Class_10 Oct 11 Trig Equations and Graphs

Thursday, October 6, 2022 2:11 PM

Tonight's Class:

- **Chapter 4 Test - next class, Thursday, Oct 13**
- **4.4 More Trig Equation Solving**
- **5.1 Sine and Cosine Graphs**

Solve these two trigonometric equations. If exact answers are possible, give those. Otherwise, round values correct to 2 decimal places.

For each equation answer:

- In which quadrants will the answers be?
- What is the size of the reference angle?
- What are the solutions?

1. $8\sin\theta - 4 = 0,\ 0 \le \theta < 2\pi$

$8\sin\theta = 4$

$\sin\theta = \frac{4}{8}$

$\sin\theta = \frac{1}{2}$

$\theta_R = \pi/6$

$\boxed{Q_1 = \pi/6 \quad Q_2 = \pi - \pi/6 = 5\pi/6}$

2. $5\tan\theta - 1 = 0,\ 0^\circ \le \theta < 360^\circ$

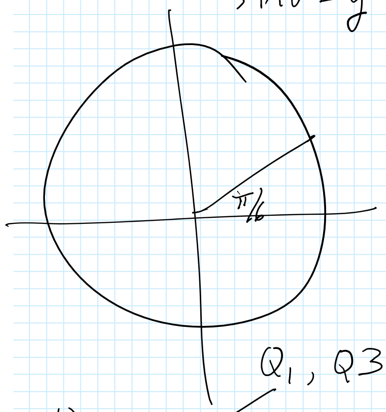
$\frac{5\tan\theta}{5} = \frac{1}{5}$

$\tan\theta = \frac{1}{5}$

$\theta_R = \tan^{-1}\left(\frac{1}{5}\right)$

$= \boxed{11.31^\circ}$

$\sin\theta = y$



$Q_1,\ Q3$

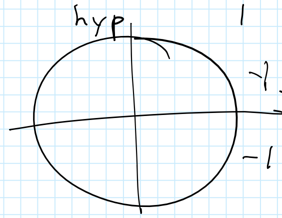
$Q3 = 180^\circ + \theta_R$

$= \boxed{191.31^\circ}$

Which of these trigonometric equations have NO SOLUTION?

1. $\sin\theta = -2$
2. $\cos\theta = 0.3$
3. $\tan\theta = 0.9$
4. $\tan\theta = 15$
5. $\sin\theta = -0.5$
6. $\cos\theta = -1.4$

$\sin\theta = \frac{opp}{hyp} = \frac{-2}{1}$



$-1 \le \cos\theta \le 1$

$-1 \le \sin\theta \le 1$

Isolate – Decide – Get Reference Angle - Solve

1) Isolate the trigonometric term. If it uses cot, sec, or csc, take the ***reciprocal*** of both sides of the equation to get a simpler-to-solve version of the equation.

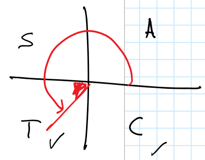
2) Decide whether the equation can be solved using

- special angles on the unit circle. Look for: $0, \pm 1, \pm\frac{1}{2}, \pm\frac{\sqrt{2}}{2}, \pm\frac{\sqrt{3}}{2}$
- the $\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$ button on the calculator
- OR, cannot be solved

3) Determine in which quadrants answers will be found.

4) Find the reference angle and use it to find all the solutions in the given domain. Use the same units (either degrees or radians) as shown in the question's domain.

Examples Solve algebraically. If possible, give exact answers. Otherwise, give answers correct to one decimal place.



a) $\sin\theta = -0.8$, for $0^\circ \le \theta < 360^\circ$

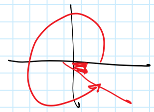
1) isolated ✓

2) calc, Q3 and Q4

3) $\theta_R = \sin^{-1}(+0.8)$

$= 53.1^\circ$ ref angle

If we calculate $\sin^{-1}(-0.8)$, the calculator gives us a negative answer. We don't want this, because the domain asks for only positive answers.
To avoid this problem, give the calculator the trigonometric ratio as a positive quantity. This guarantees the calculator will give us the ***reference angle***, which will be positive.



4) solutions:

$Q_3 = 180^\circ + \theta_R = \boxed{233.1^\circ}$

$Q_4 = 360^\circ - \theta_R = \boxed{306.9^\circ}$

b) $5\cos\theta - 2 = 2\cos\theta - 4$, for $0^\circ \le \theta < 720^\circ$

1) isolate

$-2\cos\theta \qquad -2\cos\theta$

$3\cos\theta - 2 = -4$

$+2 \qquad +2$

$\frac{3\cos\theta}{3} = \frac{-2}{3}$

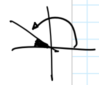
$\cos\theta = -\frac{2}{3}$

2) decide calculator, and Q2 and Q3

3) ref angle $\theta_R = \cos^{-1}\left(+\frac{2}{3}\right)$

$\doteq 48.2^\circ$

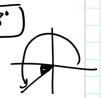
4) solve



$Q_2 = 180^\circ - \theta_R$

$= \boxed{131.8^\circ}$

and $131.8^\circ + 360^\circ = \boxed{491.8^\circ}$



$Q_3 = 180^\circ + \theta_R$

$= \boxed{228.2^\circ}$

and $228.2^\circ + 360^\circ = \boxed{588.2^\circ}$

General Solution

How many solutions a trigonometric equation has depends on the domain specified in the question. When the domain is all real numbers, there are infinitely many solutions. This is called the ***general solution***.

Example Solve algebraically. If possible, give exact answers. Otherwise, give answers correct to one decimal place.

First, get the answers between $0° \le \theta < 360°$

a) $3\cos\theta - 1 = 0$, ***general solution*** in degree measure.

1) isolate

$\frac{3\cos\theta}{3} = \frac{1}{3}$

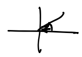
$\cos\theta = \frac{1}{3}$

2) decide calculator, Q1, Q4

3) ref. angle

$\theta_R = \cos^{-1}\left(\frac{1}{3}\right)$

$\doteq 70.5°$



4) answers: $Q_1 = \boxed{70.5°}$

$Q_4 = 360° - \theta_R = \boxed{289.5°}$

$70.5° + 360°n,\ n \in I$
$289.5° + 360°n,\ n \in I$

b) $\cot\theta + 5 = 0$, ***general solution*** in radian measure.

1) isolate

$\cot\theta = -5$

$\frac{1}{\tan\theta} = \frac{-5}{1}$

$\tan\theta = -\frac{1}{5}$

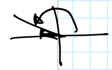
2) calculator, Q2, Q4

3) $\theta_R = \tan^{-1}\left(+\frac{1}{5}\right)$

$= 0.1973...$

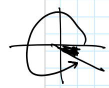
$\doteq 0.20$ (radian)

4) solve



$Q_2 = \pi - \theta_R$

$\doteq \boxed{2.9}$



$Q_4 = 2\pi - \theta_R$

$\doteq \boxed{6.1}$

general sol

$2.9 + 2\pi n,\ n \in I$
$6.1 + 2\pi n,\ n \in I$

Here's how to write the ***general solution***:

- list each answer θ, found in one full rotation, separately
- to each answer add on the appropriate amount, either

  $+2\pi n,\ n \in I$ or $+360°n,\ n \in I$

For equations using *tangent* or *cotangent*, we find that in one full rotation the two solutions are spaced exactly π or $180°$ apart. Because of this, we can just write the first solution, and add onto it:

$+\pi n,\ n \in I$ or $+180°n,\ n \in I$

Example

Suppose that for a certain equation, we are all told its solutions for $0° \le \theta < 360°$ are $\theta = 20°$ and $\theta = 160°$. What is the ***general solution***?

$20° + 360°n,\ n \in I$
$160° + 360°n,\ n \in I$

Solving Second-Degree Trigonometric Equations

When we solve equations with an exponent we usually start by factoring.

For example, solve: $2x^2 - 1 = x$

$2x^2 - x - 1 = 0$

A = 2, B = −1, C = −1

AC = −2, B = −1 } find two numbers product −2 sum −1

(−2, +1)

$2x^2 - 2x + 1x - 1 = 0$

$2x(x-1) + 1(x-1) = 0$

$(x-1)(2x+1) = 0$

$x - 1 = 0$, $\boxed{x = 1}$

$2x + 1 = 0$, $2x = -1$, $\boxed{x = -\frac{1}{2}}$

https://www.youtube.com/watch?v=gENVB6tjq_M

Example

Solve algebraically. If possible, give exact answers. Otherwise, give answers correct to one decimal place.

$2\tan^2\theta - 1 = \tan\theta$, for $0^\circ \le \theta < 360^\circ$

$2\tan^2\theta - \tan\theta - 1 = 0$

$2x^2 - x - 1 = 0$

$(x-1)(2x+1) = 0$

Factor it (we already did, up above)

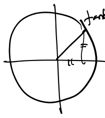
$(\tan\theta - 1)(2\tan\theta + 1) = 0$

Solve each of these equations!

$\tan\theta - 1 = 0$

isolate $\tan\theta = 1$

decide Unit circle!, Q1, Q3



$\tan\theta = \frac{y}{x} = 1$

reference angle $\theta_R = 45^\circ$

solve:

$Q_1 = \boxed{45^\circ}$

$Q_3 = 180^\circ + \theta_R$

$= \boxed{225^\circ}$

$2\tan\theta + 1 = 0$

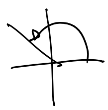
isolate $\frac{2\tan\theta}{2} = \frac{-1}{2}$

$\tan\theta = -\frac{1}{2}$

decide Calculator, Q2, Q4

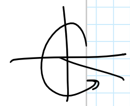
$\theta_R = \tan^{-1}\left(+\frac{1}{2}\right)$

$\doteq 26.6^\circ$



$Q_2 = 180^\circ - \theta_R$

$= \boxed{153.4^\circ}$



$Q_4 = 360^\circ - \theta_R$

$= \boxed{333.4^\circ}$

Example

Solve algebraically. If possible, give exact answers. Otherwise, give answers correct to one decimal place.

$2\cos^2\theta - 1 = 0$, for $0 \le \theta < 2\pi$

$\frac{2\cos^2\theta}{2} = \frac{1}{2}$

$\sqrt{\cos^2\theta} = \pm\sqrt{\frac{1}{2}}$

$\cos\theta = \pm\sqrt{\frac{1}{2}}$

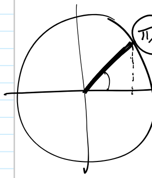
$\cos\theta = \sqrt{\frac{1}{2}}$

$\cos\theta = \frac{\sqrt{1}}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\cos\theta = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$\cos\theta = \frac{\sqrt{2}}{2}$

Q_1, Q_4



$\pi/4$

$\theta_R = \pi/4$

$Q_1 = \pi/4$

$Q_4 = \frac{4}{4}2\pi - \pi/4$

$= 8\pi/4 - \pi/4$

$= \frac{7\pi}{4}$

$\cos\theta = -\sqrt{\frac{1}{2}}$

$\cos\theta = -\frac{1}{\sqrt{2}}$

If we rationalize $\frac{1}{\sqrt{2}}$, it becomes $\frac{\sqrt{2}}{2}$

$\cos\theta = -\frac{\sqrt{2}}{2}$

✓

✓

$Q_2 = \pi - \pi/4$

$= \boxed{3\pi/4}$

$Q_3 = \pi + \pi/4$

$= \boxed{5\pi/4}$

$x^2 - 9 = 0$

$(x+3)(x-3) = 0$

$x = -3$

$x = +3$

OR

$\sqrt{x^2} = \pm\sqrt{9}$

$x = \pm 3$

(4.4) TB p 211: 2-4, 5ace, 7 all
Trigonometry Practice #4

Individual whiteboards - what is the measure of each standard-position angle below?

1. $5\pi/3$

2.

$\frac{7\pi}{4}$

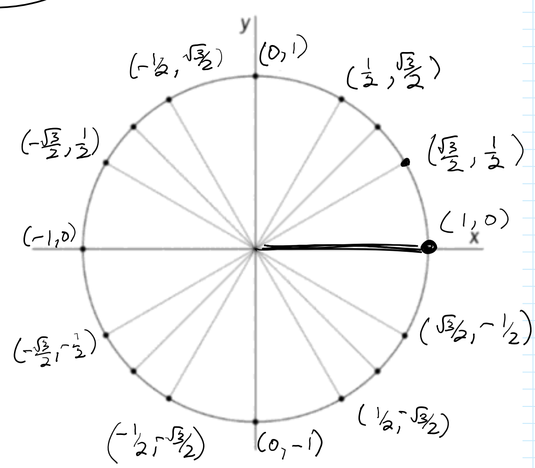
<u>Reviewing the Unit Circle</u> - fill in the tables below, without using your calculator

x-coordinate

θ	$\cos\theta$
0	1
$\frac{\pi}{6}$	$\sqrt{3}/2$
$\frac{2\pi}{6}=\frac{\pi}{3}$	$1/2$
$\frac{3\pi}{6}=\frac{\pi}{2}$	0
$\frac{4\pi}{6}=\frac{2\pi}{3}$	$-1/2$
$\frac{5\pi}{6}$	$-\sqrt{3}/2$
$\frac{6\pi}{6}=\pi$	-1
$\frac{7\pi}{6}$	$-\sqrt{3}/2$
$\frac{8\pi}{6}=\frac{4\pi}{3}$	$-1/2$
$\frac{9\pi}{6}=\frac{3\pi}{2}$	0
$\frac{10\pi}{6}=\frac{5\pi}{3}$	$1/2$
$\frac{11\pi}{6}$	$\sqrt{3}/2$
$\frac{12\pi}{6}=2\pi$	1

y-coordinate

θ	$\sin\theta$
0	0
$\frac{\pi}{6}$	$1/2$
$\frac{2\pi}{6}=\frac{\pi}{3}$	$\sqrt{3}/2$
$\frac{3\pi}{6}=\frac{\pi}{2}$	1
$\frac{4\pi}{6}=\frac{2\pi}{3}$	$\sqrt{3}/2$
$\frac{5\pi}{6}$	$1/2$
$\frac{6\pi}{6}=\pi$	0
$\frac{7\pi}{6}$	$-1/2$
$\frac{8\pi}{6}=\frac{4\pi}{3}$	$-\sqrt{3}/2$
$\frac{9\pi}{6}=\frac{3\pi}{2}$	-1
$\frac{10\pi}{6}=\frac{5\pi}{3}$	$-\sqrt{3}/2$
$\frac{11\pi}{6}$	$-1/2$
$\frac{12\pi}{6}=2\pi$	0



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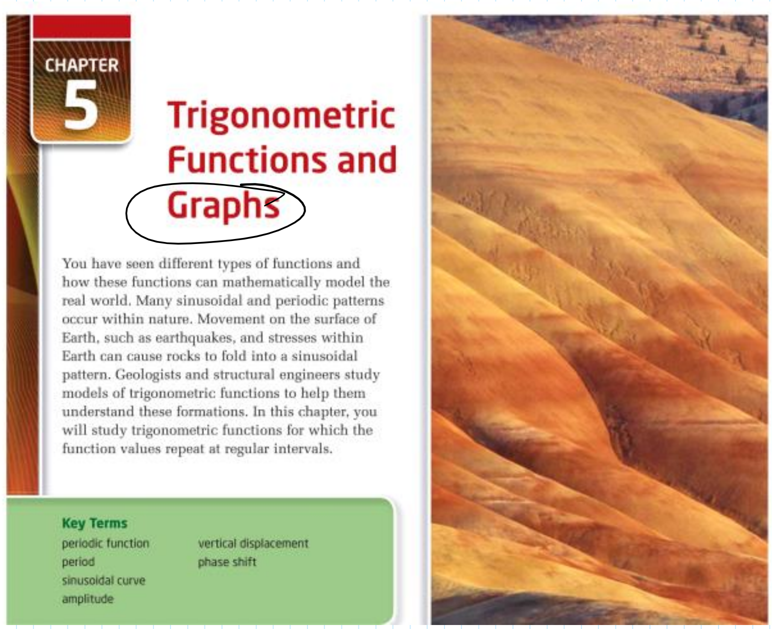


CHAPTER 5

Trigonometric Functions and Graphs

You have seen different types of functions and how these functions can mathematically model the real world. Many sinusoidal and periodic patterns occur within nature. Movement on the surface of Earth, such as earthquakes, and stresses within Earth can cause rocks to fold into a sinusoidal pattern. Geologists and structural engineers study models of trigonometric functions to help them understand these formations. In this chapter, you will study trigonometric functions for which the function values repeat at regular intervals.

Key Terms

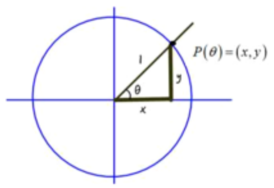
periodic function
period
sinusoidal curve
amplitude
vertical displacement
phase shift

We'll start by looking the BASIC graphs of the functions sine and cosine, and then we'll do transformations on them.

Chapter 5: Trigonometric Functions and Graphs

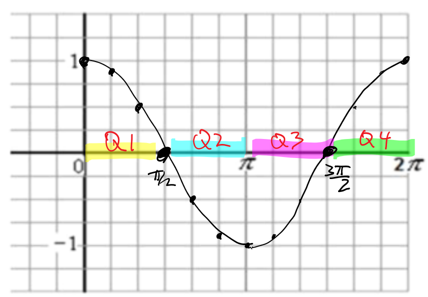
5.1 Graphing Sine and Cosine Functions

Let's track what happens to $P(\theta)$ as θ, a standard-position angle, gets larger.



		$y=\cos\theta$
Q1	As θ increases from 0 to $\frac{\pi}{2}$	cosine values (x-values) go from 1 down to 0
Q2	As θ increases from $\frac{\pi}{2}$ to π	cosine values (x-values) 0 down to −1
Q3	As θ increases from π to $\frac{3\pi}{2}$	cosine values (x-values) −1 up to 0
Q4	As θ increases from $\frac{3\pi}{2}$ to 2π	cosine values (x-values) 0 up to 1

$y=\cos\theta$ (also often written as $y=\cos x$)



θ	$\cos\theta$
0	1
$\frac{\pi}{6}$	$\sqrt{3}/2 \doteq 0.87$
$\frac{2\pi}{6}=\frac{\pi}{3}$	$1/2$
$\frac{3\pi}{6}=\frac{\pi}{2}$	0
$\frac{4\pi}{6}=\frac{2\pi}{3}$	$-1/2$
$\frac{5\pi}{6}$	$-\sqrt{3}/2 \doteq -0.87$
$\frac{6\pi}{6}=\pi$	−1
$\frac{9\pi}{6}=\frac{3\pi}{2}$	0
$\frac{12\pi}{6}=2\pi$	1

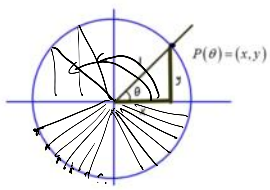
Maximum: 1 Minimum: −1 Range: $\{y \mid -1\le y\le 1\}$ Amplitude: 1

Domain: $\{\theta \mid \theta\in\mathbb{R}\}$ x-intercepts: $\frac{\pi}{2}, \frac{3\pi}{2}, \ldots$ Period: 2π Center line equation: $y=0$

$$\boxed{x=\frac{\pi}{2}+n\pi,\ n\in I}$$

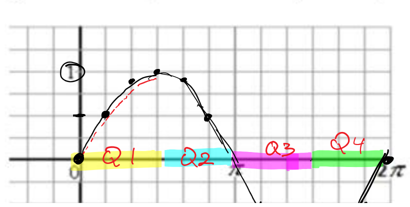
first positive one

spacing between x-intercepts



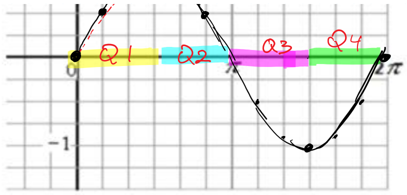
		$y=\sin\theta$
Q1	As θ increases from 0 to $\frac{\pi}{2}$	sine values (y-values) 0 up to 1
Q2	As θ increases from $\frac{\pi}{2}$ to π	sine values (y-values) 1 down to 0
Q3	As θ increases from π to $\frac{3\pi}{2}$	sine values (y-values) 0 down to −1
Q4	As θ increases from $\frac{3\pi}{2}$ to 2π	sine values (y-values) −1 up to 0

$y=\sin\theta$ (also often written as $y=\sin x$)



x's y's

θ	$\sin\theta$
0	0
$\frac{\pi}{6}$	$1/2$
$\frac{2\pi}{6}=\frac{\pi}{3}$	$\sqrt{3}/2 \doteq 0.87$
$\frac{3\pi}{6}=\frac{\pi}{2}$	1
$\frac{4\pi}{6}=\frac{2\pi}{3}$	$\sqrt{3}/2 \doteq 0.87$



$\frac{3\pi}{6} = \frac{\pi}{2}$	1
$\frac{4\pi}{6} = \frac{2\pi}{3}$	$\sqrt{3}/2 \doteq 0.87$
$\frac{5\pi}{6}$	$1/2$
$\frac{6\pi}{6} = \pi$	0
$\frac{9\pi}{6} = \frac{3\pi}{2}$	−1
$\frac{12\pi}{6} = 2\pi$	0

Maximum: 1

Minimum: −1

Range: $\{y \mid -1 \le y \le 1\}$

Amplitude: 1

largest vertical distance the graph goes away from the center line

Domain: $\{\theta \mid \theta \in \mathbb{R}\}$

x-intercepts: $0, \pi, 2\pi, \ldots$

$x = \pi n,\ n \in I$

Period: 2π

horizontal length of one cycle

Center line equation: $y = 0$

Both of these graphs are periodic. (repeat, over & over)
Both of these graphs are sinusoidal. (wave-like shape)

http://www.malinc.se/math/trigonometry/unitcircleen.php

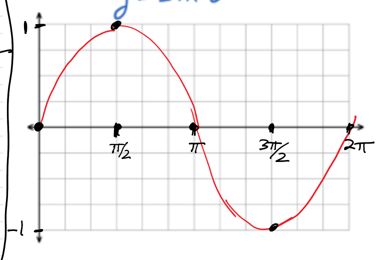
http://phet.colorado.edu/sims/html/trig-tour/latest/trig-tour_en.html

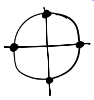
"Basic Functions" - KNOW the 5 key points for each graph!

$\sin\theta$

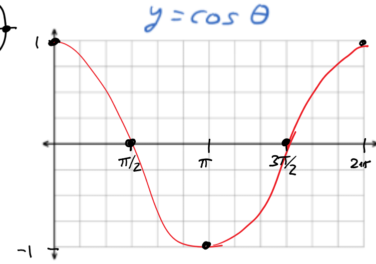
x	y
0	0
$\pi/2$	1
π	0
$3\pi/2$	−1
2π	0

$y = \sin\theta$





$y = \cos\theta$



$\cos\theta$

x	y
0	1
$\frac{\pi}{2}$	0
π	−1
$\frac{3\pi}{2}$	0
2π	1

Be sure to label the x- and y-axes

(Note - these graphs show only one period of the sine and cosine graphs. This pattern repeats over and over.)

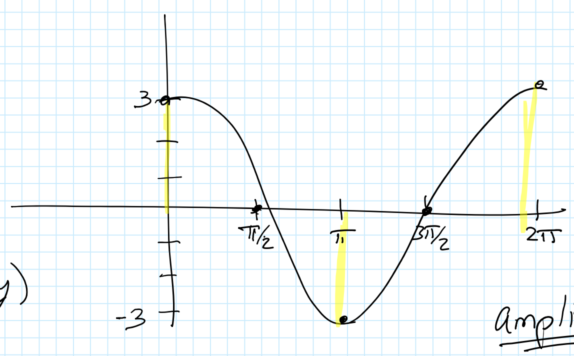
Try:

θ	$3\cos\theta$
0	3
$\pi/2$	0
π	−3
$3\pi/2$	0
2π	3

VE 3

$y = 3\cos\theta$

$(x, y) \to (x, 3y)$



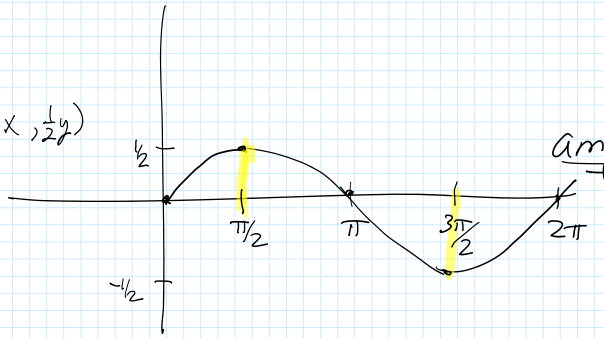
amplitude = 3

$y = \frac{1}{2}\sin\theta$

θ	$\sin\theta$
0	0
$\pi/2$	1
π	0
$3\pi/2$	-1
2π	0

θ	$\frac{1}{2}\sin\theta$
0	0
$\pi/2$	$1/2$
π	0
$3\pi/2$	$-1/2$
2π	0

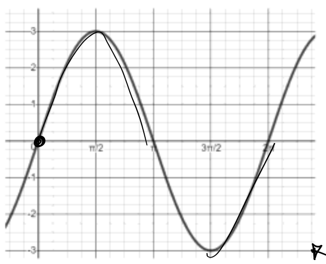
$(x, y) \to (x, \frac{1}{2}y)$



amplitude $= \frac{1}{2}$

Amplitude

is the **vertical** distance from the center line of a trigonometric graph to its maximum or minimum. The untransformed graphs of $y = \sin x$ and $y = \cos x$ have amplitude 1.



For $y = a\cos x$ or $y = a\sin x$

- vertical stretch, factor a
- amplitude $= |a|$
- if $a < 0$, graph is reflected across x-axis
- amplitude $= \frac{|\max - \min|}{2}$

Amplitude for graph shown at left? 3

Equation of the graph?

$\boxed{y = 3\sin\theta}$

What is the amplitude for each?

1. $y = \sin x \longrightarrow 1$
2. $y = 8\sin\theta \longrightarrow 8$
3. $y = -4\sin\theta \longrightarrow 4$
4. $y = \sin(2\theta) \longrightarrow 1$
5. $y = \frac{1}{5}\sin x \longrightarrow \frac{1}{5}$

What is the range for each?

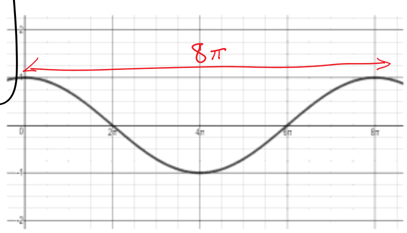
1. $y = \cos\theta$ $\quad -1 \le y \le 1$
2. $y = 5\cos x$ $\quad -5 \le y \le 5$
3. $y = -2\cos\theta$ $\quad -2 \le y \le 2$

Period

is the **horizontal** length of one complete cycle. The untransformed graphs of $y = \sin x$ and $y = \cos x$ have a period length of 2π (or 360°, if working in degree measure).

$\boxed{y = \cos 2\theta}$ period? HC $\frac{1}{2}$

period $= \pi$



For $y = \cos(bx)$ or $y = \sin(bx)$

- horizontal stretch, factor $\frac{1}{b}$
- period $= \frac{2\pi}{|b|}$ or $\frac{360^\circ}{|b|}$
- if $b < 0$, graph is reflected across y-axis

Period for graph shown at left? 8π

Equation of the graph? $y = \cos\frac{1}{4}\theta$

Since

graph's actual period $= \frac{2\pi}{|b|}$, then $|b| = \frac{2\pi}{\textit{graph's actual period}}$

If working in degrees, since

graph's actual period $= \frac{360^\circ}{|b|}$, then $|b| = \frac{360^\circ}{\textit{graph's actual period}}$

What is the horizontal stretch factor for each?

1. $y = \sin(4x)$ — HC by $\frac{1}{4}$
2. $y = 8\sin(2\theta)$ — HC by $\frac{1}{2}$
3. $y = -5\cos\left(\frac{1}{3}\theta\right)$ — HE by 3

What is the PERIOD length for each?

(usual period is 2π)

1. $y = \sin(4x)$ — (usual period) $\times \frac{1}{4} = \frac{2\pi}{4} = \pi/2$
2. $y = 8\sin(2\theta)$ — $\frac{2\pi}{2} = \pi$
3. $y = -5\cos\left(\frac{1}{3}\theta\right)$ HE by 3 — $\frac{2\pi}{1/3} = 2\pi \cdot \frac{3}{1} = 6\pi$

Coming up - Thursday, Oct 13

- **Chapter 4 Hand-in Due**
- **Chapter 4 Test - remember, there's a no-calculator portion**
- **KNOW your unit circle and how to use it)**

More Practice

- **(4.3) TB p 202: 10bc, 11acd**
- **(4.4) p 211: 2-4, 5ace, 7 all**
- **(5.1) p 233:6-8, 9ac, 10ab, 11, 14**