sin(0^\circ) = 0$ (y-coordinate)

$P\left(\frac{3\pi}{2}\right) = (0, -1)$ $\sin\left(\frac{3\pi}{2}\right) = -1$ y-coord. $\cos\left(\frac{3\pi}{2}\right) = 0$ x-coord.

Finding Approximate Values of Trigonometric Ratios

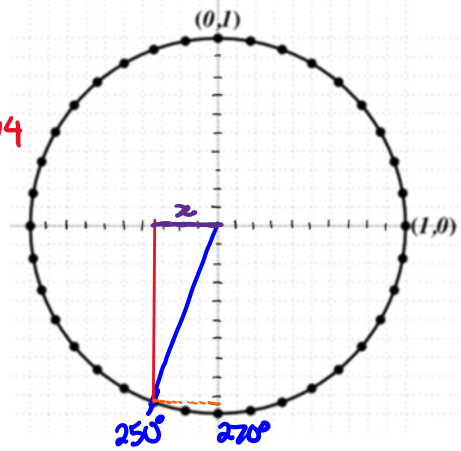
Estimate each value using the graph at right. Compare with the calculator answer, correct to 4 decimal places.

a) $\cos 250^{\circ} =$ ≈ -0.34 $\quad \sin 250^{\circ} =$ ≈ -0.94

↳ calc. = -0.34 $\quad$ ↳ calc. = -0.94

b) $\cos 500^{\circ} =$ - 0.77 $\quad \sin 500^{\circ} =$ 0.64

c) $\cos(-10^{\circ}) =$ 0.98 $\quad \sin(-10^{\circ}) =$ -0.17

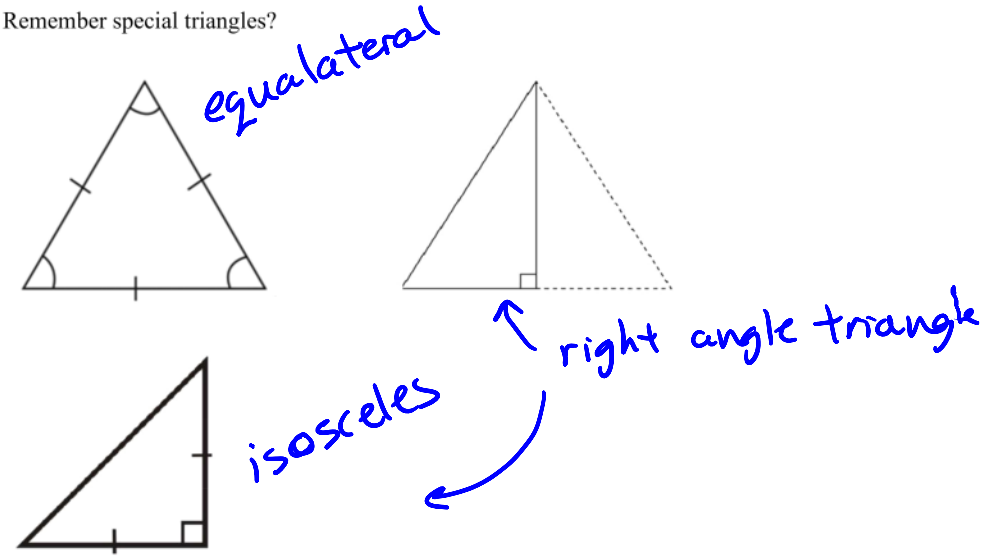


http://www.malinc.se/math/trigonometry/unitcircleen.php

Special Triangle Angles

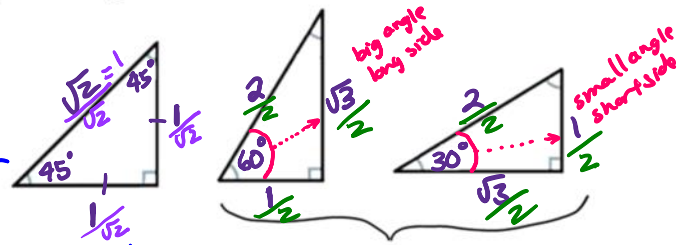
Besides the quadrantal angles, there are some other angles for which we can find ***exact*** coordinates for $P(\theta)$. These angles relate to special triangles.

Remember special triangles?



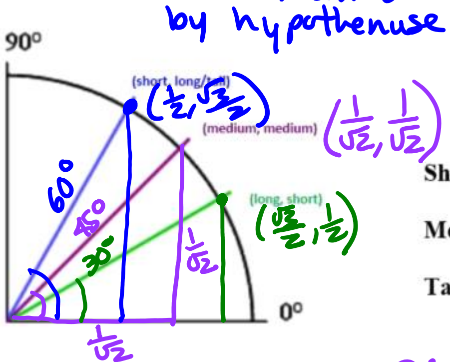
pythagorean theorem
$a^2+b^2=c^2$
$x^2+y^2=r^2$

Let's use those triangle angles in the unit circle setting. We need to adjust the size of the triangles, making their hypotenuse length = 1.



Same triangle – just turned on its side.

fit into unit circle ÷ all sides by hypothenuse.



Short side length = $\frac{1}{2}$

Medium side length = $\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)=\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$ rationalized.

Tall/long side length = $\frac{\sqrt{3}}{2}$

Recall: $30^\circ=\frac{\pi}{6}$ $45^\circ=\frac{\pi}{4}$ $60^\circ=\frac{\pi}{3}$

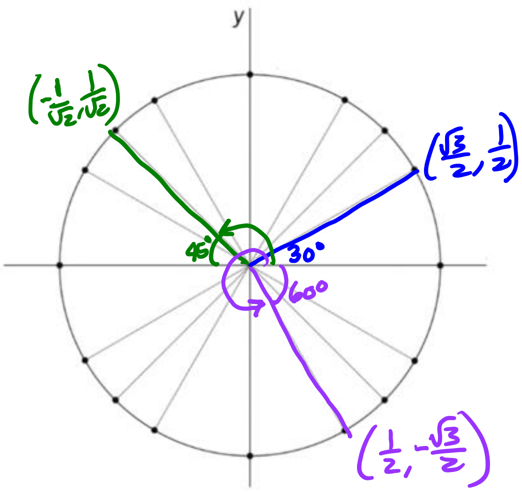
We know that the shortest side of a triangle is across from its smallest angle. This helps us label the coordinates correctly for different angles.

Try – NO calculator. Get the ***exact values***.

$\cos(30^\circ)=\frac{\sqrt{3}}{2}$ $\sin(30^\circ)=\frac{1}{2}$

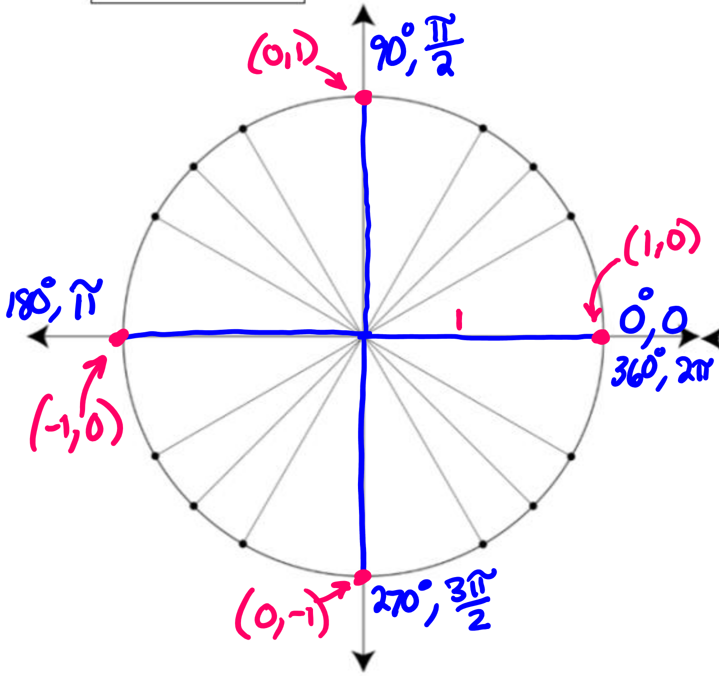
$\sin(135^\circ)=\frac{1}{\sqrt{2}}$ $\cos(135^\circ)=-\frac{1}{\sqrt{2}}$

$\cos(300^\circ)=\frac{1}{2}$ $\sin(300^\circ)=-\frac{\sqrt{3}}{2}$

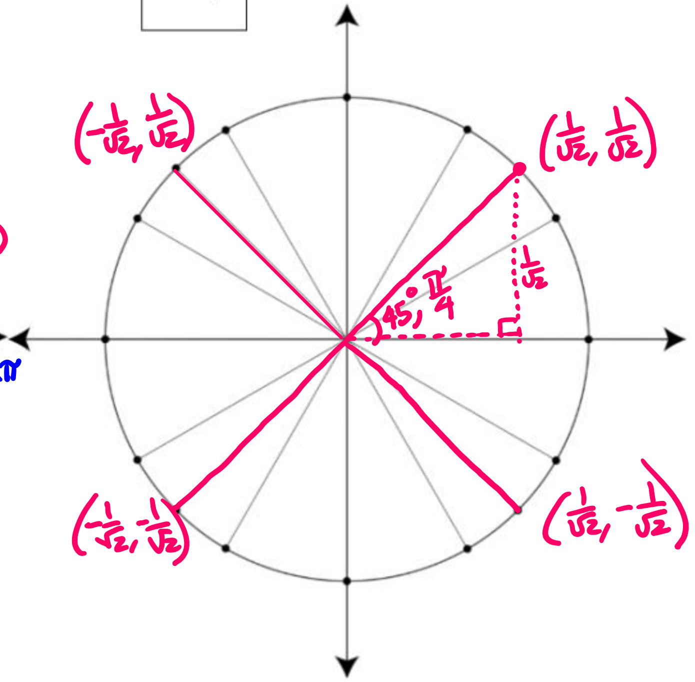


Special Angles

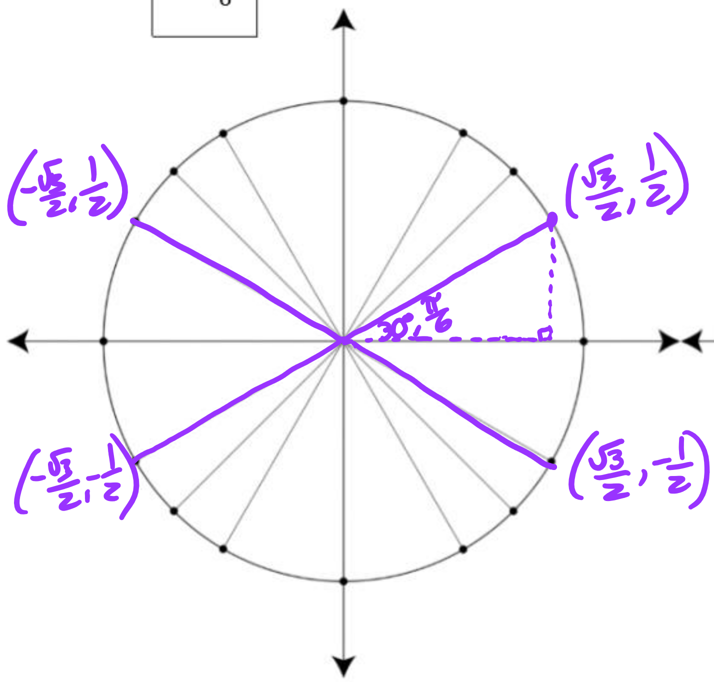
Quadrant Angles



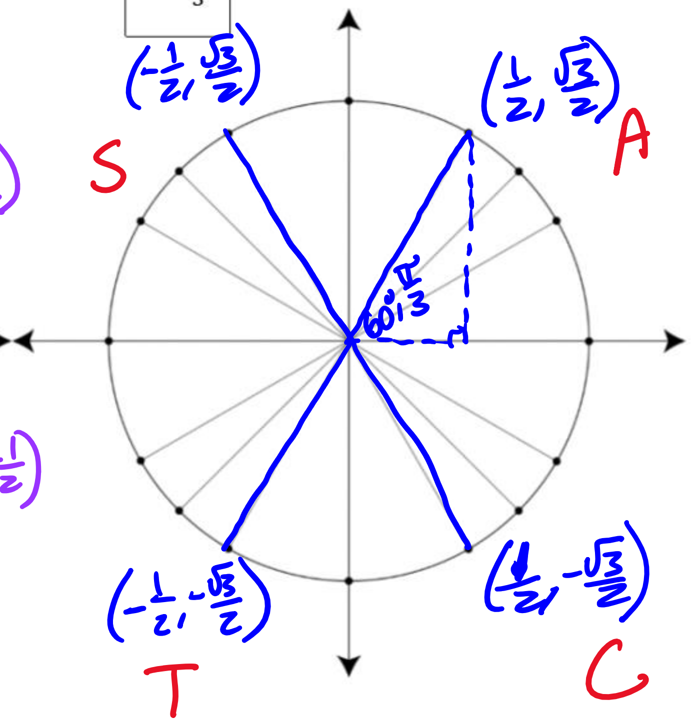
$45°, \frac{\pi}{4}$



$30°, \frac{\pi}{6}$



$60°, \frac{\pi}{3}$



$\cos\theta = x \quad \sin\theta = y \quad \tan\theta = \frac{y}{x}$

A, S, T, C

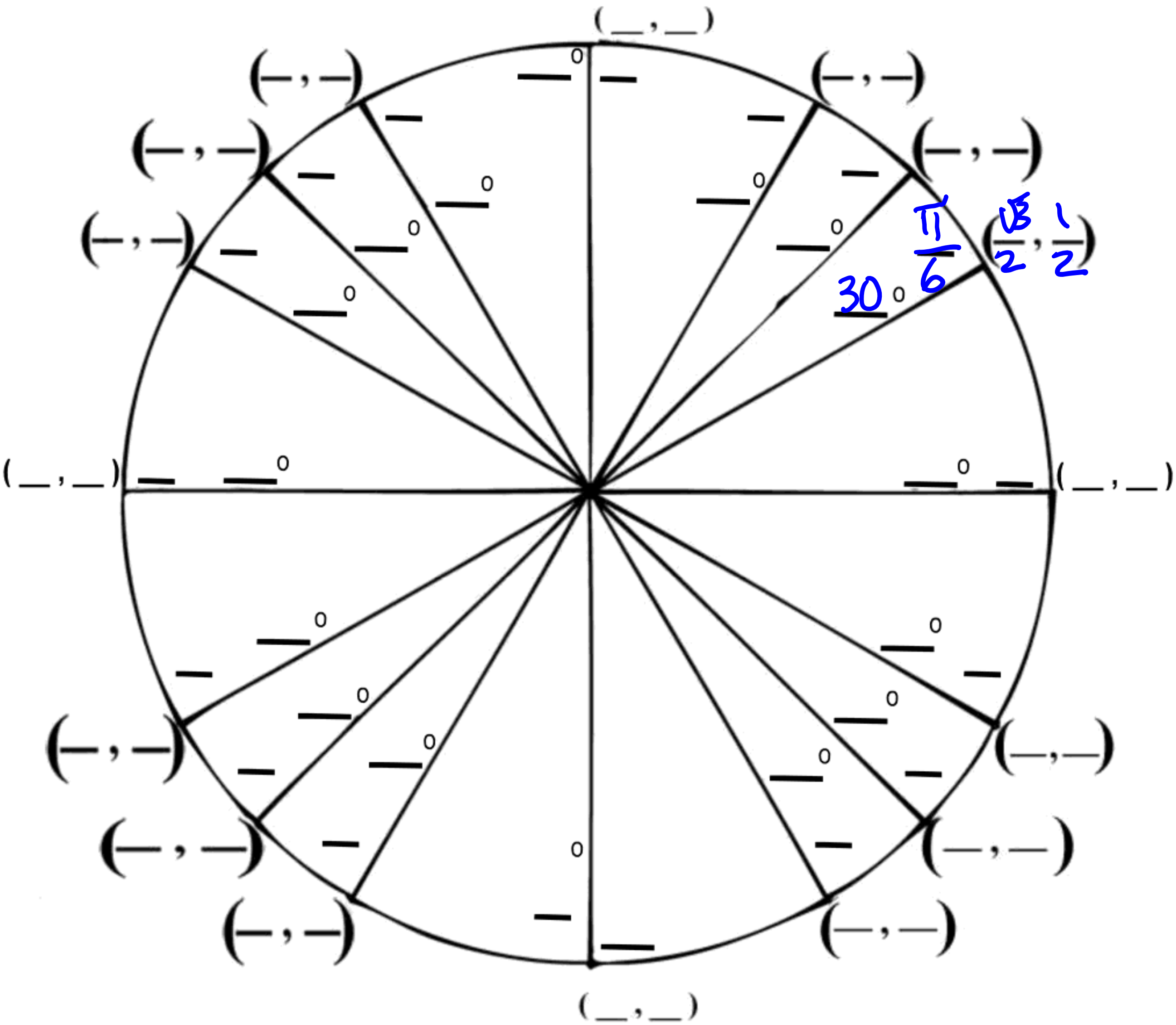
all ratios + QI

sine + in QII

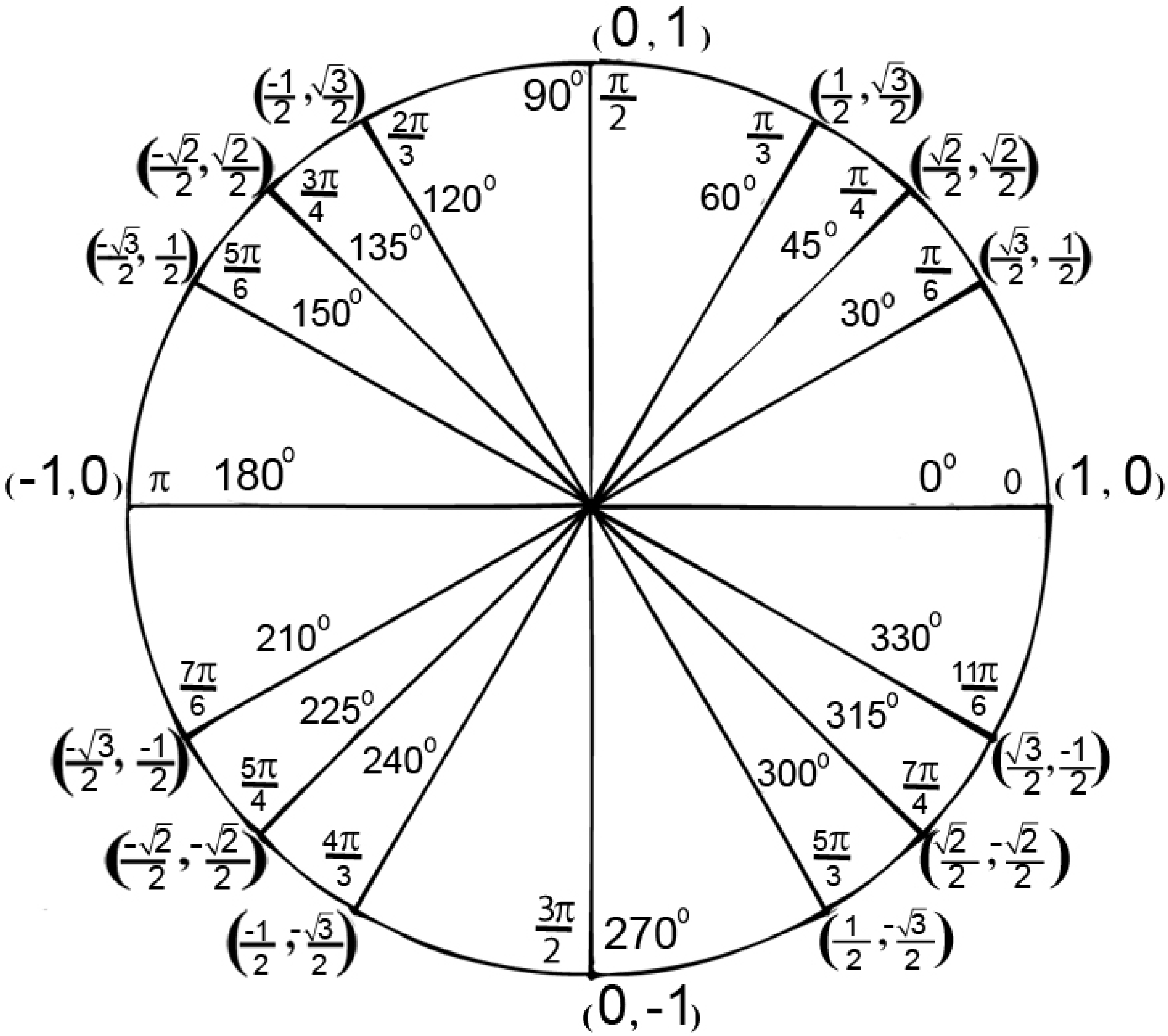
tan + in QIII

cos + in QIV

Unit Circle, Fill in the blank



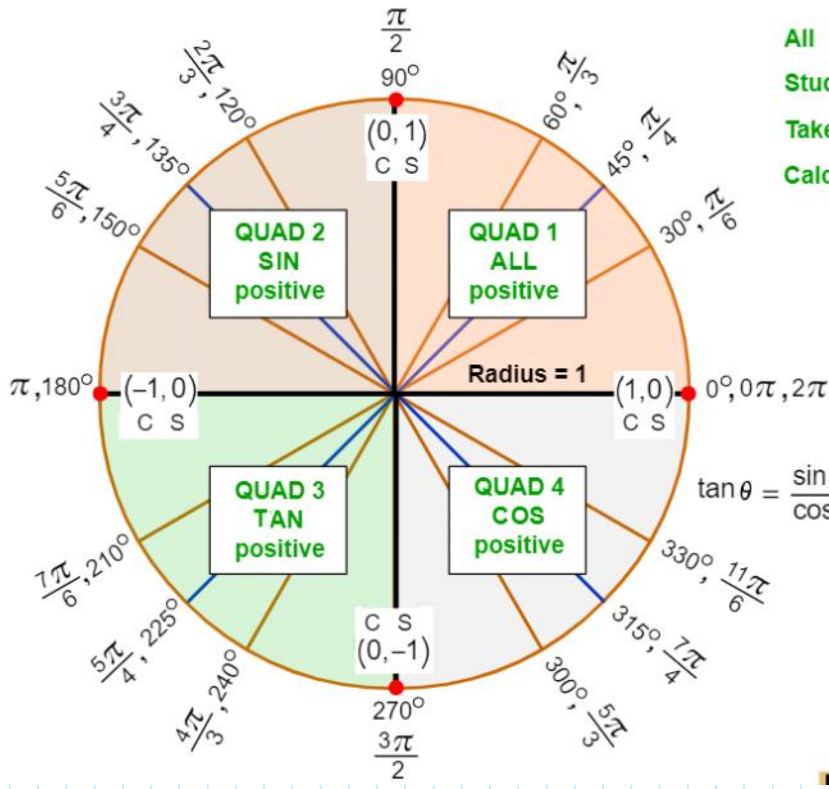
Unit Circle, Fill in the blank



www.mathwarehouse.com/unit-circle

3.6 Exact Values of Special Angles

THE TRIG CIRCLE



All

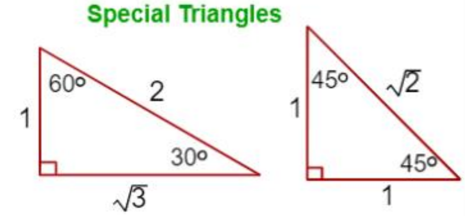
Students

Take

Calculus

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{S}{C}$

Special Triangles



$\sin = \frac{\text{opp}}{\text{hyp}}$

$\cos = \frac{\text{adj}}{\text{hyp}}$

$\tan = \frac{\text{opp}}{\text{adj}}$

	30°	45°	60°
sin	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
tan	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

Reciprocal Trig Ratios

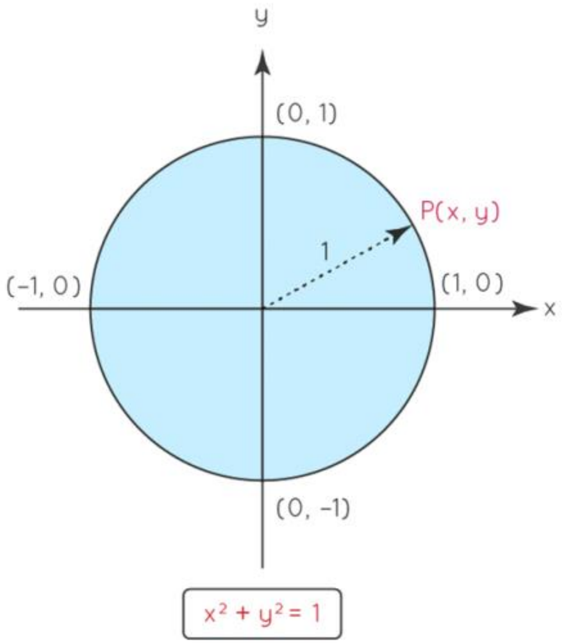
cosecant $\longrightarrow \csc\theta = \frac{1}{\sin\theta}$

secant $\longrightarrow \sec\theta = \frac{1}{\cos\theta}$

cotangent $\longrightarrow \cot\theta = \frac{1}{\tan\theta}$

4.2-4.3 Unit Circle & Trig Ratios

Equation of a Unit Circle & Coordinates in a Unit Circle



Example 1 – A Point on the Unit Circle

Show that the point $P\left(\frac{\sqrt{3}}{3}, \frac{\sqrt{6}}{3}\right)$ is on the unit circle.

Solution:
We need to show that this point satisfies the equation of the unit circle, that is, $x^2 + y^2 = 1$.

Since

$$\left(\frac{\sqrt{3}}{3}\right)^2 + \left(\frac{\sqrt{6}}{3}\right)^2 = \frac{3}{9} + \frac{6}{9} = 1$$

P is on the unit circle.

2

$$x^2 + y^2 = 1$$

$$Here, x = \frac{2}{5}$$

$$\left(\frac{2}{5}\right)^2 + y^2 = 1$$

$$\frac{2\times 2}{5\times 5} + y^2 = 1$$

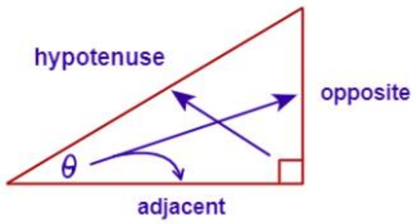
$$\frac{4}{25} + y^2 = 1$$

$$y^2 = 1 - \frac{4}{25}$$

$$y^2 = \frac{25-4}{25} = \frac{21}{25}$$

$$y = \pm\frac{\sqrt{21}}{5}$$

Review of Trig Ratios:



These definitions are only useful for acute angles.

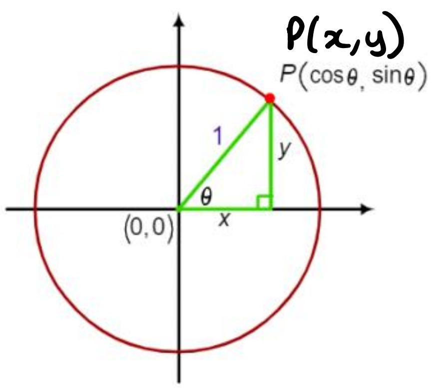
$$\sin\theta = \frac{\text{length of side opposite } \theta}{\text{length of hypotenuse}}$$ SOH

$$\cos\theta = \frac{\text{length of side adjacent } \theta}{\text{length of hypotenuse}}$$ CAH

$$\tan\theta = \frac{\text{length of side opposite } \theta}{\text{length of side adjacent } \theta}$$ TOA

Trig Ratios in a Unit Circle:

p.192



Unit Circle Definition of Sine and Cosine Functions

$$\sin\theta = \frac{\text{length of side opposite } \theta}{\text{length of hypotenuse}}$$

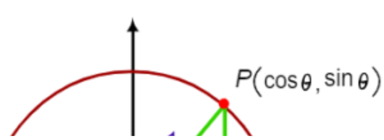
$$\sin\theta = \frac{y}{1}$$

$$\sin\theta = y$$

$$\cos\theta = \frac{\text{length of side adjacent } \theta}{\text{length of hypotenuse}}$$

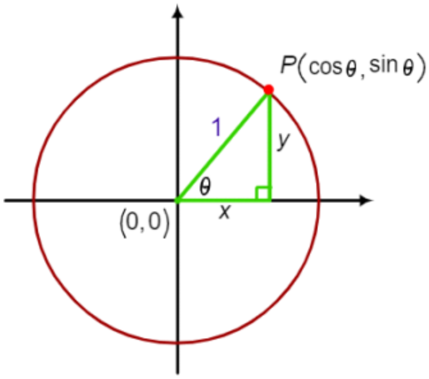
$$\cos\theta = \frac{x}{1}$$

$$\cos\theta = x$$



Unit Circle Definition of The Tangent Function

$$\tan\theta = \frac{\text{length of side opposite } \theta}{\text{length of side adjacent } \theta}$$



Unit Circle Definition of The Tangent Function

$$\tan\theta = \frac{\text{length of side opposite } \theta}{\text{length of side adjacent } \theta}$$

$$\tan\theta = \frac{y}{x} \qquad x \neq 0$$

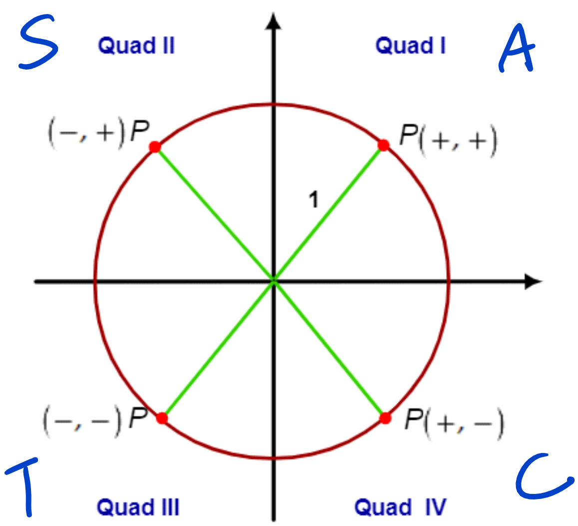
$$\cos\theta = x$$

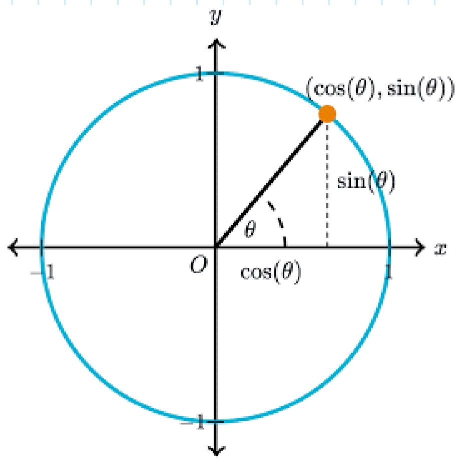
$$\sin\theta = y$$

$$\tan\theta = \frac{y}{x} = \frac{\sin\theta}{\cos\theta} \qquad \cos\theta \neq 0$$

Signs of trig ratios in each quadrant:

All Students Take Calculus





$$\tan\theta = \frac{y}{x}$$

$$\cos\theta = x$$

$$\sin\theta = y$$

Values of cosine, sine, tangent, and the reciprocals in a Unit Circle

In summary: in a unit circle.

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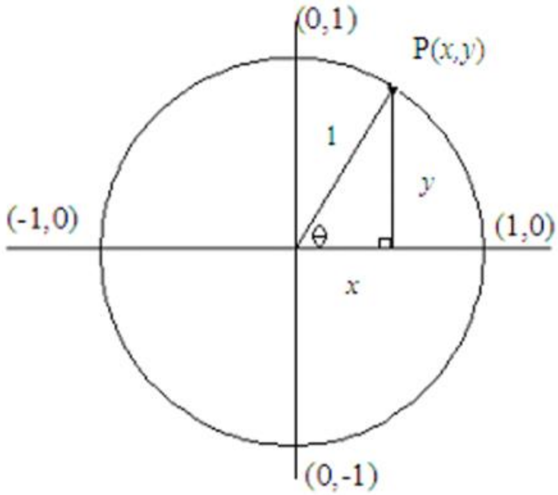
$$\sin\theta = \frac{O}{H} = \frac{y}{1} = y$$

$$\cos\theta = \frac{A}{H} = \frac{x}{1} = x$$

$$\tan\theta = \frac{0}{A} = \frac{y}{x}$$



Reciprocals:

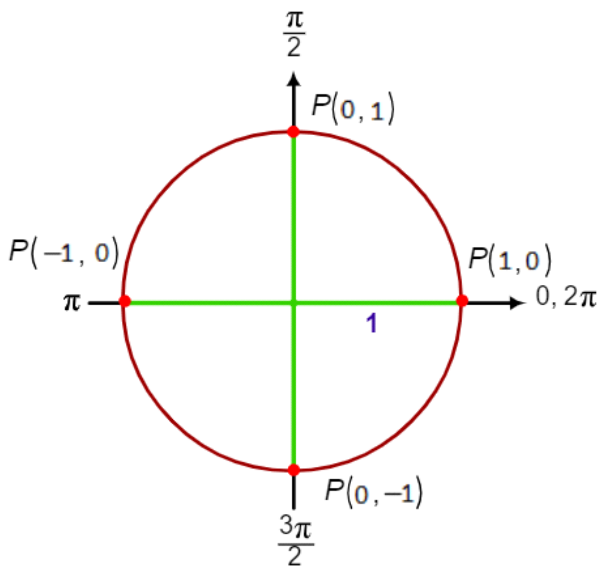
$$\csc\theta = \frac{H}{O} = \frac{1}{y}$$

$$\sec\theta = \frac{H}{A} = \frac{1}{x}$$

$$\cot\theta = \frac{A}{H} = \frac{x}{y}$$

Quadrant Angles

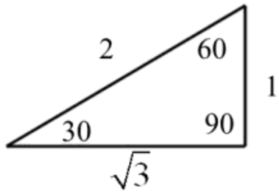
$\cos\theta = x \quad \sin\theta = y \quad \tan\theta = \frac{y}{x}$

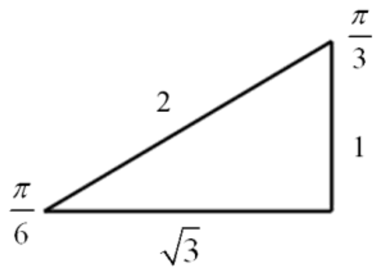


30-45-60 Special Triangles

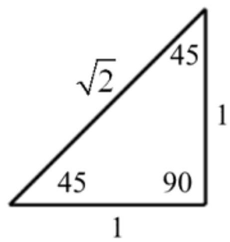
Also, Two special triangles

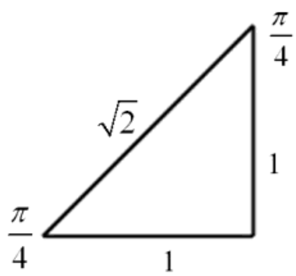
30, 60, 90 triangle

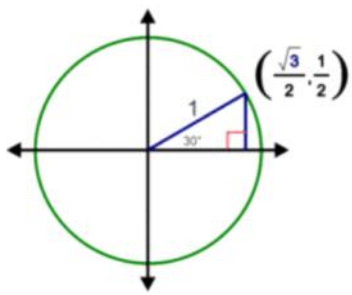


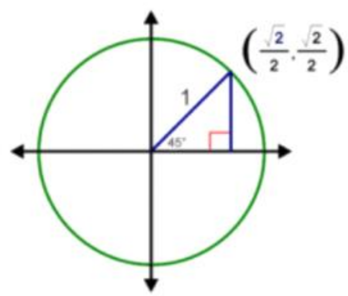


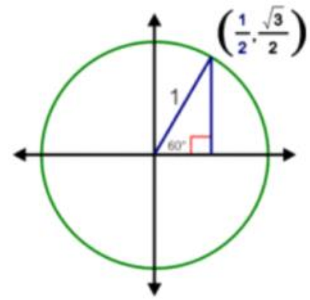
45, 45, 90 triangle





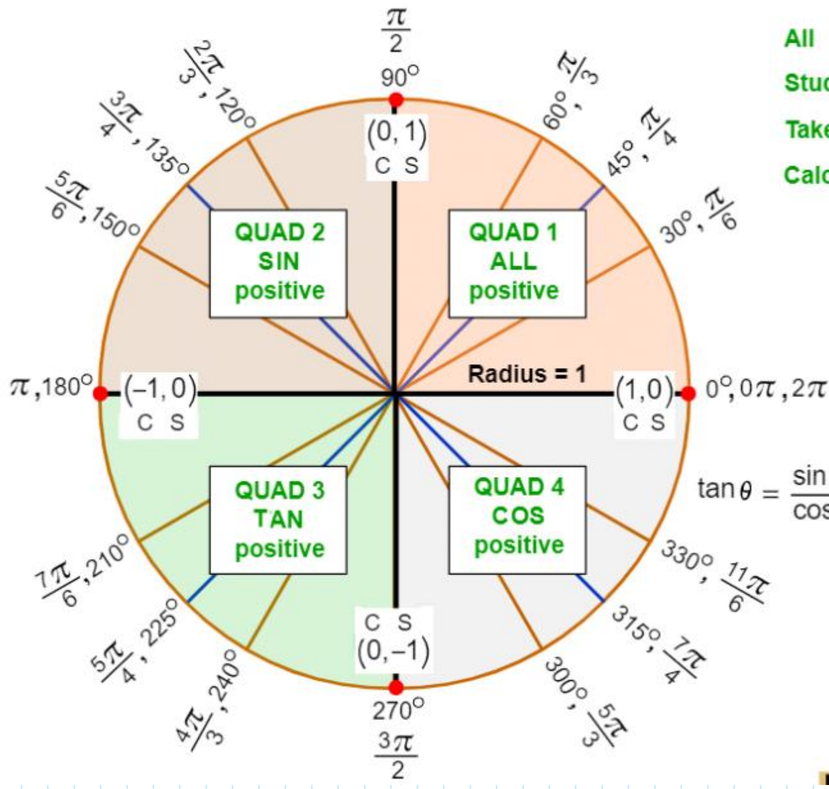






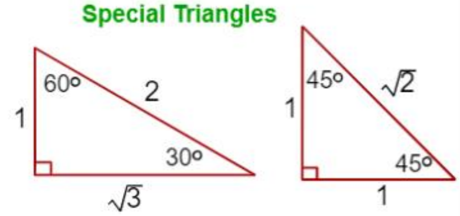
3.6 Exact Values of Special Angles

THE TRIG CIRCLE



All
Students
Take
Calculus

Special Triangles



$\sin = \frac{\text{opp}}{\text{hyp}}$

$\cos = \frac{\text{adj}}{\text{hyp}}$

$\tan = \frac{\text{opp}}{\text{adj}}$

	30°	45°	60°
sin	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
tan	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{S}{C}$$

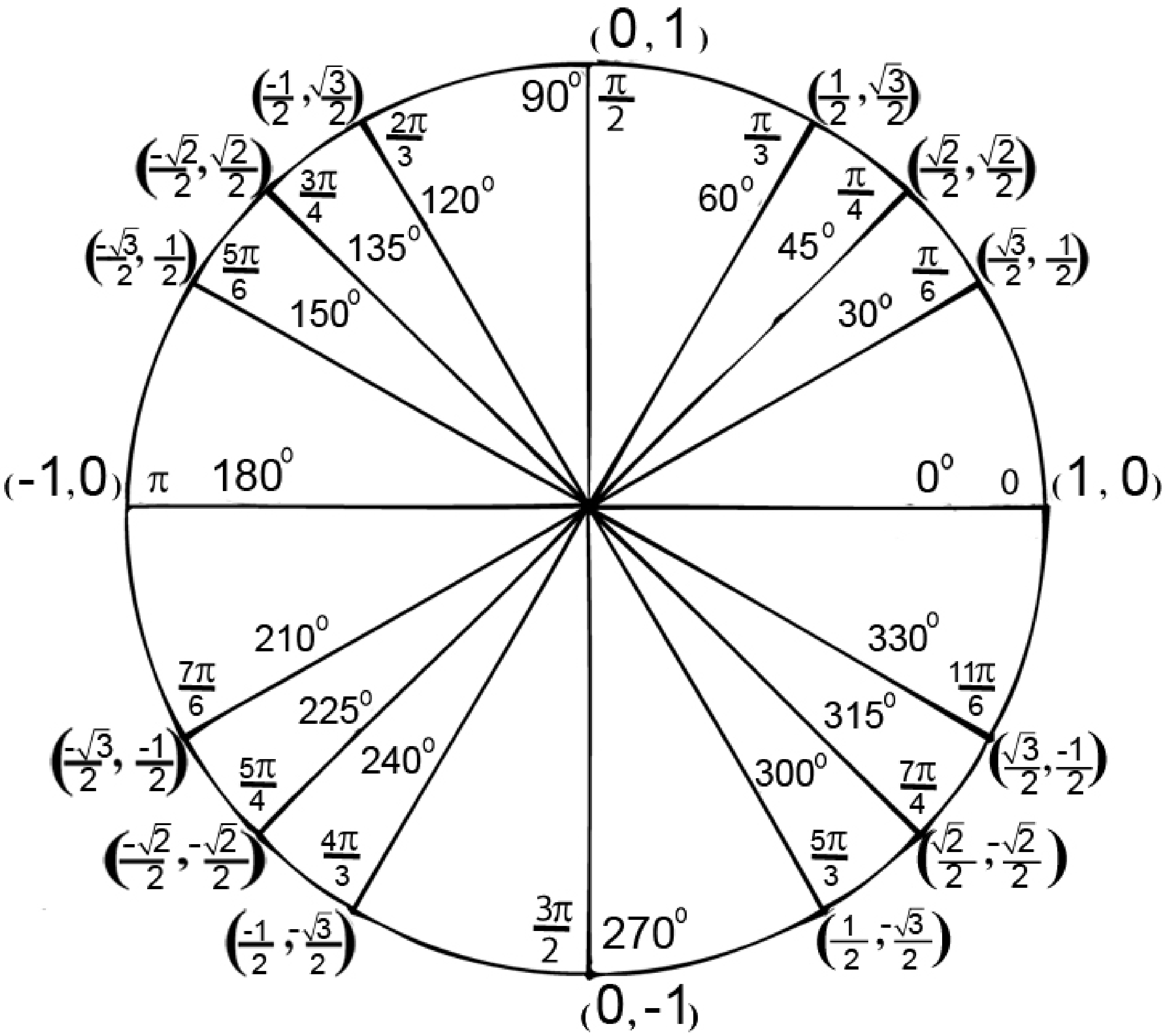
Reciprocal Trig Ratios

cosecant $\longrightarrow \csc\theta = \frac{1}{\sin\theta}$

secant $\longrightarrow \sec\theta = \frac{1}{\cos\theta}$

cotangent $\longrightarrow \cot\theta = \frac{1}{\tan\theta}$

Unit Circle, Fill in the blank



www.mathwarehouse.com/unit-circle

Coordinates of Points NOT on a Unit Circle

Ex 5
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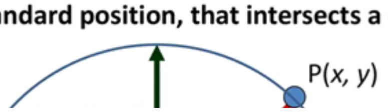
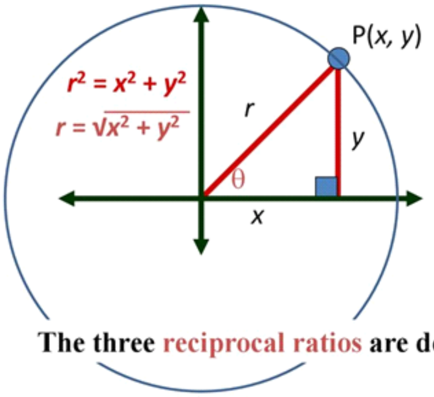
Point P(*x, y*) is the point on the terminal arm of angle θ ,an angle in standard position, that intersects a circle.



$\sin\theta = \frac{y}{}$ $\csc\theta = \frac{r}{}$

Point P(x, y) is the point on the terminal arm of angle θ ,an angle in standard position, that intersects a circle.



$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}$$

$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

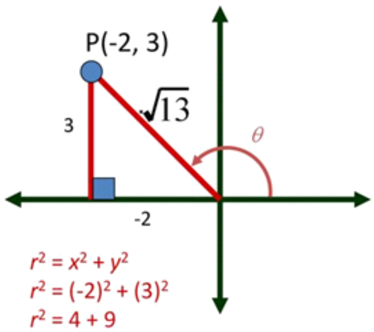
The three reciprocal ratios are defined as follows:

$$\text{cosecant} = \frac{1}{\text{sine}} \qquad \text{secant} = \frac{1}{\text{cosine}} \qquad \text{cotangent} = \frac{1}{\text{tangent}}$$

Finding the Trig Ratios of an Angle in Standard Position

The point P(-2, 3) is on the terminal arm of θ in standard position.

Does point P(-2, 3) lie on the unit circle? No, the radius of a unit circle is 1.



$r^2 = x^2 + y^2$
$r^2 = (-2)^2 + (3)^2$
$r^2 = 4 + 9$
$r^2 = 13$
$r = \sqrt{13}$

Determine the exact value of the six trigonometric ratios for angle θ.

$$\sin\theta = \frac{3}{\sqrt{13}} \qquad \csc\theta = \frac{\sqrt{13}}{3}$$

$$\cos\theta = -\frac{2}{\sqrt{13}} \qquad \sec\theta = -\frac{\sqrt{13}}{2}$$

$$\tan\theta = -\frac{3}{2} \qquad \cot\theta = -\frac{2}{3}$$