Class_11 May 23 Trig Identities

Thursday, May 18, 2023 9:37 PM

Tonight's Class:

- **Test 3 - (4.2-4.4, 5.1, 5.2, 5.4)**
- **6.1 Trigonometric Identities**

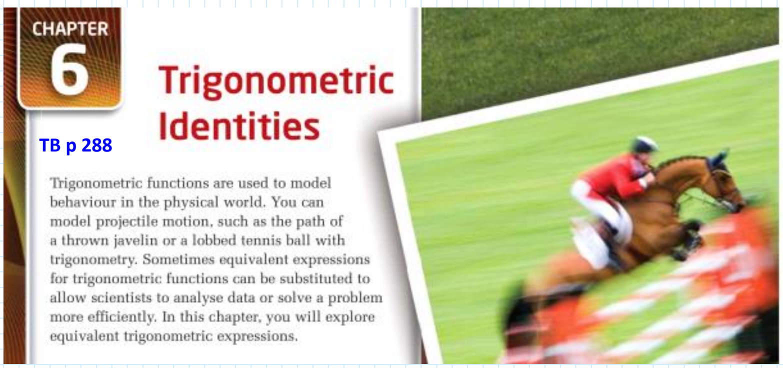
Please:

1. **Make sure your name is on your Chapter 5 Hand-in, and turn it in.**

2. **Put away your phone, calculator, and all materials except something to write with. I'll give you a formula sheet you can use on the test.**

3. **On your test, write clearly and show all necessary steps. When you are finished the non-calculator portion, raise your hand and I'll bring you the rest of the test. You can use your calculator for the second part.**

4. **While other people are still finishing, respect them by being quiet. You can leave the classroom if you wish, but be back in time for the rest of class.**



CHAPTER 6

Trigonometric Identities

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Trigonometric functions are used to model behaviour in the physical world. You can model projectile motion, such as the path of a thrown javelin or a lobbed tennis ball with trigonometry. Sometimes equivalent expressions for trigonometric functions can be substituted to allow scientists to analyse data or solve a problem more efficiently. In this chapter, you will explore equivalent trigonometric expressions.

Chapter 6: Trigonometric Identities

6.1 Trigonometric Identities

In this chapter we talk about trigonometric ***identities***. Trigonometric identities look like trigonometric equations, but there's a difference.

identities

$2x+4=2(x+2)$ always true, for any x-value

$\dfrac{(x^2-9)}{x+3}=x-3$

$\dfrac{\cancel{(x+3)}(x-3)}{\cancel{x+3}}$

$x-3 = x-3$

$\csc x=\dfrac{1}{\sin x}$

equations

$2x+3=9$ true, for a specific x-value

$2x=6$

$x=3$

$x^2+5x=-6$

$x^2+5x+6=0$

$(x+3)(x+2)=0$

$x=-3$

$x=-2$

$\sin x=0.53$

Identity - an equation that is true for **ALL** permissible values

When we are given an identity to prove, we see different expressions on the left-side and right-side of the equation. Proving the identity means we must ***change*** the expressions so that we end up with the SAME expression on both sides of the equation.

Our tools to do this are:

- algebra skills (getting common denominator, combining terms, factoring)
- basic identity substitutions

 = 

We will be verifying and proving trigonometric identities.

- ***Verifying*** an identity means we show it *seems* true. Done by:
  - *substituting* in a specific value and confirming that, for that value, the left and right sides of the identity are equal
  - *graphing* the left and right sides of the identity separately, and confirming that the graphs are exactly the same in that window
- ***Proving*** an identity means using algebra and/or Basic Identities to change the form of one or both sides of the identity, until the two sides are exactly the same.

Example

Verify the given identity, for the value $x = \frac{\pi}{5}$ $\qquad \sec x = \frac{\tan x}{\sin x}$

(by substitution)

$\sec\left(\frac{\pi}{5}\right)$	$\frac{\tan(\pi/5)}{\sin(\pi/5)}$
$\frac{1}{\cos(\pi/5)}$	$1.2360 \ldots$
$1.2360 \ldots$	

Verified

Example

Verify graphically: $\qquad \frac{\sin^3 x}{\sin x} = \sin^2 x$

graph left-hand side

graph right-hand side

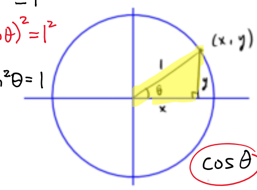
We will use these a lot in this chapter!

Basic Identities

$x^2+y^2=1^2$

$(\cos\theta)^2+(\sin\theta)^2=1^2$

$\cos^2\theta+\sin^2\theta=1$



$\tan\theta = \frac{y}{x} = \frac{\sin\theta}{\cos\theta}$

$\cos\theta = \frac{x}{1} = x$

$\sin\theta = \frac{y}{1} = y$

Pythagorean Identities

$\sin^2\theta+\cos^2\theta=1$

$\sin^2\theta = 1-\cos^2\theta$

$\cos^2\theta = 1-\sin^2\theta$

$1+\tan^2\theta=\sec^2\theta$

$1+\cot^2\theta=\csc^2\theta$

Reciprocal Identities

$\csc\theta=\frac{1}{\sin\theta}$ $\quad\sec\theta=\frac{1}{\cos\theta}$ $\quad\cot\theta=\frac{1}{\tan\theta}$

Quotient Identities $\quad\tan\theta=\frac{\sin\theta}{\cos\theta}$ $\quad\cot\theta=\frac{\cos\theta}{\sin\theta}$

(division)

Addition Identities

$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$ $\quad\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$ $\quad\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

Double Angle Identities

$\sin(2\theta)=2\sin\theta\cos\theta$

$\cos(2\theta)=\cos^2\theta-\sin^2\theta$

$\cos(2\theta)=2\cos^2\theta-1$

$\cos(2\theta)=1-2\sin^2\theta$

$\tan(2\theta)=\frac{2\tan\theta}{1-\tan^2\theta}$

Practice Using Basic Identities

Match the expressions on the left with those on the right-hand column. Put the letter of the expression that matches in the blank provided. Each gets used exactly once.

G	1. $\frac{\sin B}{\cos B} = \tan B$ Quotient	A.	1
H	2. $\frac{1}{\cos B} = \sec B$ Reciprocal	B.	$\sin^2 B$
C	3. $\csc^2 B = \cot^2 B + 1$ Pythagorean	~~C.~~	$\cot^2 B + 1$
A	4. $\sin^2 B + \cos^2 B = 1$ Pythagorean	D.	$\cos^2 B$
J	5. $\cot B \sin B$ Used a quotient identity + algebra	~~E.~~	$1 + \tan^2 B$
F	6. $\frac{\cos B}{\sin B} = \cot B$ Quotient	~~F.~~	$\cot B$
B	7. $1 - \cos^2 B = \sin^2 B$ (re-arranged) Pythag.	~~G.~~	$\tan B$
K	8. $\frac{\cos^2 B}{1 + \sin B}$	~~H.~~	$\sec B$
E	9. $\sec^2 B = 1 + \tan^2 B$ Pythag.	~~I.~~	$\csc B$
I	10. $\frac{1}{\sin B} = \csc B$ Reciprocal	J.	$\cos B$
L	11. $\frac{\cos B}{\cot B} = \sin B$	K.	$1 - \sin B$
D	12. $1 - \sin^2 B = \cos^2 B$ Pythag (re-arranged)	L.	$\sin B$

$$\frac{\cos B}{\cancel{\sin B}} \cdot \frac{\cancel{\sin B}}{1}$$

Two different trig functions in it. - try to substitute, so that ends up with only one trig function.

$$\frac{\cos^2 B}{1 + \sin B}$$

$$= \frac{1 - \sin^2 B}{1 + \sin B}$$

$$= \frac{\cancel{(1 + \sin B)}(1 - \sin B)}{\cancel{1 + \sin B}}$$

$$= 1 - \sin B$$

Now, we use ALGEBRA to help us.

$$x^2 - 9$$
$$= (x+3)(x-3)$$

$$1 - y^2$$
$$= (1+y)(1-y)$$

$$\frac{\cos B}{\cot B} = \frac{\cos B}{\frac{\cos B}{\sin B}}$$

$$= \cos B \div \frac{\cos B}{\sin B}$$

$$= \frac{\cos B}{1} \cdot \frac{\sin B}{\cos B} = \frac{\cancel{\cos B}\sin B}{\cancel{\cos B}} = \sin B$$

Try doing the rest of this page!

More Practice

Simplify each expression below. Look for substitutions you can make, using basic identities. Your final answer should contain no more than one trigonometric function.

$\sin^2\theta + \cos^2\theta = 1$

1. $\frac{\cos^2\theta}{\sin^2\theta} = \left(\frac{\cos\theta}{\sin\theta}\right)^2$

$= (\cot\theta)^2$

$= \cot^2\theta$

2. $\tan\theta\sec\theta\cos\theta$

$= \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\cancel{\cos\theta}} \cdot \frac{\cancel{\cos\theta}}{1}$

$= \frac{\sin\theta}{\cos\theta} = \boxed{\tan\theta}$

3. $1 - \cos^2\theta \; = \; \sin^2\theta$

or $(\sin^2\theta + \cancel{\cos^2\theta}) - \cancel{\cos^2\theta}$

$= \sin^2\theta$

4. $\cos^2\theta - 1$

5. $1 + \tan^2\theta$

6. $\underbrace{\sin^2\theta + \cos^2\theta}_{} + 1$

$= 1 + 1$

$= 2$

7. $\csc^2\theta - \cot^2\theta$

8. $\sin^2\theta + \cos^2\theta + \tan^2\theta$

9. $\frac{\sin^2\theta + \sin\theta}{\cos\theta + \cos\theta\sin\theta}$

10. $\frac{\sqrt{1-\sin^2 x}}{\sqrt{1+\tan^2 x}}$

11. $1 - \sec^2 x$

12. $\sec^2 x - 1$