Class_10 Feb 7 - filled in Solving Quadratic Equations

Wednesday, February 8, 2023 11:38 AM

Tonight's Class:

- **Unit 1 Test return and rewrite sign-up**
- **Working through 3.3-3.6**
  - **Solving quadratic equations by factoring**
  - **Solving quadratic equations by square-rooting**
  - **Solving quadratic equations using quadratic formula**

Here is a filled-in version of what I had planned to do with the class on Tuesday, February 7.

3.3 Solving Quadratic Equations by Factoring

Focus: apply factoring strategies to solve quadratic equations

The next few sections look at ways to solve quadratic equations

- **What is a quadratic equation?**

Quadratic Equation

A quadratic equation is any equation that can be written in the form $ax^2 + bx + c = 0$, where a, b, and c are constants and $a \neq 0$.

- **We know that solving is finding values of "x" that make the equation true. We solve linear equations by collecting the constants, collecting the variables, and isolating. The process for solving quadratic equations is different.**

For example, look at this quadratic equation: $x^2 - 5x - 36 = 0$

If we try to isolate "x", we run into trouble. We might get one of these two results, but neither gives us what we need:

Not helpful: $x = \dfrac{x^2 - 36}{5}$ OR $x = \pm\sqrt{5x + 36}$

Instead, we will

- **collect all terms to one side of the equation, so zero is on the other side**
- **FACTOR**
- **Set each factor equal to zero, and solve for x**

WT, page 205

Example 1 **Solving Quadratic Equations by Factoring**

Solve each equation, then verify the solution.

a) $x^2 - 2x - 8 = 0$ **b)** $(2x - 3)(x + 1) = 3$

a) $x^2 - 2x - 8 = 0$

$(x + 2)(x - 4) = 0$

[If $ab = 0$, we know $a = 0$, $b = 0$, or both a and $b = 0$. This is called the ZERO Property.]

$\Rightarrow$ $x + 2 = 0$ $x - 4 = 0$

$\boxed{x = -2}$ $\boxed{x = 4}$

Like always, we can check solutions by substituting back into the original equation: $x^2 - 2x - 8 = 0$

$(-2)^2 - 2(-2) - 8 = 4 + 4 - 8 = 0$ ✓

and $(4)^2 - 2(4) - 8 = 16 - 8 - 8 = 0$ ✓

They work ✓

b) $(2x - 3)(x + 1) = 3$

expand left-side first: $(2x - 3)(x + 1) = 3$

$2x^2 + 2x - 3x - 3 = 3$

$2x^2 - x - 3 = 3$

collect ALL terms to one side

$2x^2 - x - 3 = 3$

$-3 \qquad -3$

$2x^2 - x - 6 = 0$

now, FACTOR

$AC = 2(-6)$
$= -12$
Product $= -12$
Sum $= -1$

$\left.\right\}$ $-4, 3$

$2x^2 - 4x + 3x - 6 = 0$

$2x(x-2) + 3(x-2) = 0$

$(x-2)(2x+3) = 0$

set each factor equal to zero and solve for x

$x - 2 = 0$

$\boxed{x = 2}$

$2x + 3 = 0$
$\quad -3 \quad -3$

$\frac{2x}{2} = \frac{-3}{2}$

$\boxed{x = -\frac{3}{2}}$

Can check by substituting in.

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Example 2 **Solving Quadratic Equations Involving a Common Factor**

Solve each equation, then verify the solution.

a) $2x^2 + 18 = 12x$ **b)** $2x^2 = 4x$

a) $2x^2 + 18 = 12x$

COLLECT all terms to one side:

$2x^2 + 18 = 12x$
$\quad -12x \quad -12x$

$2x^2 - 12x + 18 = 0$

FACTOR

$2(x^2 - 6x + 9) = 0$

remember to factor out GCF when you can!

$2(x-3)(x-3) = 0$

SET each factor equal to zero

"2" cannot equal zero, no answer from this factor

$x - 3 = 0$

$\boxed{x = 3}$

b) $2x^2 = 4x$

COLLECT

$2x^2 = 4x$
$\quad -4x \quad -4x$

$$2x^2 - 4x = 0$$

FACTOR

$$2x(x-2) = 0$$

SET to Zero

$$\frac{2x}{2} = \frac{0}{2} \qquad x - 2 = 0$$

$$\boxed{x = 0} \qquad \boxed{x = 2}$$

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Example 3 **Solving a Radical Equation**

Solve this equation, then verify the solution: $\sqrt{x+3} - 1 = x$

We've done this before. We start as usual:

1) find restrictions, using radicand ≥ 0
2) isolate radical
3) square both sides

$$\sqrt{x+3} - 1 = x$$

restrictions

radicand ≥ 0

$x + 3 \geq 0$

$\boxed{x \geq -3}$

isolate, then square

$$\sqrt{x+3} - 1 = x$$
$$+1 \qquad +1$$
$$\left(\sqrt{x+3}\right)^2 = (x+1)^2$$
$$x + 3 = (x+1)(x+1)$$
$$x + 3 = x^2 + x + x + 1$$
$$x + 3 = x^2 + 2x + 1$$

☆ CAREFUL when squaring the right hand side!

Our result is a quadratic equation, so we now need to

$$x + 3 = x^2 + 2x + 1$$

Our result is a quadratic equation, so we now need to

COLLECT

$$x+3 = x^2+2x+1$$
$$-x \; -3 \qquad -x \; -3$$
$$0 = x^2+x-2$$

or

FACTOR

$$x^2+x-2=0$$
$$(x-1)(x+2)=0$$

SET to 0

$$x-1=0 \qquad x+2=0$$
$$x=1 \qquad x=-2$$

We've done everything correctly, but it's 100% necessary to check these solutions in the original. The reason is that when we square both sides, we can often end up with EXTRANEOUS roots. These are answers that are solutions to the equation we get after squaring, but not solutions of the original equation.

VERIFY in original equation $\qquad \sqrt{x+3}-1=x$

$x=1$

LS	RS
$\sqrt{1+3}-1$	1
$\sqrt{4}-1$	✓
$2-1$	
1 ✓	

yes

$\boxed{x=1}$ is a solution

$x=-2$

LS	RS
$\sqrt{-2+3}-1$	-2
$\sqrt{1}-1$	
$1-1$	
0	

no

~~$x=-2$~~ is an extraneous root

3.4 Using Square Roots to Solve Quadratic Equations

Focus: use strategies of determining square roots to solve quadratic equations

We are looking at only one small part of this section. It also talks about

something called "completing the square" which we will not do until Chapter 4.

Some quadratic equations can be written very simply, in the form

$$\boxed{}^{\,2} = \text{number}$$

These equations can be solved in two different ways:

1) **Expanding the left side, collecting all terms to one side and factoring**

   OR

2) **Square-rooting both sides**

$$x^2 - 64 = 0$$

Factoring Method

$$x^2 - 64 = 0$$
$$(x+8)(x-8) = 0$$
$$x+8=0 \qquad x-8=0$$
$$x=-8 \qquad x=8$$

Square-rooting Method

$$x^2 - 64 = 0$$
$$x^2 = 64$$
$$\sqrt{x^2} = \sqrt{64}$$
$$x = \pm 8$$

which means $x = 8$ and $x = -8$

important to include the $\pm$

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Example 1 **Solving Quadratic Equations by Determining Square Roots**

we'll use the square-root method

Solve each equation. Verify the solution.

a) $2x^2 - 1 = 5$ **b)** $(x-4)^2 = 12$

a) $2x^2 - 1 = 5$

$ +1 \quad +1$

isolate squared term

$$\frac{2x^2}{2} = \frac{6}{2}$$

$$x^2 = 3$$

square-root

leave in radical form,

$$x^2 = 3$$

square-root both sides

$$\sqrt{x^2} = \pm\sqrt{3}$$

Solutions: $x = \sqrt{3}$, $x = -\sqrt{3}$ or just $\boxed{x = \pm\sqrt{3}}$

leave in radical form, not as a decimal approximation

b) $(x-4)^2 = 12$

isolate squared term (already is!)

square-root both sides

$$\sqrt{(x-4)^2} \quad \pm\sqrt{12}$$

$$x - 4 = \pm\sqrt{12}$$

isolate x

$$x - 4 = \pm\sqrt{12}$$
$$+4 \qquad +4$$
$$\boxed{x = 4 \pm \sqrt{12}}$$

this is clearer than writing $x = \pm\sqrt{12} + 4$

☆ only doing #4, p222-223

3.5 Using the Quadratic Formula to Solve Quadratic Equations

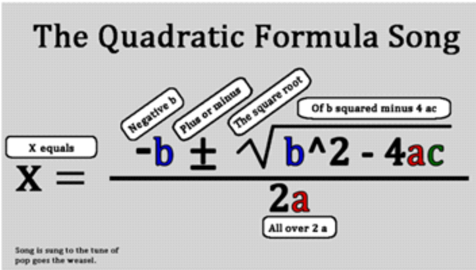
Focus: use the quadratic formula to solve quadratic equations

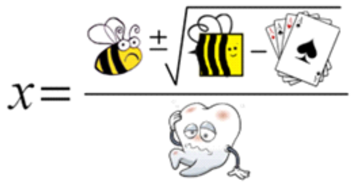
When a quadratic equation does not factor, we can solve it by using the quadratic formula. **(This formula is derived on page 232 of the work text.)**

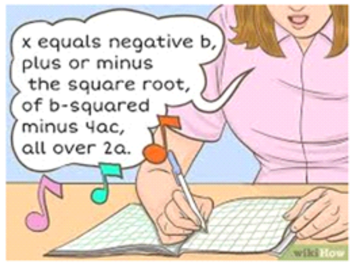
Quadratic Formula

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$







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Example 1 **Solving a Quadratic Equation of the Form $x^2 + bx + c = 0$**

Solve each equation.

a) $x^2 + 4x - 1 = 0$

b) $x^2 - x + 4 = 0$

Check Your Understanding

1. Solve each equation.

a) $x^2 + 2x + 7 = 0$

b) $x^2 - 2x - 7 = 0$

a) $x^2 + 4x - 1 = 0$

$a = 1$

$b = 4$

$c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{(4)^2 - (4)(1)(-1)}}{2(1)}$$

use brackets to help avoid errors

$$= \frac{-4 \pm \sqrt{16 - (-4)}}{2}$$

$$= \frac{-4 \pm \sqrt{20}}{2}$$

simplify radical, if possible

$$= \frac{-4 \pm \sqrt{4 \cdot 5}}{2}$$

$$= \frac{-4 \pm 2\sqrt{5}}{}$$

if all the numbers have a

except the radicand we can

$$2$$

$$= \frac{-4 \pm 2\sqrt{5}}{2}$$

if all the numbers except the radicand have a common factor, we can reduce

$$= \frac{\cancel{2}(-2 \pm 1\sqrt{5})}{\cancel{2}(1)}$$

$$= \boxed{-2 \pm \sqrt{5}}$$

these are the exact solutions
If we are asked for approximate solutions, we use the calculator to get: $x \doteq 0.236, -4.236$

b) $x^2 - x + 4 = 0$

$a=1$
$b=-1$
$c=4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - (4)(1)(4)}}{2(1)}$$

$$= \frac{1 \pm \sqrt{1 - 16}}{2}$$

$$= \frac{1 \pm \sqrt{-15}}{2}$$

not a real number

$\boxed{\text{no solution, no real roots}}$

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US

Check Your Understanding

2. Solve each equation.

a) $(2x + 1)(x - 1) = 5x$

b) $\frac{1}{2}x^2 - \frac{5}{4}x = 3$

Example 2 Solving a Quadratic Equation of the Form $ax^2 + bx + c = 0$

Solve each equation.

a) $2x = 3(x - 1)(x + 1)$ **b)** $\frac{2}{3}x^2 + 1 = \frac{5}{6}x$

a) $2x = 3(x-1)(x+1)$

$2x = 3(x^2 + \cancel{x} - \cancel{x} - 1)$

a) $2x = 3(x-1)(x+1)$

$2x = 3(x^2 + \cancel{x} - \cancel{x} - 1)$

distribute, collect all terms to one side

$2x = 3(x^2 - 1)$

$2x = 3x^2 - 3$

$-2x \qquad -2x$

$0 = 3x^2 - 2x - 3$

$a = 3$
$b = -2$
$c = -3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - (4)(3)(-3)}}{2(3)}$$

$$= \frac{2 \pm \sqrt{4 + 36}}{6}$$

$$= \frac{2 \pm \sqrt{40}}{6}$$

$$= \frac{2 \pm \sqrt{4 \cdot 10}}{6}$$

$$= \frac{2^{\div 2} \pm 2^{\div 2}\sqrt{10}}{6^{\div 2}}$$

$$= \boxed{\frac{1 \pm \sqrt{10}}{3}}$$

b) $\frac{2}{3}x^2 + 1 = \frac{5}{6}x$

$-\frac{5}{6}x \qquad -\frac{5}{6}x$

$\frac{2}{3}x^2 - \frac{5}{6}x + 1 = 0$

Notice that the a-value and b-value are both fractions, which makes computations harder.
It's always true that if we multiply both sides of an equation by the same number, the resulting equation has the same solutions as the original equation.

Let's multiply each side of the equation by "6" This is the number that would have been a common denominator for the fractions. Multiplying by 6 will get rid of all the fractions.

$$6\left[\frac{2}{3}x^2 - \frac{5}{6}x + 1\right] = 6\left[0\right]$$

$$\frac{12}{3}x^2 - \frac{30}{6}x + 6 = 0$$

$$4x^2 - 5x + 6 = 0$$

$a = 4$

$b = -5$

$c = 6$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - (4)(4)(6)}}{2(4)}$$

$$= \frac{5 \pm \sqrt{25 - 96}}{8}$$

$$= \frac{5 \pm \sqrt{-71}}{8}$$

not a real number

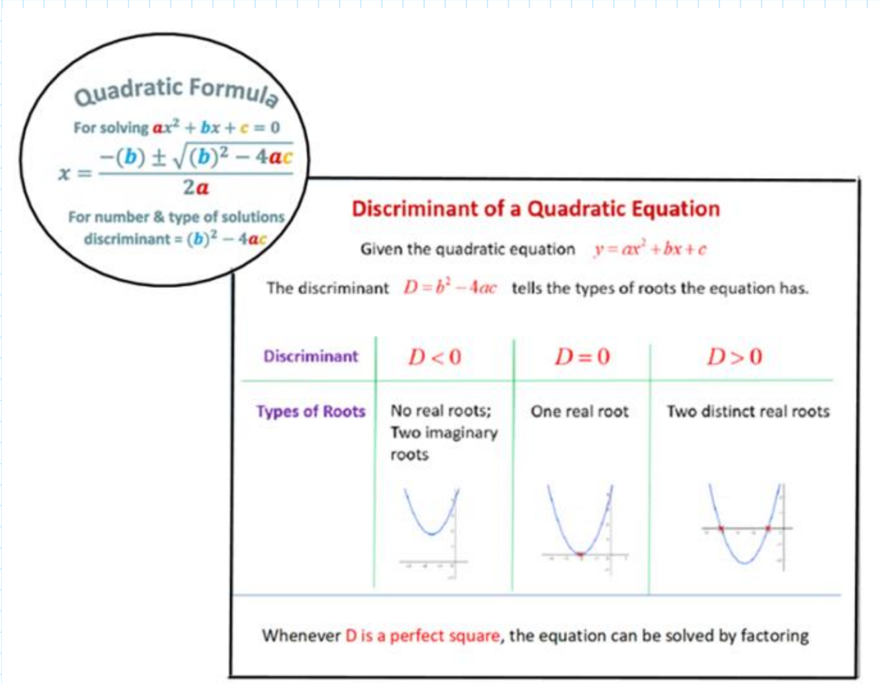
$\boxed{\text{no solution, no real roots}}$

3.6 Interpreting the Discriminant

Focus: determine the number of solutions a quadratic equation has, without solving the equation

In the quadratic formula, the part under the square root is called the discriminant.

It identifies HOW MANY real solutions/roots a quadratic equation will have.



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Example 1 **Determining the Number of Roots of a Quadratic Equation**

Without solving, determine whether the equation $5x^2 - 8x + 6 = 0$ has one, two, or no real roots.

$a = 5$
$b = -8$
$c = 6$

Check Your Understanding

1. Without solving, determine whether the equation $9x^2 - 6x + 1 = 0$ has one, two, or no real roots.

$$D = b^2 - 4ac$$
$$= (-8)^2 - (4)(5)(6)$$
$$= 64 - 120$$
$$= -56$$

$-56 < 0$, so $\boxed{\text{no real roots}}$

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RM, US

Check Your Understanding

2. a) Determine the values of k for which $2x^2 + 7x + k = 0$ has no real roots.

b) Use one value of k to write an equation that has no real roots.

Example 2 **Creating an Equation with a Given Number of Roots**

a) Determine the values of k for which the equation $3x^2 - 5x + k = 0$ has two real roots.

b) Use one value of k to write an equation that has two real roots.

a) $3x^2 - 5x + k = 0$

To have two real roots, we need $D > 0$, or $b^2 - 4ac > 0$

$a = 3$
$b = -5$
$c = k$

$$\Rightarrow (-5)^2 - (4)(3)(k) > 0$$
$$25 - 12k > 0$$
$$-25 \quad -25$$
$$\frac{-12k}{-12} > \frac{-25}{-12}$$

$\boxed{\text{If } k < \frac{25}{12}, \text{ it has 2 real roots}}$

b) Here is one possible equation with 2 real roots

$$3x^2 - 5x + 1 = 0$$

We can choose any number smaller than $\frac{25}{12}$

We can choose any number smaller than $\frac{25}{12}$

For next class

- **Work on the worktext questions for 3.3-3.6**
- **For section 3.4, ONLY do question #4**
- **Start working on Chapter 3 Hand-in assignment, due Feb 14**