Class_10 May 18 Trig Graphs and Applications

Wednesday, May 17, 2023 10:03 PM

Tonight's Class:

- **Chapter 4 Hand-in Due**
- **5.2 Transforming Trig Graphs (continued)**
- **5.4 Trig Applications**

Convert $1040° \times \frac{\pi}{180°} = \frac{104\cancel{0}\pi}{18\cancel{0}} = \frac{104\pi}{18} = \frac{52\pi}{9}$

From the notes package, page 28:

reflected

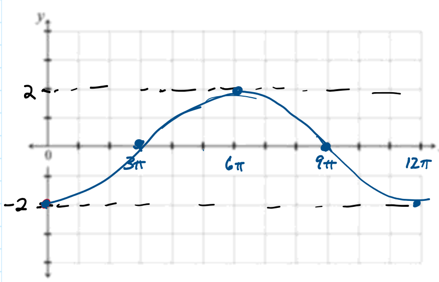
amplitude = 2

b) $y = -2\cos\left(\frac{1}{6}x\right)$

HE by 6

period $= 2\pi \times 6$

$= 12\pi$



x	y	
0	-2	min
3π	0	CL
6π	2	MAX
9π	0	CL
12π	-2	min

$\frac{12\pi}{4}$

$= 3\pi$

spacing

Summary

$p = \frac{2\pi}{b}$ or Horizontal stretch of $\frac{1}{b}$

$$y = a\sin b(x - c) + d$$

amplitude (vert. stretch

phaseshift (horiz. translation)

vertical displacement (vert. translation) midline/centre line $y = d$

Try

Determine the amplitude, period, phase shift and vertical displacement.

a) $y = 5\cos\left(3\left(x + \frac{\pi}{4}\right)\right) + 2$

period $= \frac{2\pi}{b} = \frac{2\pi}{3}$

amp $= 5$

period $= \frac{2\pi}{3}$

p. s. $= \pi/4$ left or $-\pi/4$

v. disp $=$ up 2

b) $y = \frac{1}{2}\cos\left(x + 80^\circ\right) + 5$

amp $= \frac{1}{2}$

period $= 360^\circ$

p.s. $= 80^\circ$ left, or -80°

v disp $=$ up 5

c) $y = -2\sin\left(5x - \frac{\pi}{3}\right) + 4$

$5\left(x - \frac{\pi}{15}\right)$

$5x \quad -\frac{5\pi}{15}$

$\frac{\pi}{3} \div 5$

$\frac{\pi}{3} \cdot \frac{1}{5}$

$\frac{\pi}{15}$

amp $= +2$

period $= \frac{2\pi}{5}$

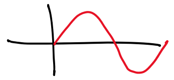
p. s. $= \frac{\pi}{15}$ right

v disp $=$ up 4

Sketching a Sinusoidal Graph

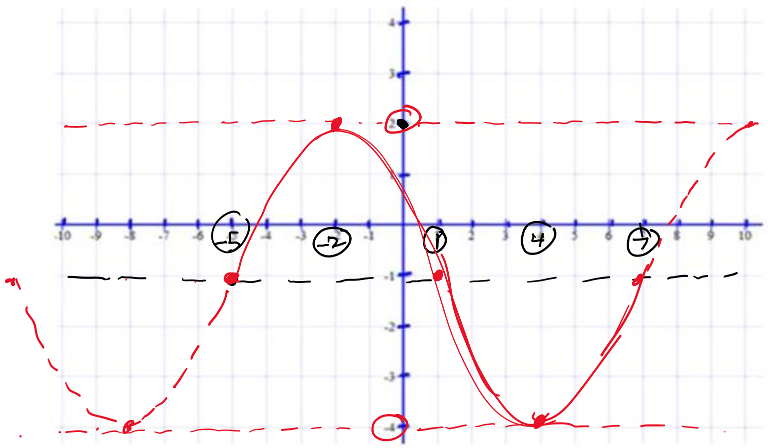
Consider the equation: $y = 3\sin\left(\frac{2\pi}{12}(x+5)\right) - 1$

a) Key features:

basic sine shape	vertical displacement	equation of center line
	down 1	$y = -1$
amplitude	**maximum**	**minimum**
3	$-1+3 = 2$	$-1-3 = -4$
period	**spacing** period $\div 4$	**phase shift**
$\frac{\cancel{2\pi}}{1} \cdot \frac{12}{\cancel{2\pi}} = 12$	$= 12 \div 4 = 3$	5 left

1) plot center line
2) plot max + min on y-axis
3) label x-axis

b) Accurately sketch one period of the graph. Give the coordinates of 5 key points. Include the center line on your sketch.



start table at p.shift location

x	y	
−5	−1	CL
−2	2	Max
1	−1	CL
4	−4	min
7	−1	CL

use spacing to get my other x-values

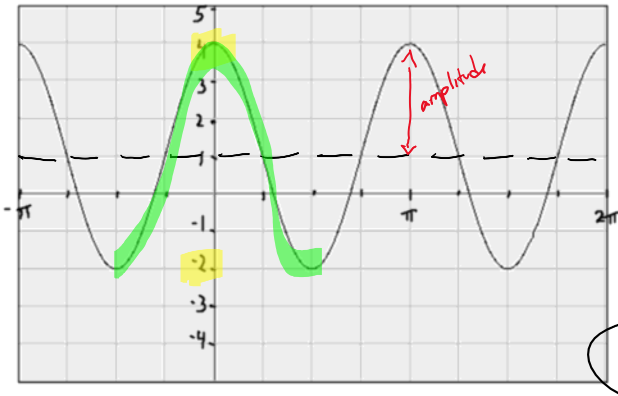
Finding the Equation of a Graph

Sine and cosine graphs are both called ***sinusoidal graphs***.

- For any sinusoidal graph, it is possible to write a sine equation that creates that graph, and a cosine equation that creates that same graph.
- There are many different equations that generate the same sinusoidal graph.

Example

Give two different equations that create this graph.



Maximum: 4

Minimum: -2

Center line: $y = 1$

(average heights) $\frac{4+-2}{2} = \frac{2}{2} = 1$

Vertical displacement: up 1

Amplitude: 3

Period: π

(horizontal distance from MAX to next MAX)

b value: 2

$b = \frac{\text{normal period}}{\text{graph's period}}$

$b = \frac{2\cancel{\pi}}{\cancel{\pi}} = 2$

Possible sine equation:

$$y = 3\sin\left(2\left(x + \frac{\pi}{4}\right)\right) + 1$$

$$y = -3\sin\left(2\left(x - \frac{\pi}{4}\right)\right) + 1$$

Possible cosine equation:

$$y = 3\cos(2x) + 1$$

$$y = -3\cos\left(2\left(x + \frac{2\pi}{4}\right)\right) + 1$$

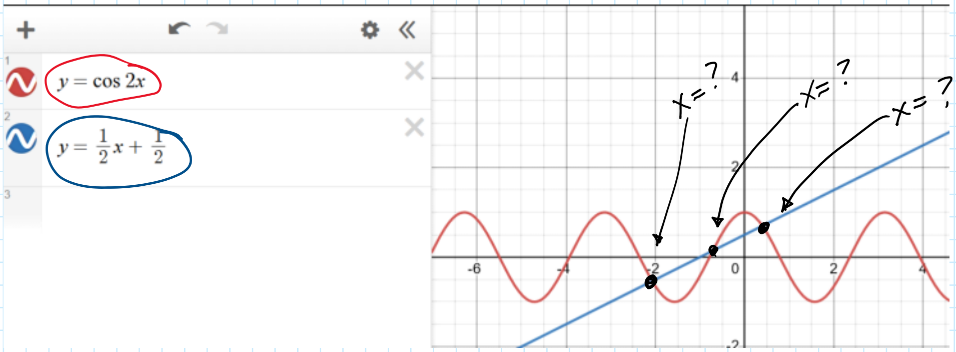
or

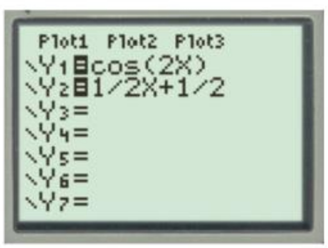
$$y = -3\cos\left(2\left(x + \frac{\pi}{2}\right)\right) + 1$$

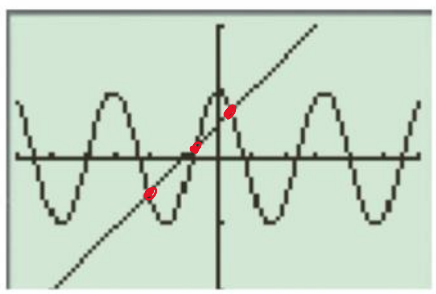
5.4 Trig Equations and Application Questions

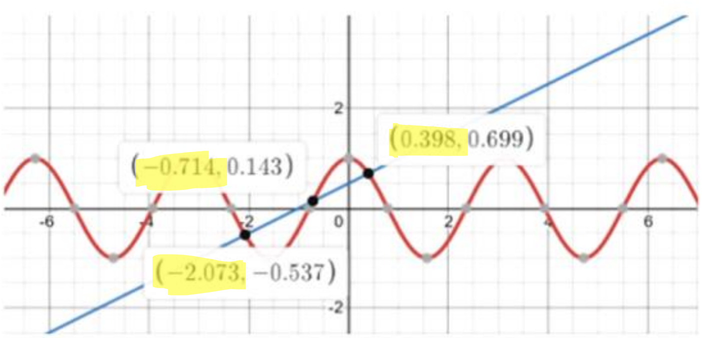
Some equations cannot be solved algebraically. For this reason, we want to understand how to solve equations graphically.

Example: Solve: $\cos 2x = \frac{1}{2}x + \frac{1}{2}$, for $0 \le x < 2\pi$.







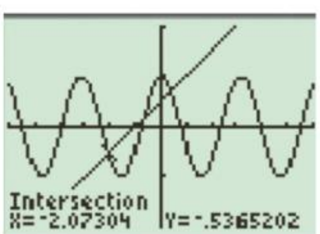


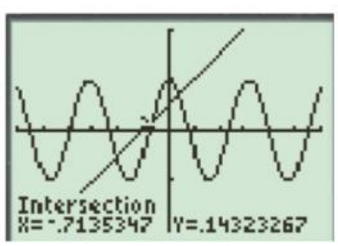
Find the solutions to the equation

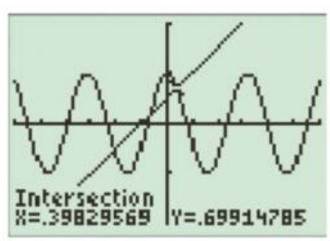
$$\cos(2x) = \frac{1}{2}x + \frac{1}{2}, \text{ for } 0 \le x < 2\pi$$

$x = -0.714$ $x = 0.398$

$x = -2.073$

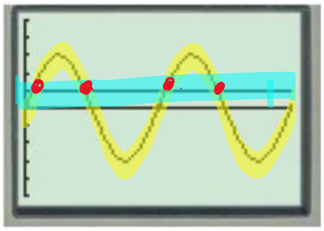


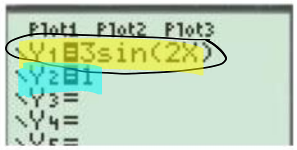




5.4 Equations and Graphs of Trigonometric Functions

Below we see how we can solve a trigonometric equation, ***graphically***.





a) What is the equation that is being solved? $3\sin 2x = 1$

~~b) The window has been restricted to match the domain for this question. What is that domain?~~

c) How many solutions are there, in this domain? Mark them on the calculator graph screenshot, shown above. 4, they are the intersections of the 2 graphs

Remember, there are two ways to solve equations GRAPHICALLY

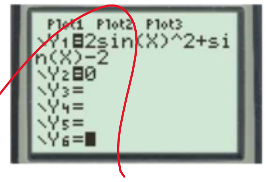
Intersection Method
1) Enter the LHS of the equation as Y_1
2) Enter the RHS of the equation as Y_2
3) Set the *Xmin* and *Xmax* values using the given domain.
4) Use the "intersect" feature to find each place where the LHS = RHS. The *x*-values of the intersections are the equation's solutions.

X-intercepts (zeroes) Method
1) Collect all terms of the equation on one side of the equals sign, so it looks like $\boxed{}$ = 0.
2) Enter the equation as Y_1
3) Set the *Xmin* and *Xmax* values using the given domain.
4) Use the "Zero" feature to find each *x*-intercept. These *x*-values are the equation's solutions.

Try

Solve ***graphically***, correct to 1 decimal place. Include a sketch of the graph with the solutions marked on it.

$$2\sin^2 x + \sin x - 2 = 0, \text{ for } 0^\circ \le x \le 720^\circ$$



Example

Consider the trigonometric equation $6\sin\left(\frac{\pi}{4}x\right)+8=10$

a) Solve ***graphically*** for $0 \le x < 2\pi$, correct to 4 decimal places. Include a sketch of the graph with the solutions marked on it.

This affects the length of the period!

b) Find the general solution, ***algebraically***, correct to 4 decimal places.

$$6\sin\left(\frac{\pi}{4}x\right)+8=10$$
$$-8 \qquad -8$$

$$\frac{6\sin\left(\frac{\pi}{4}x\right)}{6}=\frac{2}{6}$$

$$\sin\left(\frac{\pi}{4}x\right)=\frac{1}{3}$$

$$\frac{4}{\pi}\left(\frac{\pi}{4}x = \sin^{-1}\left(\frac{1}{3}\right)\right)$$

$$x = \frac{4}{\pi}\cdot\sin^{-1}\left(\frac{1}{3}\right)$$

$$\boxed{x \doteq 0.4327}$$

Q_1 answer.

$$\frac{\pi}{4}x = \sin^{-1}\left(\frac{1}{3}\right)$$

$$x = \frac{\sin^{-1}\left(\frac{1}{3}\right)}{\pi/4}$$

$$x \doteq 0.4327$$

$$\sin\theta = \frac{1}{3}$$

$$\theta_R = \sin^{-1}\left(\frac{1}{3}\right)$$

General solution:

$x = 0.4327 + (\text{period})(n)$

$x = 0.4327 + 8n,\ n \in I$

$x = 3.5673 + 8n,\ n \in I$

Q_2 answer:

$$\frac{\pi}{4}x = \pi - \theta_R$$

$$\frac{\cancel{\pi/4}\, x}{\cancel{\pi/4}} = \frac{\pi - \sin^{-1}\left(\frac{1}{3}\right)}{\pi/4}$$

$$x = \frac{\pi - \sin^{-1}(1/3)}{\pi/4} = 3.5673$$

$y = \sin\left(\frac{\pi}{4}x\right)$ What is the period?

$$\frac{2\pi}{b} = \frac{2\pi}{\pi/4}$$

$$= \frac{2\pi}{1} \cdot \frac{4}{\pi} = \frac{8\cancel{\pi}}{\cancel{\pi}} = 8$$

Example of an application question

The depth of water, h meters, at a certain port, at time t hours, is given by this equation, where $t = 0$ represents midnight:

$$h(t) = 1.4\sin\left(\frac{2\pi}{12.2}(t - 0.8)\right) + 2.7$$

How deep will the water be at 2:00 AM? At 4:00 PM?

substitute $t=2$

$h(2) = 1.4\sin\left(\frac{2\pi}{12.2}(2-0.8)\right) + 2.7$

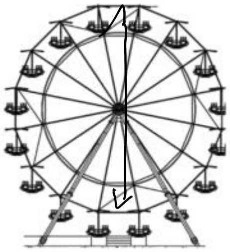
$\boxed{h(2) \doteq 3.5 \text{ m}}$

substitute $t=16$, $\boxed{h(16) \doteq 4.1 \text{ m}}$

WB - Creating a Sinusoidal Graph and Equation

Example

Suppose the pictured Ferris wheel has diameter 40 meters, and the height of the seat where you first get on is 2 meters above the ground. This wheel takes 2 minutes to rotate and travels at a constant speed.



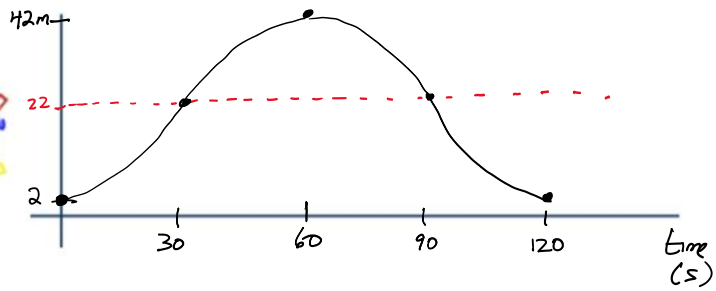
- Minimum height? 2 m
- Maximum height? 42m
- Center line height? 22 m
- Period length in seconds? 120 seconds

George Washington Gale Ferris Jr.

Born	February 14, 1859 Galesburg, Illinois
Died	November 22, 1896 (aged 37) Pittsburgh, Pennsylvania
Cause of death	Typhoid fever
Education	Rensselaer Polytechnic Institute (1881)
Known for	The original Chicago Ferris Wheel and the Ferris wheel concept

Centr line $\frac{42+2}{2} = \frac{44}{2} = 22$

a) Sketch a complete period of the graph, showing the height of a passenger above the ground as a function of time, in seconds. Give the coordinates of 5 key points.



t	*h*
0	2
30	22
60	42
90	22
120	2

b) Create a sinusoidal equation for this graph.

$b = \frac{\text{regular period}}{\text{graph's period}} = \frac{2\pi}{120}$

(either sine or cosine is fine)

$$h = -20\cos\left(\frac{2\pi}{120}t\right) + 22$$

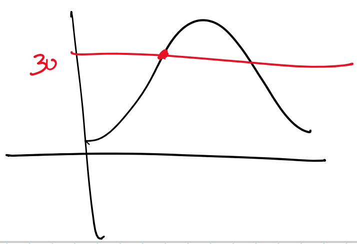
c) How high above the ground is a passenger 12 seconds after getting on, correct to one decimal place?

$$h = -20\cos\left(\frac{2\pi}{120}(12)\right) + 22$$

(radian mode)

$h \doteq 5.8$ m

d) During the first rotation of the Ferris wheel, what is the first time that the passenger reaches a height of 30 meters above the ground? ~~Solve this graphically.~~ Solve algebraically

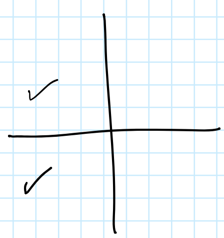


$$30 = -20\cos\left(\frac{2\pi}{120}t\right) + 22$$
$$-22 \qquad\qquad -22$$

$$\frac{8}{-20} = \frac{\cancel{-20}}{\cancel{-20}}\cos\left(\frac{2\pi t}{120}\right)$$

$$\cos\left(\frac{2\pi t}{120}\right) = \frac{8}{-20} = \frac{2}{-5}$$

$$\cos\left(\frac{2\pi t}{120}\right) = \frac{2}{-5}$$



$$\theta_R = \cos^{-1}\left(+\frac{2}{5}\right)$$
$$= 1.15927...$$

$$Q_2 \text{ answer} = \pi - 1.15927....$$
$$= 1.982313173$$

Q_2 answer $= \pi - 1.15927\ldots$

$= 1.982313173$

$$\frac{\cancel{\frac{2\pi t}{120}}}{\cancel{\left(\frac{2\pi}{120}\right)}} = \frac{1.982313173}{\left(\frac{2\pi}{120}\right)}$$

$$\boxed{t \doteq 37.9 \text{ seconds}}$$

Coming Up

- **No class on Monday, May 22 (Victoria Day)**
- **Chapter 5 Hand-in assignment - omit question #9. Due Tuesday, May 23**

- **Test 3 on Tuesday, May 23 (on 4.2-5.4, omit 5.3 and 6.1). It includes a NO calculator section**
  - **Know how to use the unit circle to get exact values**
  - **Given a sinusoidal equation, be able to sketch it without using technology**
  - **Given a sinusoidal graph, be able to figure out its equation without using technology**
  - **Know how to find period, phase shift, amplitude, vertical displacement, spacing between key points, coordinates of key points.**
  - **Know how to algebraically solve an equation similar to the example in the notes, page 36. (Or, like #12b in the hand-in assignment)**
  - **Understand the METHOD for graphically solving trigonometric equations**
  - **Be able to create a circular motion equation and solve it. (similar to TB p 279, #19)**