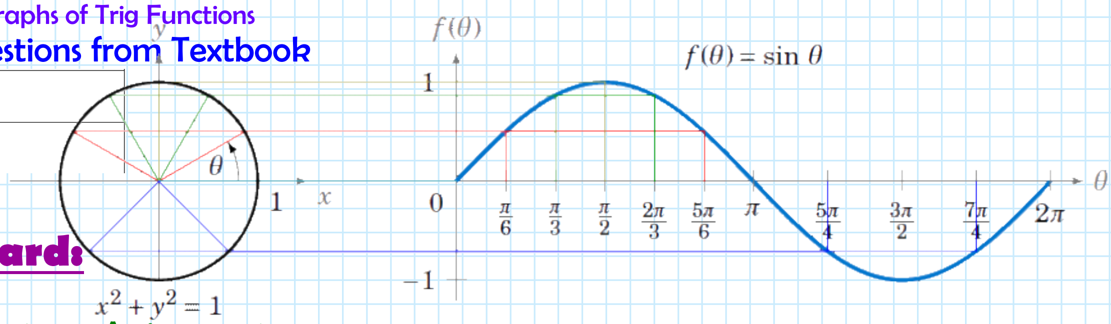
Class_09 May 17 Graphing Transformations of Sine and Cosine

Plan For Today:

1. Question about anything from last class? 4.3-4.4
2. Start Chapter 5:
   - 5.1: Graphing Sine and Cosine
   - 5.2: Graphing Transformations on Sinusoidal Functions
   - ~~5.3: Graphing Tangent Function~~
   - 5.4: Equations and Graphs of Trig Functions
3. Work on practice questions from Textbook

Page 233: #6-8, 9ac, 10ab, 11, 14
Page 250: #3-7, 10, 14, 15ac, 16ac



Plan Going Forward:

1. Finish working on Chapter 4 Assignment.

- **CHAPTER 4 ASSIGNMENT DUE THURSDAY, MAY 18TH**

2. Finish going through practice question from 5.1-5.2 in the textbook.
   Work on the 5.1-5.2 Practice graphing handout - key posted on website.

3. You will finish Chapter 5 Trigonometry and start 6.1 Identities on Thursday (tomorrow). Have a look through the last sections in ch5 to prepare for tomorrow.

SCHOOL CLOSED ON MONDAY, MAY 22ND
FOR VICTORIA DAY LONG WEEKEND

- **CHAPTER 5 ASSIGNMENT DUE TUESDAY, MAY 23RD**
- **TEST 3 ON TUESDAY, MAY 23RD (ON 4.2-5.4 OMIT 5.3, 6.1)**

Please let us know if you have any questions or concerns about your progress in this course.
The notes from today will be posted at egolfmath.weebly.com after class.
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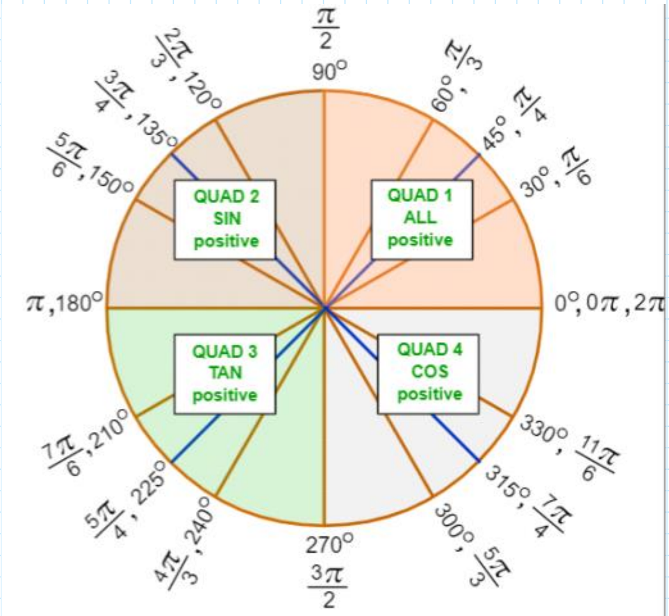
4.4 Review



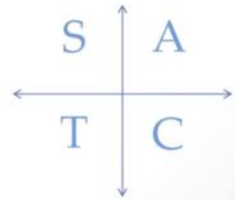
Solving Trig Equations

- When solving trig equations, you will need to get the ***trig function*** isolated (by itself).
- **Ex:** 2sin x = 1 ← **Divide both sides by 2**

$\sin x = \frac{1}{2}$ ← **Use unit circle or inverse function on calculator to find angle**

We will limit our solutions to $[0, 2\pi)$, and all answers must be in **RADIANS** **(π form)**

$30^o \text{ and } 150^o = \frac{\pi}{6} \text{ and } \frac{5\pi}{6}$



- **Ex:** sin x - $\sqrt{2}$ = - sin x ← **now add sinx to both sides.**

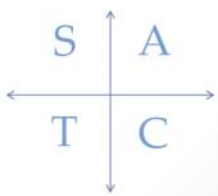
2sin x - $\sqrt{2}$ = 0 ← **add $\sqrt{2}$ to both sides.**

2sin x = $\sqrt{2}$ ← **divide by 2 (keep as fraction)**

sin x = $\frac{\sqrt{2}}{2}$ ← **find angle with sin**

- Look only at the interval from $[0, 2\pi)$ to find angles

- Answers: $\frac{\pi}{4}$ *and* $\frac{3\pi}{4}$



- **Ex: $4sin^2x$ – 3 = 0** ← **now add 3 both sides.**

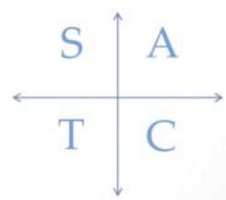
$4sin^2x = 3$ ← **divide by 4.**

$sin^2x = \frac{3}{4}$ ← **take the √ of both sides.**

$\sin x = \pm\sqrt{\frac{3}{4}}$ ← **use unit circle or calculator**

Which angles have a sine value that is $\pm\frac{\sqrt{3}}{2}$?

- Answers: $\frac{\pi}{3}$, $\frac{2\pi}{3}$, $\frac{4\pi}{3}$, $\frac{5\pi}{3}$



- **How do you solve a problem involving factoring?**

(Set equation equal to zero and then factor)

Ex: $sin^2x = 2\sin x$ ← **subtract 2 sin x**

$sin^2x - 2\sin x = 0$ ← **factor out sin x**

$\sin x(\sin x - 2) = 0$ ← **set each factor = 0**

$\sin x = 0$ $\quad$ $\sin x - 2 = 0$ ← **solve both**

$\sin x = 2$

Look only at the interval from $[0, 2\pi)$

- **Answers: 0 , π *only*** **since sin x ≠ 2 ever**

Factoring Trigonometric Expressions

We can apply three basic factoring techniques:

- common factor
- difference of two squares
- factoring trinomials of the form $ax^2 + bx + c$

NOTE

Solving Quadratic Trigonometric Equations

$(\sin x)^2$ is often written $\sin^2 x$

Example FACTORISE!

Solve $7\sin^2 x + 3\sin x - 4 = 0$

for $0° \leqslant x \leqslant 360°$

$(7\sin x + 4)(\sin x - 1) = 0$

$7\sin x + 4 = 0$ or $\sin x - 1 = 0$

$\sin x = -\frac{4}{7}$ S | A, T | C (T ✓, C ✓) $\sin x = 1$

acute angle $\approx 34.8°$ $x = 90°$

$\therefore\ x \approx 180° + 34.8°$ or $x \approx 360° - 34.8°$

$\approx 214.8°$ $\approx 325.2°$

- **How do you solve problems of a quadratic type?**
  (Set equation equal to zero and then factor OR... quadratic formula)

Ex: $2sin^2x - 3sinx + 1 = 0$ ← factor

$(2\sin x - 1)(\sin x - 1) = 0$ ← set each factor = 0

$(2\sin x - 1) = 0$ *and* $(\sin x - 1) = 0$

$2\sin x = 1$ $\sin x = 1$

$\sin x = \frac{1}{2}$

Look only at the interval from $[0, 2\pi)$ ***to find angles***

For sin ½ → $\frac{\pi}{6}, \frac{5\pi}{6}$ For sin 1 → $\frac{\pi}{2}$

Solving Second-Degree Trigonometric Equations

When we solve equations with an exponent we usually start by factoring.

For example, solve: $2x^2 - 1 = x$

Example

Solve algebraically. If possible, give exact answers. Otherwise, give answers correct to one decimal place.

$2\tan^2\theta - 1 = \tan\theta$, for $0^\circ \le \theta < 360^\circ$

$-\tan\theta \quad -\tan\theta$

$2\tan^2\theta - \tan\theta - 1 = 0$ $\quad AC = -2$

$2\tan^2\theta - 2\tan\theta + \tan\theta - 1 = 0$ $\quad -2, +1 \quad -1$

$2\tan\theta(\tan\theta - 1) + (\tan\theta - 1) = 0$

$(2\tan\theta + 1)(\tan\theta - 1) = 0$

$2\tan\theta + 1 = 0 \qquad \tan\theta - 1 = 0$

$\tan\theta = -\frac{1}{2} \qquad \tan\theta = 1$

$\frac{y}{x}$ (−) QII, IV

no special angles b/c $\tan\theta = -\frac{1}{2}$ is not special

use calc.

$\theta_R = \tan^{-1}\left(\frac{1}{2}\right)$

$\theta_R = 26.6^\circ$

QII $180^\circ - 26.6^\circ = 153.4^\circ$

QIV: $360 - 26.6^\circ = 333.4^\circ$

$\frac{y}{x} = 1$ $\quad y = x$ at $\frac{\pi}{4}, 45^\circ$ b/c $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ $x = y$

QI $\theta = 45^\circ$

QIII $\theta = 180^\circ + 45^\circ$

$\theta = 225^\circ$

$\theta = 45^\circ, 153.4^\circ, 225^\circ, 333.4^\circ$

Example

Solve algebraically. If possible, give exact answers. Otherwise, give answers correct to one decimal place.

$$2\cos^2\theta - 1 = 0,\ \text{for } 0 \le \theta < 2\pi$$

+1

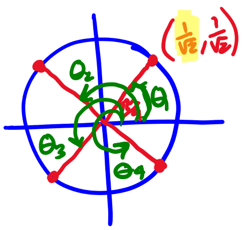
$2\cos^2\theta = 1$

$\sqrt{\cos^2\theta} = \sqrt{\frac{1}{2}}$

$\cos\theta = \pm\frac{1}{\sqrt{2}}$

★ always write ± when square rooting to solve.

$\frac{1}{\sqrt{2}}$ goes with $\frac{\pi}{4}$



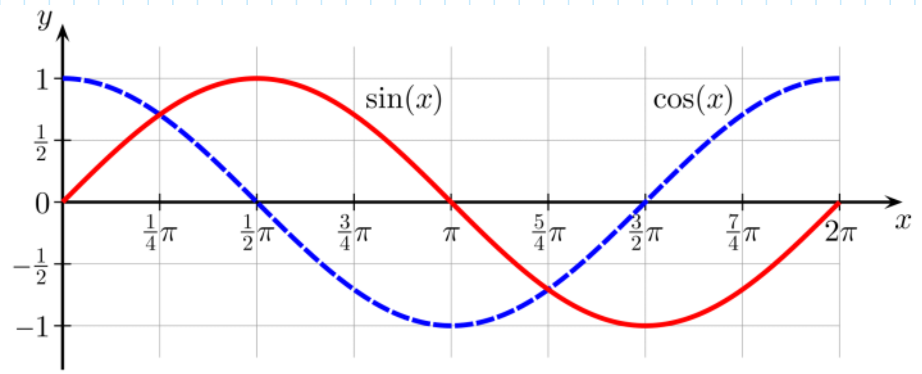
(+) QI QIV $\theta = \frac{\pi}{4}, \frac{7\pi}{4}$

(−) QII QIII $\theta = \frac{3\pi}{4}, \frac{5\pi}{4}$

$$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

5.1-5.2 Graphing Sine & Cosine & Transformations

Explore Learning Gizmo: (5 min per day)
https://tinyurl.com/yckn3e3p



Sine Function

Domain: $(-\infty, \infty)$ **Range:** $[-1, 1]$

y-intercept: 0

x-intercepts: $n\pi, n \in \mathbb{Z}$

Continuity: continuous on $(-\infty, \infty)$

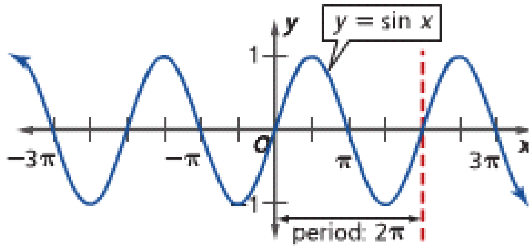
Symmetry: origin (odd function)

Extrema: maximum of 1 at $x = \frac{\pi}{2} + 2n\pi, n \in \mathbb{Z}$

minimum of −1 at $x = \frac{3\pi}{2} + 2n\pi, n \in \mathbb{Z}$

End Behavior: $\lim_{x \to -\infty} \sin x$ and $\lim_{x \to \infty} \sin x$ do not exist.

Oscillation: between −1 and 1



Cosine Function

Domain: $(-\infty, \infty)$ **Range:** $[-1, 1]$

y-intercept: 1

x-intercepts: $\frac{\pi}{2}n, n \in \mathbb{Z}$

Continuity: continuous on $(-\infty, \infty)$

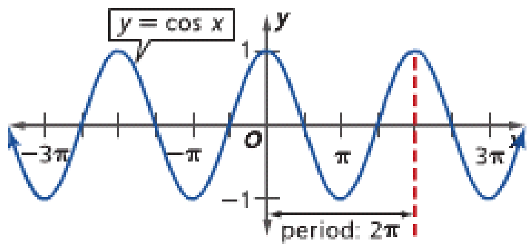
Symmetry: y-axis (even function)

Extrema: maximum of 1 at $x = 2n\pi$, $n \in \mathbb{Z}$

minimum of −1 at $x = \pi + 2n\pi$, $n \in \mathbb{Z}$

End Behavior: $\lim_{x \to -\infty} \cos x$ and $\lim_{x \to \infty} \cos x$ do not exist.

Oscillation: between −1 and 1



Term	Definition/Explanation
Amplitude	Half the vertical distance from the maximum height to the minimum height of the function.
Interval	The domain of one cycle; written as $[x_b, x_e]$, where x_b is the beginning and x_e is the end.
Period	The horizontal length of one repeating pattern of the function.
Phase Shift or Horizontal Shift	The horizontal distance a function is moved.
Vertical Shift	The vertical distance a function is moved.
Interval	The horizontal starting point and ending point of one complete period of a cyclical trigonometric function.

5.1 p.223

Graphing Transformations on the Sine and Cosine Function

Steps:

1. Make sure the equation is written in standard transformation form:
   - $y = a\sin b(x-c)+d$ and $y = a\cos b(x-c)+d$
2. List all characteristics (in radians or degrees depending on the question):
   - Amplitude = a
   - Vertical Displacement = d
   - Period = 2pi/b
   - Phase Shift = c
3. Determine the Midline, Maximum and Minimum for the y-axis scale based on the amplitude and vertical displacement.

Amplitude

-the distance from the central axis to the minimum or maximum value of the periodic function.

$$a = \frac{Max - Min}{2}$$

$$a = \frac{2-(-2)}{2} = \frac{4}{2} = 2$$

$y = a\sin x$ amplitude $= |a|$

Note: the same results occur for the function y = cosx

Vertical Displacement

$$d = \frac{Max + Min}{2}$$ max $= -1$ min $= -3$

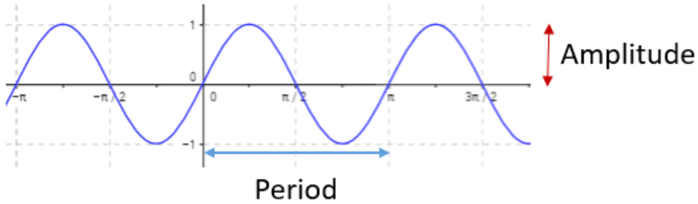
$$d = \frac{-1+(-3)}{2} = \frac{-4}{2} = -2$$

Note: the same results occur for the function y = sinx

4. Determine the period and phase shift for the x-axis scale.
   - Use 4,6,8,12,16 squares on the grid to equal the length of the period
   - Divide the period by the number of squares to determine the length of one square then label the x-axis to easily find the phase shift
5. Place the first point at the beginning of the cycle at the phase shift.
   - On the midline for sine
   - On the max point for +cosine
   - On the min point for -cosine
6. Divide the number of squares you gave for the length of the period by 4, then count that many squares for each max, midline and min point for one complete cycle.
7. Continue the pattern to graph at least 2 cycles.

Transformation of Trigonometric Graphs

$$y = A\sin\left[B\left(x - C\right)\right] + D$$



$|A|$ is the amplitude

The period is $\frac{2\pi}{B}$

Phase (horizontal) shift is C

Vertical shift is D

The same applies for the Cosine Function.

For the Tangent Function, the period is $\frac{\pi}{B}$

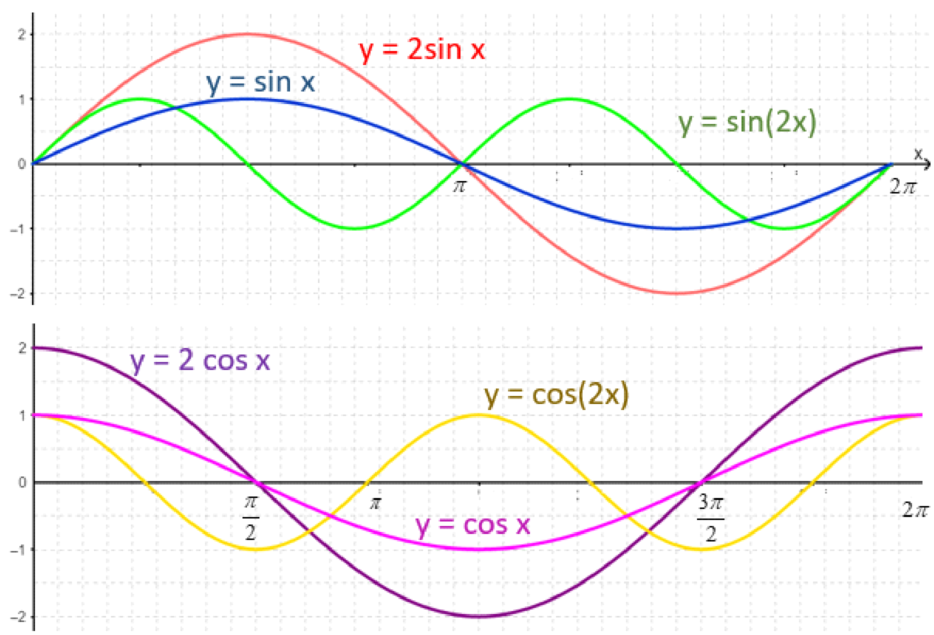
Transform Sine and Cosine Graphs

y = A sin(B(x-k)) + c
y = A cos(B(x-k)) + c

The amplitude is |A|

The period is $\frac{2\pi}{|B|}$

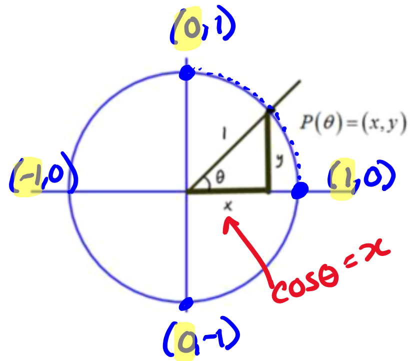
k is horizontal shift
c is vertical shift



Chapter 5: Trigonometric Functions and Graphs

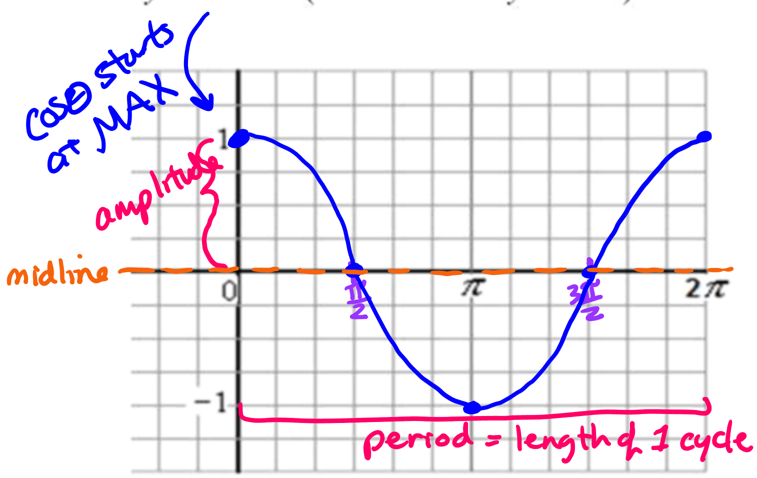
5.1 Graphing Sine and Cosine Functions

Let's track what happens to $P(\theta)$ as θ, a standard-position angle, gets larger.



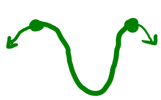
		$y=\cos\theta$
Q1	As θ increases from 0 to $\frac{\pi}{2}$	cosine values (x-values) decreases from 1 to 0
Q2	As θ increases from $\frac{\pi}{2}$ to π	cosine values (x-values) decreases from 0 to -1
Q3	As θ increases from π to $\frac{3\pi}{2}$	cosine values (x-values) increases from -1 to 0
Q4	As θ increases from $\frac{3\pi}{2}$ to 2π	cosine values (x-values) increases from 0 to 1

$y=\cos\theta$ (also often written as $y=\cos x$)

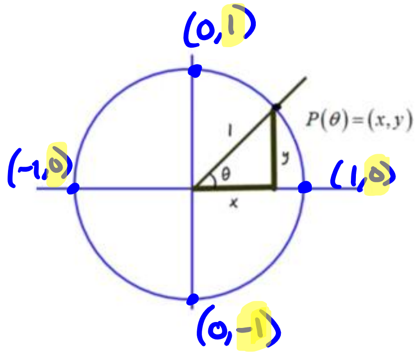


θ	$\cos\theta$	
0	1	(0,1)
$\frac{\pi}{6}$		
$\frac{2\pi}{6}=\frac{\pi}{3}$		
$\frac{3\pi}{6}=\frac{\pi}{2}$	0	$(\frac{\pi}{2},0)$
$\frac{4\pi}{6}=\frac{2\pi}{3}$		
$\frac{5\pi}{6}$		
$\frac{6\pi}{6}=\pi$	-1	$(\pi,-1)$
$\frac{9\pi}{6}=\frac{3\pi}{2}$	0	$(\frac{3\pi}{2},0)$
$\frac{12\pi}{6}=2\pi$	1	$(2\pi,1)$

Maximum: 1 Minimum: −1 Range: $\{y \mid -1 \le y \le 1, y \in \mathbb{R}\}$ Amplitude: 1

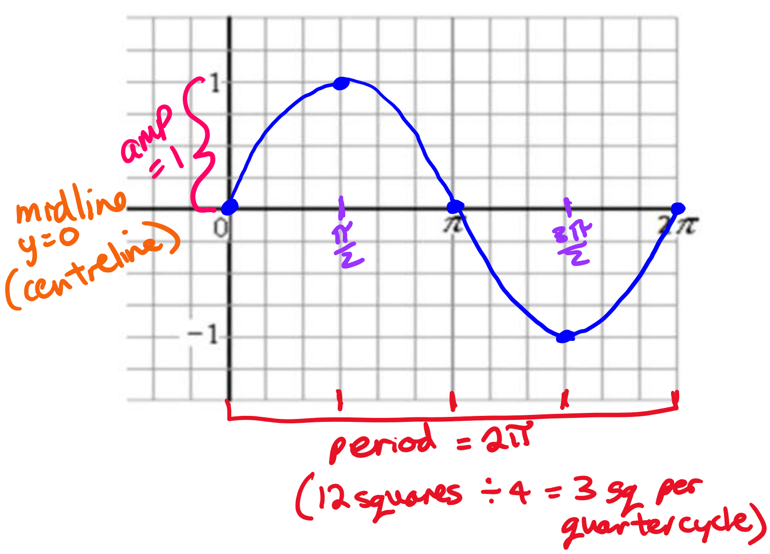


Domain: $\{x \mid x \in \mathbb{R}\}$ x-intercepts: $\frac{\pi}{2}, \frac{3\pi}{2}$ Period: 2π Center line equation: (midline) x-axis $\therefore y=0$



		$y=\sin\theta$
Q1	As θ increases from 0 to $\frac{\pi}{2}$	sine values (y-values) increases from 0 to 1
Q2	As θ increases from $\frac{\pi}{2}$ to π	sine values (y-values) decreases from 1 to 0
Q3	As θ increases from π to $\frac{3\pi}{2}$	sine values (y-values) decreases from 0 to -1
Q4	As θ increases from $\frac{3\pi}{2}$ to 2π	sine values (y-values) increases from -1 to 0

$y=\sin\theta$ (also often written as $y=\sin x$)



θ	$\sin\theta$	
0	0	(0,0)
$\frac{\pi}{6}$		
$\frac{2\pi}{6}=\frac{\pi}{3}$		
$\frac{3\pi}{6}=\frac{\pi}{2}$	1	$(\frac{\pi}{2},1)$
$\frac{4\pi}{6}=\frac{2\pi}{3}$		
$\frac{5\pi}{6}$		
$\frac{6\pi}{6}=\pi$	0	$(\pi,0)$
$\frac{9\pi}{6}=\frac{3\pi}{2}$	-1	$(\frac{3\pi}{2},-1)$
$\frac{12\pi}{6}=2\pi$	0	$(2\pi,0)$

Maximum: 1

Minimum: -1

Range: $\{y \mid -1 \le y \le 1, y \in \mathbb{R}\}$

Amplitude: 1

Domain: $\{x \mid x \in \mathbb{R}\}$

x-intercepts: $0, \pi, 2\pi$

Period: 2π

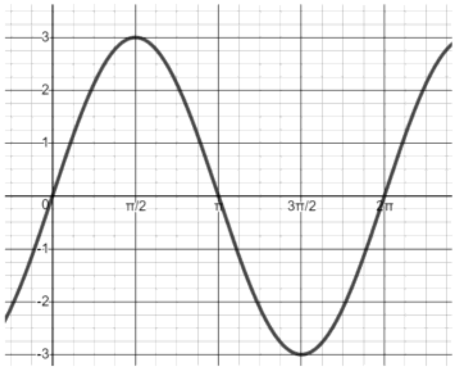
Center line equation: $y=0$

$\theta = x \therefore \sin\theta = \sin x$

$y = \sin\theta$ same as $y = \sin x$

Amplitude

is the **vertical** distance from the center line of a trigonometric graph to its maximum or minimum. The untransformed graphs of $y = \sin x$ and $y = \cos x$ have amplitude 1.



For $y = a\cos x$ or $y = a\sin x$

- vertical stretch, factor a
- amplitude = $|a|$
- if $a < 0$, graph is reflected across x-axis
- amplitude = $\frac{|\max - \min|}{2}$

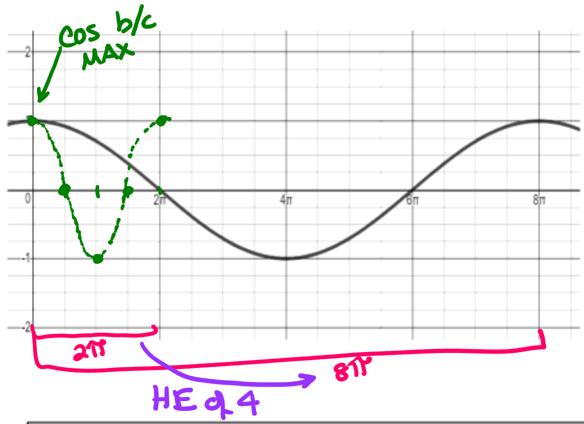
Amplitude for graph shown at left? amp=3

Equation of the graph?

$y = 3\sin x$

Period

is the **horizontal** length of one complete cycle. The untransformed graphs of $y = \sin x$ and $y = \cos x$ have a period length of 2π (or 360°, if working in degree measure).



For $y = \cos(bx)$ or $y = \sin(bx)$

- horizontal stretch, factor $\frac{1}{b}$
- period = $\frac{2\pi}{|b|}$ or $\frac{360^\circ}{|b|}$ } formula
- if $b < 0$, graph is reflected across y-axis

$P = \frac{2\pi}{b} \rightarrow b = \frac{2\pi}{P}$

Period for graph shown at left? 8π → $b = \frac{2\pi}{8\pi}$, $b = \frac{1}{4}$

HE of 4, $b = \frac{1}{4}$

Equation of the graph?

$y = \cos\left(\frac{1}{4}x\right)$

Since graph's actual period = $\frac{2\pi}{\lvert b \rvert}$, then $\lvert b \rvert = \frac{2\pi}{graph's\ actual\ period}$ If working in degrees, since graph's actual period = $\frac{360^\circ}{\lvert b \rvert}$, then $\lvert b \rvert = \frac{360^\circ}{graph's\ actual\ period}$

When sketching a period of a trigonometric function graph, we

- multiply the period length by ¼, to determine the spacing between key points
- plot key points: maximum, minimum, and center-line
- connect key points smoothly, getting a sinusoidal shape

normal period = 2π ÷ 4 $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$, 2π

Try

1a) $y = -3\sin(5x)$

amplitude: 3 period: $\frac{2\pi}{5}$ key point spacing: $\frac{2\pi}{5} \div 4 \rightarrow \frac{2\pi}{5} \times \frac{1}{4} = \frac{\pi}{10}$

b) $y = -\frac{1}{4}\sin\left(\frac{1}{3}x\right)$

amplitude: $\frac{1}{4}$ period: $2\pi \times 3 = 6\pi$ key point spacing: $6\pi \div 4 = \frac{3\pi}{2}$

2. Write the equation of a function with these characteristics.

a) sine function; amp = 3, period = π

$a = 3$, $b = \frac{2\pi}{p}$, $b = \frac{2\pi}{\pi}$, $b = 2$

$y = 3\sin 2x$

b) cosine function, amp =2.4, period = 10π

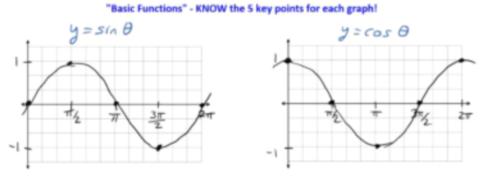
$2\pi \rightarrow 10\pi$, $b = \frac{1}{5}$

$a = 2.4$, $b = \frac{2\pi}{10\pi} = \frac{1}{5}$

$y = 2.4\cos\frac{1}{5}x$

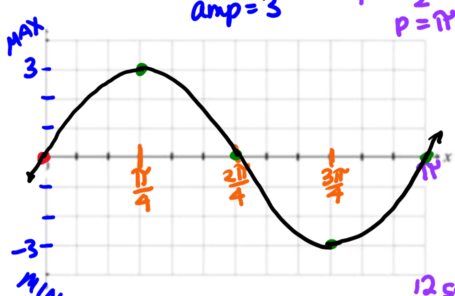
"Basic Functions" - KNOW the 5 key points for each graph!

3. For each equation below, accurately sketch one period of its graph. Give the coordinates of 5 key points.



a) $y = 3\sin(2x)$

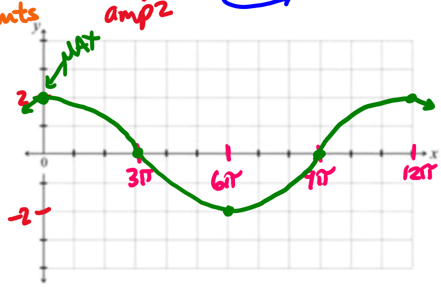
amp = 3, $p = \frac{2\pi}{2}$ or $2\pi \times \frac{1}{2}$, $p = \pi \div 4 \rightarrow$ key points $\frac{\pi}{4}$



x	y
0	0
$\frac{\pi}{4}$	3
$\frac{\pi}{2}$	0
$\frac{3\pi}{4}$	-3
π	0

b) $y = -2\cos\left(\frac{1}{6}x\right)$

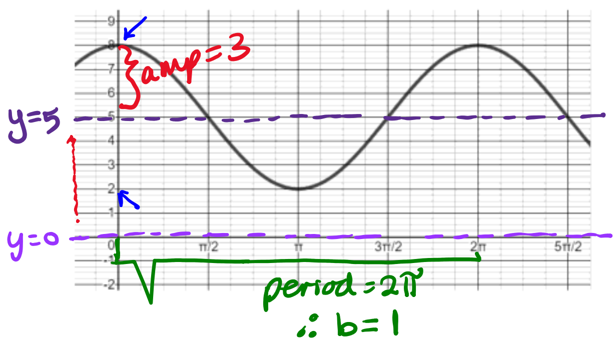
amp 2, $p = 2\pi \times \frac{6}{1} = 12\pi \div 4$, key points 3π



x	y
0	2
3π	0
6π	-2
9π	0
12π	2

5.2 More Transformations of Sinusoidal Functions

Vertical Displacement
is the amount of vertical translation (up/down) a sinusoidal graph moves



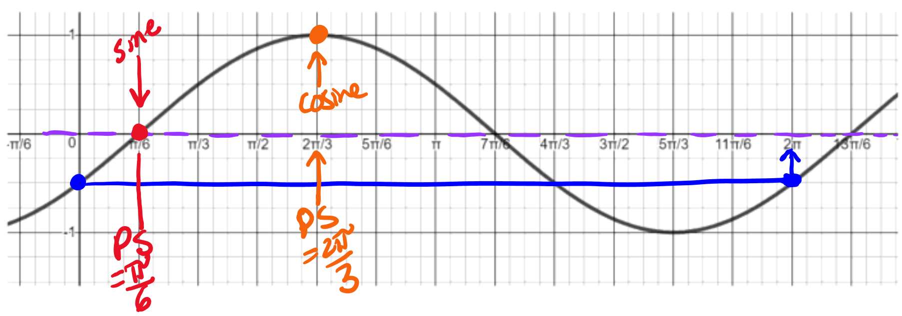
For $y = a\cos x + d$ or $y = a\sin x + d$
- vertical displacement, d units
- center line is located at $y = d$
- when we have no equation, we can figure out the vertical displacement from the graph:

$$vertical\ disp = \frac{\max + \min}{2}$$

Vertical displacement for this graph? 5 up = midline

Equation of this graph? $y = 3\cos x + 5$

Phase Shift
is the amount of horizontal translation (left/right) a sinusoidal graph moves



For $y = \cos(x - c)$ or $y = \sin(x - c)$
- phase shift, c units
- when we have no equation, we use the graph to find the phase shift. Choose a period of either sine or cosine that begins near the y-axis. Identify how much it has moved left/right compared to the basic (untransformed) graph.

For the graph above, find its equation in the form: $y = \sin(x - c)$ → $y = \sin\left(x - \frac{\pi}{6}\right)$

For the graph above, find its equation in the form: $y = \cos(x - c)$ → $y = \cos\left(x - \frac{2\pi}{3}\right)$

Summary

$p = \frac{2\pi}{b}$ or Horizontal stretch of $\frac{1}{b}$

$$y = a\sin b(x - c) + d$$

vertical displacement (vert. translation) midline/centre line $y = d$

amplitude (vert. stretch

phaseshift (horiz. translation)

Try

Determine the amplitude, period, phase shift and vertical displacement.

TRY

a) $y = 5\cos\left(3\left(x + \frac{\pi}{4}\right)\right) + 2$

b) $y = \frac{1}{2}\cos\left(x + 80^\circ\right) + 5$

c) $y = -2\sin\left(5x - \frac{\pi}{3}\right) + 4$

Sketching a Sinusoidal Graph

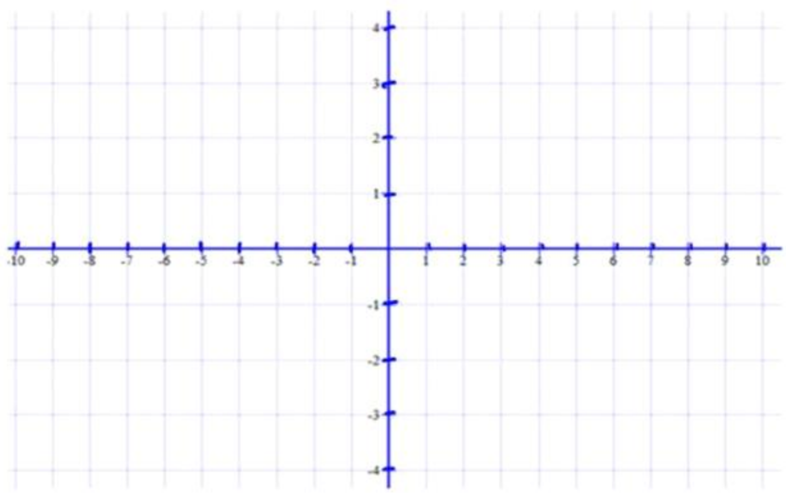
✻ rational periods.

Consider the equation: $y = 3\sin\left(\frac{2\pi}{12}(x+5)\right) - 1$

a) Key features:

basic sine shape	vertical displacement	equation of center line
amplitude	maximum	minimum
period	spacing	phase shift

b) Accurately sketch one period of the graph. Give the coordinates of 5 key points. Include the center line on your sketch.



x	y

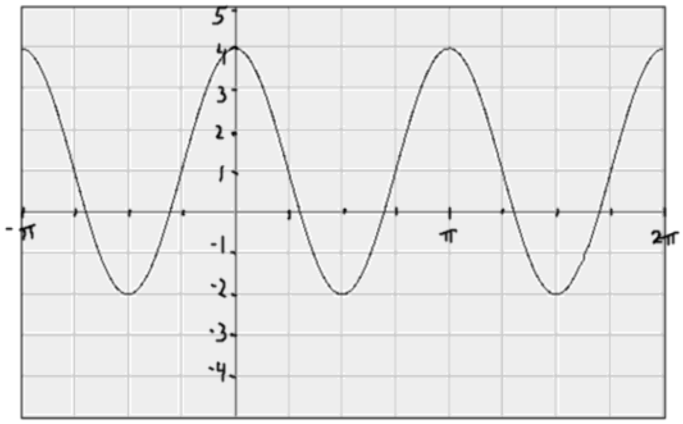
Finding the Equation of a Graph

Sine and cosine graphs are both called ***sinusoidal graphs***.

- For any sinusoidal graph, it is possible to write a sine equation that creates that graph, and a cosine equation that creates that same graph.
- There are many different equations that generate the same sinusoidal graph.

Example

Give two different equations that create this graph.



Maximum:

Minimum:

Center line:

Vertical displacement:

Amplitude:

Period:

b value:

Possible sine equation:

Possible cosine equation: