Class_08 Jan 31 - Factoring

Saturday, January 28, 2023 6:28 PM

Tonight's Class:

- **Any questions from Chapter 2?**
- **Chapter 2 Test - closed book, but can use both foldables**
- **Working through sections 3.1 and 3.2**
  - **Factoring trinomials, leading coefficient not equal to 1**
  - **Factoring polynomial expressions**

Recap question

b) $$\frac{(3\sqrt{5}+\sqrt{2})}{(\sqrt{5}+\sqrt{3})} \cdot \frac{(\sqrt{5}-\sqrt{3})}{(\sqrt{5}-\sqrt{3})} = \frac{3\sqrt{5}\sqrt{5} - 3\sqrt{5}\sqrt{3} + \sqrt{2}\sqrt{5} - \sqrt{2}\sqrt{3}}{\sqrt{5}\sqrt{5} - \sqrt{5}\sqrt{3} + \sqrt{3}\sqrt{5} - \sqrt{3}\sqrt{3}}$$

$$= \frac{15 - 3\sqrt{15} + \sqrt{10} - \sqrt{6}}{5 - \cancel{\sqrt{15}} + \cancel{\sqrt{15}} - 3}$$

$$= \frac{15 - 3\sqrt{15} + \sqrt{10} - \sqrt{6}}{2}$$

p122 #6b)

$$\frac{1}{5\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{5\sqrt{3}\sqrt{3}} = \frac{\sqrt{3}}{5\cdot 3} = \frac{\sqrt{3}}{15}$$

$$\frac{1}{5\sqrt{3}} \cdot \frac{5\sqrt{3}}{5\sqrt{3}} = \frac{5\sqrt{3}}{25\cdot 3} = \frac{5^{\div 5}\sqrt{3}}{75_{\div 5}} = \frac{\sqrt{3}}{15}$$

p125, 10b)

$$\frac{(5\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})} \cdot \frac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})} = \frac{\overset{15}{5\cdot 3} + 5\sqrt{6} + \sqrt{6} + 2}{3 + \cancel{\sqrt{6}} - \cancel{\sqrt{6}} - 2}$$

$$= \frac{17 + 6\sqrt{6}}{1}$$

$$= 17 + 6\sqrt{6}$$

d) $$\frac{(6\sqrt{3}-2)}{(5+4\sqrt{2})} \cdot \frac{(5-4\sqrt{2})}{(5-4\sqrt{2})} = \frac{30\sqrt{3} - 24\sqrt{6} - 10 + 8\sqrt{2}}{25 - \cancel{20\sqrt{2}} + \cancel{20\sqrt{2}} - 16\underbrace{\sqrt{4}}_{2}}$$

d) $\frac{(6\sqrt{3}-2)}{(5+4\sqrt{2})}\cdot\frac{(5-4\sqrt{2})}{(5-4\sqrt{2})} = \frac{30\sqrt{3}-24\sqrt{6}-10+8\sqrt{2}}{25-\cancel{20\sqrt{2}}+\cancel{20\sqrt{2}}-\underbrace{16\sqrt{4}}_{32}}$

$= \frac{30\sqrt{3}-24\sqrt{6}-10+8\sqrt{2}}{-7}$

$= \frac{-30\sqrt{3}+24\sqrt{6}+10-8\sqrt{2}}{+7}$

Recap

$\frac{2}{\sqrt[4]{27}}\cdot\frac{\sqrt[4]{3}}{\sqrt[4]{3}} = \frac{2\sqrt[4]{3}}{\sqrt[4]{81}} = \boxed{\frac{2\sqrt[4]{3}}{3}}$

$2^4=16$

$3^4=81$

perfect $(\ )^4$

p 127 13a)

$\frac{1}{\sqrt{5}}-\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} - \frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{3}{3}\cdot\frac{\sqrt{5}}{5} - \frac{\sqrt{3}}{3}\cdot\frac{5}{5}$

$= \frac{3\sqrt{5}}{15} - \frac{5\sqrt{3}}{15}$

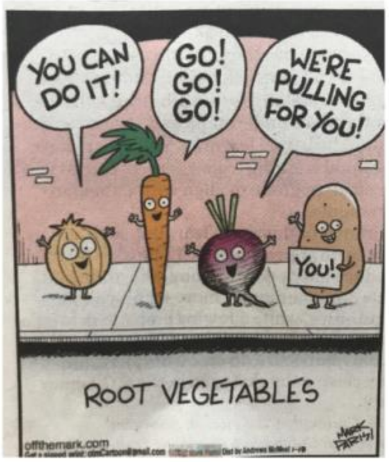
$= \frac{3\sqrt{5}-5\sqrt{3}}{15}$

$\frac{(5\sqrt{2}-4)}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{6}-4\sqrt{3}}{3}$

Please:

- **Make sure your name is on your <u>Chapter 2 Hand-in</u>, and turn it in.**
- **Put away your phone and all materials except for the "foldables," a calculator, and something to write with.**
- **On your test, write clearly and show all necessary steps - including on multiple-choice questions! When you are finished, please look over your test before handing it in.**
- **While other people are still finishing, respect them by being quiet. You can leave the classroom if you wish, but be back in time for the rest of class.**

They're back....



Preview 8

3.1 Factoring Trinomials, Leading Coefficient not 1

Focus: write trinomial as the product of two binomials, or of a constant and two binomials

Factor – "30" means write it as a product:

$3 \cdot 10$
$5 \cdot 6$
$2 \cdot 15$

these are all ways to factor 30.

factor completely – break it down into a product of prime numbers

$30 = 3 \cdot 10$

$30 = \boxed{3 \cdot 5 \cdot 2}$

$5 \cdot 6$

$= \boxed{5 \cdot 2 \cdot 3}$

$2 \cdot 15$

$= \boxed{2 \cdot 3 \cdot 5}$

Factoring polynomials, including trinomials (3 terms) is our focus here.

Greatest Common Factor (GCF) (always the first step!)

Factor Out the GCF

The first step to factoring is to factor out the greatest common factor (GCF) from each term.

Example: $12y^3 - 15y^2 + 6y$

$2\times2\times3\times y\times y\times y \qquad 5\times3\times y\times y \qquad 2\times3\times y$

$3y(4y^2 - 5y + 2)$

check: $12y^3 - 15y^2 + 6y$

$$\frac{12y^3}{3y} - \frac{15y^2}{3y} + \frac{6y}{3y}$$

$$4y^2 - 5y + 2$$

To determine the Greatest Common Factor of a polynomial, find the largest monomial that divides evenly into each term.

FIND THE GCF:

$$\frac{6x^2y}{2xy} + \frac{14xy^2}{2xy} + \frac{42xy}{2xy} + \frac{2x^2y^2}{2xy}$$

$$2xy(3x + 7y + 21 + xy)$$

How do we find the monomial?

- Break each term up into its prime factors
- Look for factors that appear in EACH term
- Multiply them together = GCF

$6x^2y = 2\cdot3\cdot x\cdot x\cdot y$

$14xy^2 = 2\cdot7\cdot x\cdot y\cdot y$

$42xy = 2\cdot3\cdot7\cdot x\cdot y$

$2x^2y^2 = 2\cdot x\cdot x\cdot y\cdot y$

42
6 7
2 3

After finding the GCF, divide it out of each term to get the factored form of the polynomial:

$$6x^2y + 14xy^2 + 42xy + 2x^2y^2$$

$$= 2xy(3x + 7y + 21 + xy)$$

Try: $9x^2y^3 + 18x^3y^2 + 15xy^2$

Factor out the GCF

$$3xy^2(3xy + 6x^2 + 5)$$

$$\frac{9x^2y^3}{3xy^2} = 3xy$$

check: $9x^2y^3 + 18x^3y^2 + 15xy^2$

Factoring Trinomials often, will be in this format:

$$ax^2 + bx + c$$

where a, b, and c are numbers

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review how we multiply binomials

$$(x+7)(x-5)$$
$$= x^2 - 5x + 7x - 35$$
$$= x^2 + 2x - 35$$

← this is a trinomial!

$$x^2 + 7x + 12 = (x+3)(x+4)$$

AND add to make 7

what two numbers multiply to make 12

12

Product	Sum
1, 12	
2, 6	
3, 4	7

Trinomials with leading coefficient = 1

Factoring Trinomials with a =1

	$x^2 + bx + c$	$x^2 + 2x - 8$
Find the two numbers that will make these equations true.	$\square \times \square = c$ $\square + \square = b$	$\boxed{4} \times \boxed{-2} = -8$ $\boxed{4} + \boxed{-2} = 2$
Put the two numbers in the expression.	$(x+\square)(x+\square)$	$(x+\boxed{4})(x+\boxed{-2})$

Try factoring these:

a) $x^2 - 5x - 24 = (x+3)(x-8)$

S = -5	P = -24
	1, 24
	2, 12

$\boxed{3, -8}$

factoring these:

$S=-5$ | $P=-24$
1, 24
2, 12
3, 8
4, 6

$\boxed{3,-8}$
$-3, 8$

b) $x^2-8x+12=(x-2)(x-6)$

$S=-8$ | $P=12$
1, 12
2, 6
3, 4

$-2,-6$

ALWAYS check if there's a GCF

$-3x^2+6x+105$

leading coefficient $=1$

$=-3(1x^2-2x-35)$

$=-3(x+5)(x-7)$

it's easier to do the rest of it, if we factor out a negative

$S=-2$ | $P=-35$
1, 35
5, 7

$5,-7$

$-3(x^2-7x+5x-35)$

$-3(x^2-2x-35)$

$=-3x^2+2x+105$ ✓

Sometimes we'll be stuck with a leading coefficient not equal to 1.
Two methods for factoring this type **(there are even more methods!)**

- **Systematic trial, or "Guess and Check"**
- **AC method, or "Decomposition Method"**

<u>Guess and Check Method</u>

Trinomials with leading coefficient ≠ 1

Example: $2x^2-x-21$

Product -21
1, 21
3, 7

guess + check: ~~$(2x\quad 1)(x\quad 21)$~~

~~$(2x\quad 21)(x\quad 1)$~~

~~$(2x\quad 3)(x\quad 7)$~~

$(2x-7)(x+3)$

Decomposition/AC Method

$ax^2 + bx + c$

$2x^2 + 5x - 7$

$2x^2 - 2x + 7x - 7$

$2x(x - 1) + 7(x - 1)$

$(x-1)(2x + 7)$

AC method

1) multiply "A" and "C"

$2 \times -7 = -14$

2) find two numbers that have product of AC and sum of B

product of -14	sum of 5
1, 14	
$-2, 7$	

3) split up the middle term into 2 terms, using the numbers you just got as coefficients.

4) factor the GCF out of the first two terms and out of the second two terms

Try: $3x^2 - 2x - 5$

1) $AC = -15$

2) what are the 2 numbers we need? (product = AC, sum = B)

$S = -2$	$P = -15$
	$-5, 3$

3) split up the middle term

4) do the GCF step

$3x^2 - 5x + 3x - 5$

$x(3x - 5) + 1(3x - 5)$

$= (3x-5)(x + 1)$

OR

$3x^2 + 3x - 5x - 5$

$3x(x + 1) - 5(x+1)$

$(x+1)(3x - 5)$

Factor by Grouping/Decomposition

$3x^2 - 10x + 8$

The **product** is $3 \times 8 = 24$.
The **sum** is -10.

Factors of 24	
1	24
2	12
3	8
4	6

$3x^2 - 6x - 4x + 8$

Rewrite the middle term of the polynomial using -6 and -4. ($-6x - 4x$ is just another way of expressing $-10x$.)

$3x(x - 2) - 4(x - 2)$

$(x - 2)(3x - 4)$

Factor by grouping.

1) Find AC
2) Find two numbers that multiply to = AC,
   And add to make "B"

3) Split up the middle term in the original question
   into two parts, using the numbers you found in step 2

4) Factor the GCF out of the first 2 terms
5) Factor the GCF out of the last 2 terms
6) Write final factored answer

3.2 Factoring Polynomial Expressions

Focus: factor polynomial expressions that contain functions

subtraction

Factoring a Difference of Squares

Difference of Squares

$$a^2 - b^2 = (a+b)(a-b) = a^2 - ab + ab - b^2$$
$$= a^2 - b^2$$

Examples:

$9x^2 - 16$
$= (3x)^2 - 4^2$
$= (3x+4)(3x-4)$

$4x^2 - 81y^2$
$= (2x)^2 - (9y)^2$
$= (2x+9y)(2x-9y)$

WT p 193

Example 4 **Factoring Using the Difference of Squares Pattern**

Factor each polynomial expression.

a) $4x^2 - 25y^2$ **b)** $(2x - 1)^2 - (y + 4)^2$

c) $32(x + 2)^2 - 18(2y - 3)^2$

a) $4x^2 - 25y^2 = (2x)^2 - (5y)^2$
$= (2x + 5y)(2x - 5y)$

b) $(2x-1)^2 - (y+4)^2$

do a substitution
$A = 2x-1$
$B = y+4$

$= A^2 - B^2$
$= (A + B)(A - B)$
$= (2x-1 + y+4)(2x-1-(y+4))$
$= (2x - 1 + y + 4)(2x - 1 - y - 4)$
$= (2x + y + 3)(2x - y - 5)$

For next class

- Finish worktext questions for 3.1 and 3.2

that match the ones we did

- More factoring practice worksheets available on website
- Prepare for the Unit 1 Test, next class
  - Includes concepts from Chapter 1 and Chapter 2
  - Out of 30-35 marks, something like that
  - You will be permitted to use both foldables (Exponent Rules and Rationals) during the test