Class_09 Oct 6 Trig Ratios and Equations

Tuesday, October 4, 2022 4:18 PM

Tonight's Class:

- **Grab a white board, pen and eraser**
- **4.3 Trig Ratios**
- **4.4 Trigonometric Equations**

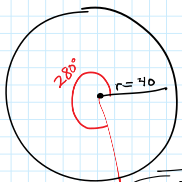
Approximate dates for Chapter Tests and Unit Tests

Please note, these dates may change during the semester!

	Topic	Test Date
	Function Transformations (Ch 1)	Tuesday, Sep 20
Unit 1	**Transformations & Polynomial Functions**	**Thursday, Sep 29**
	Trigonometry & the Unit Circle (Ch 4)	~~Tuesday, Oct 11~~ **Thursday, Oct 13**
	Trig Functions & Graphs (Ch 5)	Tuesday, Oct 25

Quick Check-in - individual whiteboard

A circle has radius 40 cm. What is the arc length subtended by a central angle that measures 280 degrees? Give answer correct to 1 decimal place.



$$(2\pi)(40)\left[\frac{280^\circ}{360^\circ}\right] = \left(\frac{a}{\cancel{2\pi(40)}}\right)\cancel{(2\pi)(40)}$$

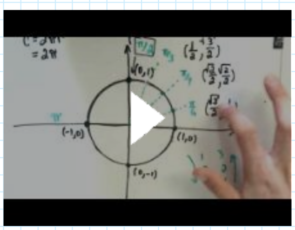
$$a = \frac{(2\pi)(40)(280)}{360}$$

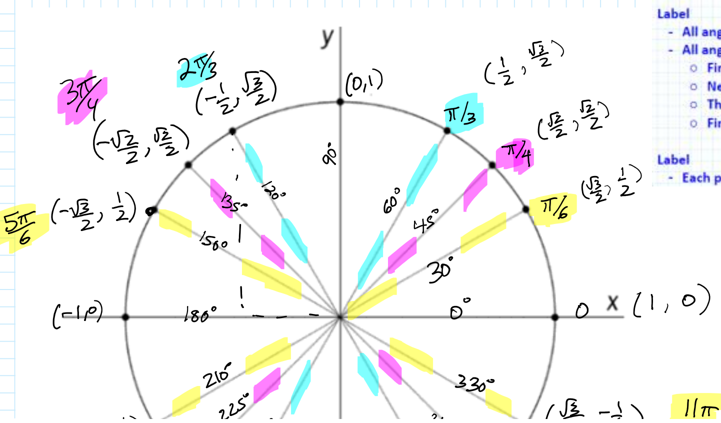
$$= \boxed{195.5 \text{ cm}}$$

$$a = r\theta$$

$$a = 40 \times \frac{280\pi}{180}$$

[A Trick to Remember Values on The Unit Circle](#)



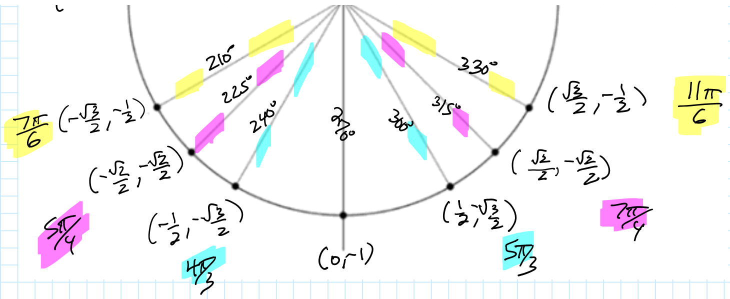


Label

- **All angles, in degree measure**
- **All angles in radians,**
    - **First, ones with denominator "6"**
    - **Next, ones with denominator "4"**
    - **Then, ones with denominator "3"**
    - **Finally, ones with denominator "2"**

Label

- **Each point in Quadrant 1 with its x- and y-coordinates**

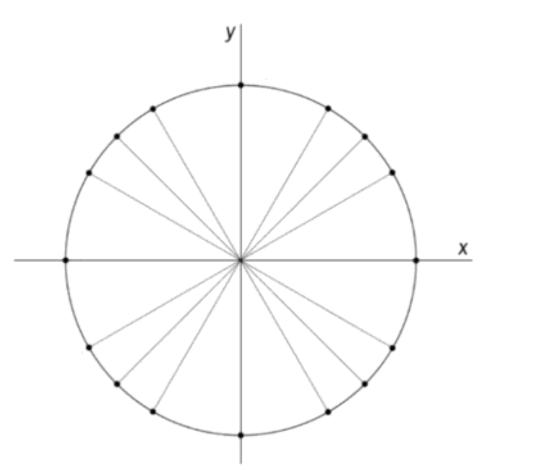


WB -

Use your unit circle to find these values exactly. Don't use a calculator.

$\sin\theta = y$ coordinate

1. $\sin 30^\circ = \frac{1}{2}$
2. $\sin 225^\circ = -\frac{\sqrt{2}}{2}$
3. $\sin 240^\circ = -\frac{\sqrt{3}}{2}$
4. $\sin 45^\circ = \frac{\sqrt{2}}{2}$
5. $\sin 150^\circ = \frac{1}{2}$
6. $\sin 300^\circ = -\frac{\sqrt{3}}{2}$
7. $\sin 0^\circ = 0$
8. $\sin 390^\circ = \frac{1}{2}$
9. $\sin 270^\circ = -1$
10. $\sin 210^\circ = -\frac{1}{2}$
11. $\sin 180^\circ = 0$
12. $\sin 330^\circ$ $-\frac{1}{2}$

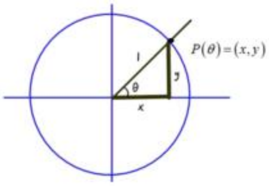


$\cos\theta = x$-coordinate

1. $\cos 0 = 1$
2. $\cos\left(\frac{7\pi}{6}\right) = -\frac{\sqrt{3}}{2}$
3. $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$
4. $\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$
5. $\cos\left(\frac{3\pi}{2}\right) = 0$
6. $\cos\left(\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}$
7. $\cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$
8. $\cos\left(\frac{4\pi}{3}\right) = -\frac{1}{2}$
9. $\cos\left(\frac{11\pi}{6}\right) = \frac{\sqrt{3}}{2}$
10. $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$
11. $\cos(\pi) = -1$
12. $\cos\left(\frac{7\pi}{4}\right) = \frac{\sqrt{2}}{2}$

4.3 Trigonometric Ratios

We can use the unit circle diagram to get definitions for all six of the trigonometric ratios.



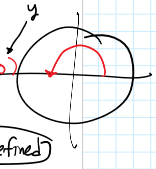
Primary Ratios	***Reciprocal Ratios***
$\sin\theta = \frac{y}{1} = y$	$\csc\theta = \frac{1}{y}$
$\cos\theta = \frac{x}{1} = x$	$\sec\theta = \frac{1}{x}$
$\tan\theta = \frac{y}{x}$	$\cot\theta = \frac{x}{y}$

More Exact Values - Use the unit circle to find each ***exact value – no calculator!***

a) $\cos(\pi) = -1$ (x-coord)

$\sin(\pi) = 0$

$\tan(\pi) = \frac{y}{x} = \frac{0}{-1} = 0$



$\sec(\pi) = \frac{1}{-1} = -1$

$\csc(\pi) = \frac{1}{0} =$ undefined

$\cot(\pi) = \frac{x}{y} = \frac{-1}{0}$ undefined

b) $\cos\left(\frac{7\pi}{4}\right) = \frac{\sqrt{2}}{2}$ x-coord

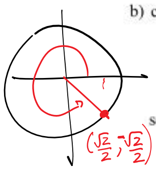
$\sin\left(\frac{7\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

$\tan\left(\frac{7\pi}{4}\right) = \frac{y}{x} = \frac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = -1$

$-\frac{\sqrt{2}}{2} \div \frac{\sqrt{2}}{2}$

$= -\frac{\sqrt{2}}{2} \cdot \frac{2}{\sqrt{2}}$

$= \frac{-\cancel{2}\cancel{\sqrt{2}}}{\cancel{2}\cancel{\sqrt{2}}} = -1$



$\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$\sec\left(\frac{7\pi}{4}\right) = \frac{1}{\frac{\sqrt{2}}{2}} = 1 \cdot \frac{2}{\sqrt{2}} = \frac{2}{\sqrt{2}}$

$\csc\left(\frac{7\pi}{4}\right) = -\frac{2}{\sqrt{2}}$

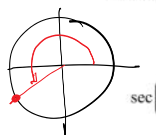
$\cot\left(\frac{7\pi}{4}\right) = \frac{x}{y} = \frac{\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = -1$

c) $\cos\left(\frac{7\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

$\sin\left(\frac{7\pi}{6}\right) = -\frac{1}{2}$

$\tan\left(\frac{7\pi}{6}\right) = \frac{y}{x} = \frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = -\frac{1}{2} \cdot -\frac{2}{\sqrt{3}}$

$= \frac{\cancel{2}}{\cancel{2}\sqrt{3}} = \frac{1}{\sqrt{3}}$



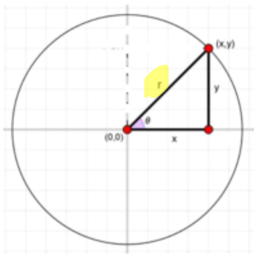
$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

$\sec\left(\frac{7\pi}{6}\right) = -\frac{2}{\sqrt{3}}$ (reciprocal of cosine)

$\csc\left(\frac{7\pi}{6}\right) = \frac{-2}{1} = -2$

$\cot\left(\frac{7\pi}{6}\right) = \frac{x}{y} = \frac{\sqrt{3}}{1} = \sqrt{3}$

Sometimes the radius is not 1. Here are the ratio definitions that work for any *r* value.



Primary Ratios	***Reciprocal Ratios***
$\sin\theta = \frac{y}{r}$	$\csc\theta = \frac{r}{y}$
$\cos\theta = \frac{x}{r}$	$\sec\theta = \frac{r}{x}$
$\tan\theta = \frac{y}{x}$	$\cot\theta = \frac{x}{y}$

Know these definitions!

Example
The terminal arm of a standard position angle θ contains the point $(-2,-5)$. Find the value of all six trigonometric ratios for angle θ. You do not need to find the size of angle θ. Leave answers in ***exact fractional form***.

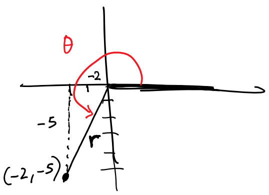
The angle terminates in quadrant 3

$x =$ -2

$y =$ -5

$r =$ $\sqrt{29}$

$$x^2 + y^2 = r^2$$
$$(-2)^2 + (-5)^2 = r^2$$
$$4 + 25 = r^2$$
$$29 = r^2$$
$$r = \sqrt{29}$$



$$\sin\theta = \frac{y}{r} = \frac{-5}{\sqrt{29}} \times \frac{\sqrt{29}}{\sqrt{29}} = \frac{-5\sqrt{29}}{29}$$

$$\cos\theta = \frac{x}{r} = \frac{-2}{\sqrt{29}}$$

$$\tan\theta = \frac{y}{x} = \frac{-5}{-2} = \frac{5}{2}$$

$$\csc\theta = \frac{r}{y} = \frac{\sqrt{29}}{-5}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{29}}{-2}$$

$$\cot\theta = \frac{x}{y} = \frac{-2}{-5} = \frac{2}{5}$$

Finding the Signs of the Trigonometric Ratios

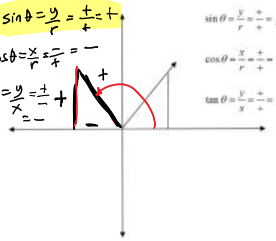
How can we predict whether a specific trigonometric ratio will be positive or negative?

r - positive in every quadrant
x - depends on the quadrant, can be + or –
y - depends on the quadrant, can be + or –

$\sin\theta = \frac{y}{r} = \frac{+}{+} = +$

$\cos\theta = \frac{x}{r} = \frac{-}{+} = -$

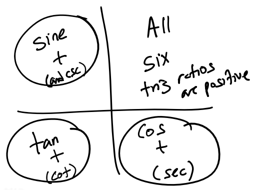
$\tan\theta = \frac{y}{x} = \frac{+}{-} = -$



$\sin\theta = \frac{y}{r} = \frac{+}{+} = positive$

$\cos\theta = \frac{x}{r} = \frac{+}{+} = positive$

$\tan\theta = \frac{y}{x} = \frac{+}{+} = positive$

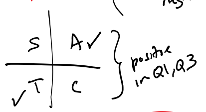


Example

Given the information below, decide in which quadrant (or quadrants) angle θ can terminate.

a) $\tan\theta < 0$ (tangent is negative)

negative in: Q2 Q4



b) $\csc\theta > 0$ in the same quadrants as $\sin\theta > 0$.

Q1 Q2

c) $\sin\theta < 0$ and $\cot\theta > 0$

$\tan\theta > 0$

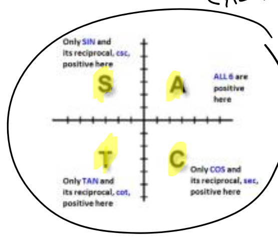
must be Q3

d) $\sec\theta > 0$ and $\tan\theta < 0$

$\cos\theta > 0$

Q4

"CAST"

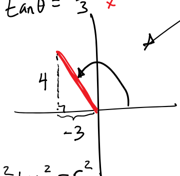


All Students Take Calculus

Example

Find the value of $\sec\theta$, if we know $\boxed{\tan\theta = -\frac{4}{3}}$ and $\sin\theta > 0$.

$\tan\theta = -\frac{4}{3} = \frac{y}{x}$



$\Rightarrow$ angle is in Q2

$x^2 + y^2 = r^2$
$(-3)^2 + (4)^2 = r^2$
$9 + 16 = r^2$
$25 = r^2$, $r = 5$

$\sec\theta = \frac{r}{x}$

$\boxed{\sec\theta = \frac{5}{-3}}$

$\cos\theta = \frac{x}{r}$

Approximate Values

For angles not related to special angles, calculators can give us accurate approximations.

Try

Evaluate each of the following ratios correct to 4 decimal places, using a calculator.

a) $\tan(-65^\circ) =$ -2.1445

b) $\sec 417^\circ = \frac{1}{\cos 417^\circ}$ $\doteq 1.8361$

c) $\cot\left(\frac{3\pi}{5}\right) = \frac{1}{\tan\left(\frac{3\pi}{5}\right)}$ $\doteq -0.3249$

d) $\cos(4) \doteq$ -0.6536

radians, not degrees!

e) $\sin(-200^\circ) \doteq$ 0.3420

f) $\csc\left(\frac{4\pi}{7}\right) = \frac{1}{\sin\left(\frac{4\pi}{7}\right)}$ $\doteq 1.0257$

Use the correct mode – either degrees or radians, matching the angle's units.

Use the correct mode – either degrees or radians, matching the angle's units.

Be careful when evaluating reciprocal ratios.
Don't take the reciprocal of the angle!
Instead, use these relationships:

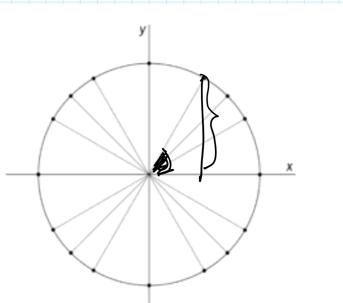
$$\csc(\mathit{angle}) = \frac{1}{\sin(\mathit{angle})}$$

$$\cot(\mathit{angle}) = \frac{1}{\tan(\mathit{angle})}$$

$$\sec(\mathit{angle}) = \frac{1}{\cos(\mathit{angle})}$$

TB 4.3 p 201: 1-2(acegik), 3ace, 6ace, 9ace, 12ac

Use your unit circle to solve these equations:



1. $\sin\theta = \frac{\sqrt{3}}{2}$, $0^\circ \le \theta < 90^\circ$

y-coord.

60°

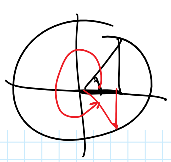
2. $\sin\theta = \frac{\sqrt{3}}{2}$, $0^\circ \le \theta < 360^\circ$

60° and 120°

3. $\cos\theta = \frac{\sqrt{2}}{2}$, $0^\circ \le \theta < 90^\circ$

x-coord.

45°

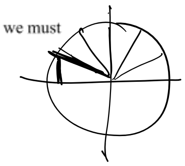


4. $\cos\theta = \frac{\sqrt{2}}{2}$, $0^\circ \le \theta < 360^\circ$

45°
315°

4.4 Solving Trigonometric Equations

We now know how to find trigonometric ratio values for any angle. Sometimes we must settle for an approximation, other times we can give an exact value.



For example:

$\sin 47^\circ \doteq$ 0.73135 $\qquad \cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

Now we look at the opposite situation. Given the value of a trigonometric ratio, how do we find out the size of the *angle*? This is a process that works well:

Isolate – Decide – Get Reference Angle - Solve

See page 21.

Isolate – Decide – Get Reference Angle - Solve

1) Isolate the trigonometric term. If it uses cot, sec, or csc, take the ***reciprocal*** of both sides of the equation to get a simpler-to-solve version of the equation.

2) Decide whether the equation can be solved using
- special angles on the unit circle. Look for: $0, \pm 1, \pm\frac{1}{2}, \pm\frac{\sqrt{2}}{2}, \pm\frac{\sqrt{3}}{2}$
- the $\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$ button on the calculator
- OR, cannot be solved

3) Determine in which quadrants answers will be found.

4) Find the reference angle and use it to find all the solutions in the given domain. Use the same units (either degrees or radians) as shown in the question's domain.

Examples Solve algebraically. If possible, give exact answers. Otherwise, give answers correct to one decimal place.

domain tells us which mode to use

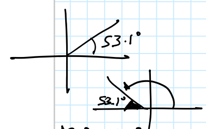
a) $\sin\theta - 0.8 = 0$, for $0^\circ \le \theta < 360^\circ$

1) isolate trig function: $\sin\theta - 0.8 = 0$
$+0.8 \quad +0.8$
$\sin\theta = 0.8$

2) decide [in which quadrants can this happen? Q1, Q2
[calc. or unit circle

3) reference angle: $\sin^{-1}(\sin\theta) = \sin^{-1}(0.8)$
$\theta_R = \sin^{-1}(0.8)$ degree mode $\doteq \boxed{53.1^\circ}$

4) answers
$Q_1 = \boxed{53.1^\circ}$
$Q_2 = \boxed{126.9^\circ}$

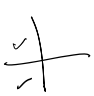


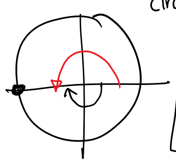
$180^\circ - 53.1^\circ$

b) $\cos\theta = -1$, for $-\pi \le \theta < 2\pi$

isolate ✓

decide Q2, Q3 unit circle





$\boxed{\pi \quad \text{also} \quad -\pi}$

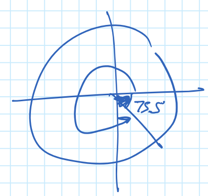
Try: $4\cos\theta - 1 = 0$, $0 \le \theta < 360^\circ$

isolate $4\cos\theta = 1$
$\cos\theta = \frac{1}{4}$

decide Q1, Q4 use calc.

ref. $\theta_R = \cos^{-1}\left(\frac{1}{4}\right)$
$= 75.522\ldots^\circ$

solve: $\boxed{Q_1 \doteq 75.5^\circ}$ $\boxed{Q_4 = 360^\circ - \theta_R = 284.5^\circ}$



radian mode

c) $3\cos\theta - 12 = 0$, for $0 \le \theta < 2\pi$

isolate $\frac{3\cos\theta}{3} = \frac{12}{3}$
$\cos\theta = 4$

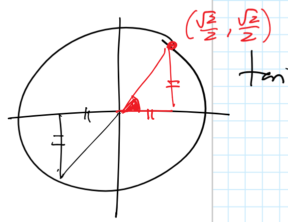
ref. angle $\theta_R = \cos^{-1}(4)$
impossible! $\boxed{\text{no solution}}$

decide Q1, Q4, use calc.

$\cos\theta = \frac{adj}{hyp}$ $\frac{4}{1}$

d) $3\tan\theta - 3 = 0$, for $0 \le \theta < 2\pi$ radians

isolate $\frac{3\tan\theta}{3} = \frac{3}{3}$
$\tan\theta = 1$



$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$\tan\theta = 1 = \frac{y}{x}$

decide Q1, Q3 unit circle:

solve $\boxed{Q_1 = \pi/4}$ $\boxed{Q_3 = 5\pi/4}$

e) $3\sec\theta - 10 = 0$, for $0^\circ \le \theta < 720^\circ$ degrees

coterminal

$3\sec\theta = 10$

e) $3\sec\theta - 10 = 0$, for $0 \le \theta < 720$ degrees

isolate $\frac{3\sec\theta}{3} = \frac{10}{3}$

$\sec\theta = \frac{10}{3}$

$\frac{1}{\cos\theta} = \frac{10}{3}$

$\cos\theta = \frac{3}{10}$

decide Q1, Q4 CAST

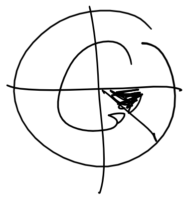
ref $\theta_R = \cos^{-1}\left(\frac{3}{10}\right)$

$= 72.54\ldots$

$\boxed{Q_1 \doteq 72.5°}$

$Q_4 \doteq 360° - \theta_R$

$= \boxed{287.5°}$



Coterminal

other Q_1: $72.5°$
$+360°$
$= \boxed{432.5°}$

other Q_4 answer $= 287.5°$
$+360°$
$= \boxed{647.5°}$

Coming up

- **Starting Chapter 5 next class (trigonometric graphs)**
- **Complete the Chapter 4 hand-in, except for #14befg, please wait on those.**
- **Prepare for the Chapter 4 Test, Thursday, Oct 13**

More Practice

Worksheet: More Chapter 4 Review, found on website (with solutions)
Worksheet: Trig Practice #4, on website (with solutions)
(4.3) p 201: 1-2(acegik), 3ace, 6ace, 9ace, 10all, 11 all, 12ac
(4.4) p 211: 3-4, 5ace

Chapter 4 Test Thursday, Oct 13 - includes a "no-calculator" section.

Make sure you can:

- **Convert between radians and degrees**
- **Draw angles (in radians or degrees) in standard position**
- **Decide which quadrant an angle belongs to**
- **Determine coterminal angles**
- **Determine reference angles**
- **Solve problems involving arc length**
- **Use special triangle values to find EXACT values**
- **Use the unit circle**
- **Find exact values of trig ratios**
- **Solve trigonometric equations**

You will have a sheet like this when you write the test.

CHAPTER 4 – formula sheet

$\csc\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$ $\tan\theta = \frac{\sin\theta}{\cos\theta}$

Quadratic Formula:

For $ax^2 + bx + c = 0,\ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Arc Length Formula: $a = r\theta$