Class_11 Oct 13 More Graphs and Applications

Tuesday, October 11, 2022 5:40 PM

Tonight's Class:
- **Warm-up**
- **Chapter 4 Test**
- **5.2 Sinusoidal Graphs**

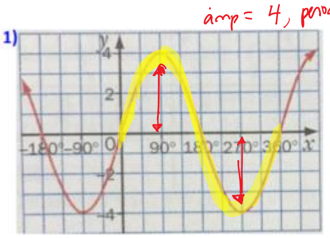
Please:
1. **Make sure your name is on the Chapter 4 Hand-in worksheet and hand it in.**
2. **Put away your calculator and all other materials.**
3. **On your test, write clearly and show all necessary steps. When you are finished the non-calculator portion, raise your hand and I'll bring you the rest of the test. You can use your calculator for the second part.**
4. **While other people are still finishing, respect them by being quiet. You can leave the classroom if you wish, but be back in time for the rest of class.**

Individual WB - warm-up

For each graph, state its
- **Amplitude**
- **Period**
- **Equation**

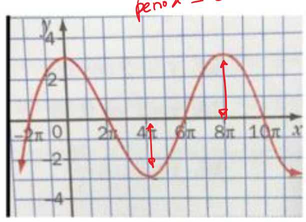
amp = 4, period = 360°

1)

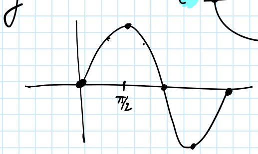


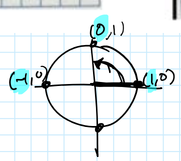
amp = 3
period = 8π

2)

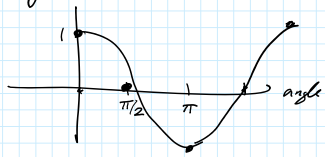


$y = \sin\theta$





$y = \cos\theta$



When sketching a period of a trigonometric function graph, we
- multiply the period length by ¼, to determine the spacing between key points
- plot key points: maximum, minimum, and center-line
- connect key points smoothly, getting a sinusoidal shape

Base graphs
- amplitude = 1
- period = 2π

Try

1a) $y = -3\sin(5x)$ HC $\frac{1}{5}$

amplitude: $|-3| = 3$

period: $2\pi \times \frac{1}{5} = \frac{2\pi}{5}$

key point spacing: $\frac{2\pi}{5} \times \frac{1}{4} = \frac{2\pi}{20} = \frac{\pi}{10}$

b) $y = -\frac{1}{4}\sin\left(\frac{1}{3}x\right)$ HE 3

amplitude:

period: $\quad \frac{6\pi}{}$

key point spacing: $\quad \frac{6\pi}{} = \frac{3\pi}{}$

$\sin(b\theta)$

$2\pi \times \frac{}{5} = \frac{}{5}$ $\quad \frac{}{5} \times \frac{}{4} = \frac{}{20}$ $\quad \frac{}{10}$

b) $y = -\frac{1}{4}\sin\left(\frac{1}{3}x\right)$ HE 3

amplitude: $\left|-\frac{1}{4}\right| = \frac{1}{4}$

period: $2\pi \times 3 = 6\pi$

key point spacing: $\frac{6\pi}{1} \cdot \frac{1}{4} = \frac{6\pi}{4} = \frac{3\pi}{2}$

2. Write the equation of a function with these characteristics.

a) sine function; amp = 3, period = π

$y = 3\sin(2\theta)$ $\quad b = \frac{2\pi}{\pi}$ $\quad b = 2$

b) cosine function, amp =2.4, period = 10π

$y = 2.4\cos\left(\frac{1}{5}\theta\right)$

$b = \frac{2\pi}{\text{actual period}} = \frac{2\pi}{10\pi} = \frac{2}{10}$ or $\frac{1}{5}$

$y = a\sin(b\theta)$

b $\quad$ actual period $= \frac{2\pi}{b} \cdot b$

$b\,(\text{period}) = \frac{2\pi}{\text{period}}$

$$\boxed{b = \frac{2\pi}{\text{actual period}}}$$

$y = 4\sin\left(\frac{1}{3}\theta\right)$

period $= \frac{2\pi}{\frac{1}{3}} = 2\pi \cdot \frac{3}{1} = 10\pi$

2a) amp = 3

period of π

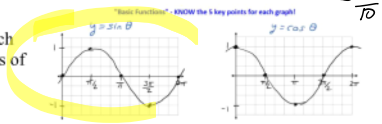
"b"

$y = 3\sin(2\theta)$

$b = \frac{2\pi}{\pi}$

3. For each equation below, accurately sketch one period of its graph. Give the coordinates of 5 key points.

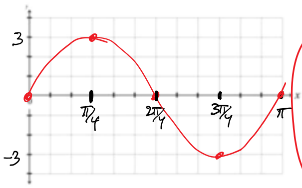
"Basic Functions" - KNOW the 5 key points for each graph!



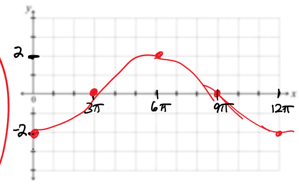
a) $y = 3\sin(2x)$ HC ½

x	y
0	
π/2	
π	
3π/2	
2π	

b) $y = -2\cos\left(\frac{1}{6}x\right)$



x	y	
0	0	CL
π/4	3	M
2π/4	0	CL
3π/4	-3	m
4π/4 = π	0	CL



x	y	
0	-2	m
3π	0	CL
6π	+2	M
9π	0	CL
12π	-2	m

amp = 3

period $= \frac{2\pi}{2} = \pi$

Spacing $= \frac{1}{4} \times \pi = \pi/4$

amp = +2

period $= \frac{2\pi}{1/6} = 2\pi \times \frac{6}{1} = 12\pi$

spacing $= \frac{1}{4} \times \frac{12\pi}{1} = \frac{12\pi}{4} = 3\pi$

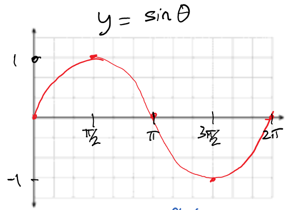
(5.1) p 233:6-8, 9ac, 10ab, 11, 14

Try these:

Sinusoidal Graphs

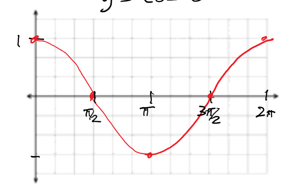
Sketch one cycle of each function below. Label the x-axis and y-axis, and fill in the table with coordinates of 5 key points.

$y = \sin\theta$



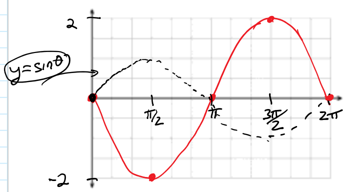
x	y
0	0
π/2	1
π	0
3π/2	−1
2π	0

$y = \cos\theta$



x	y
0	1
π/2	0
π	−1
3π/2	0
2π	1

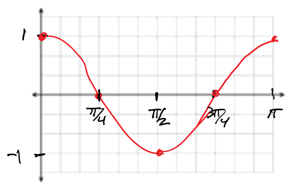
$y = -2\sin\theta$ reflects across x-axis, VE by 2



x	~~y~~ -2y
0	0
π/2	-2
π	0
3π/2	+2
2π	0

$(x, y) \rightarrow (x, -2y)$

$y = \cos(2x)$ HC by ½ each x-value gets multiplied by ½



½x	y
0	1
π/4	0
π/2	−1
3π/4	0
π	1

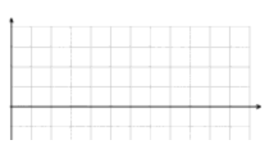
$(x, y) \rightarrow \left(\frac{1}{2}x, y\right)$

$\frac{\pi}{2} \times \frac{1}{2} = \frac{\pi}{4}$

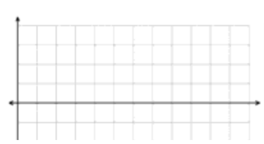
$\frac{\pi}{1} \times \frac{1}{2} = \frac{\pi}{2}$

$\frac{3\pi}{2} \times \frac{1}{2} = \frac{3\pi}{4}$

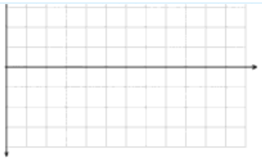
$\frac{2\pi}{1} \times \frac{1}{2} = \frac{2\pi}{2} = \pi$

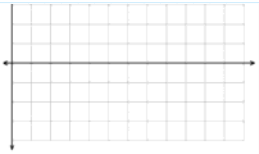


x	y



x	y





(5.1) p 233: 6-8, 9ac, 10ab, 11, 14

Textbook, page 238

5.2

Transformations of Sinusoidal Functions

Focus on…

- graphing and transforming sinusoidal functions
- identifying the domain, range, phase shift, period, amplitude, and vertical displacement of sinusoidal functions
- developing equations of sinusoidal functions, expressed in radian and degree measure, from graphs and descriptions
- solving problems graphically that can be modelled using sinusoidal functions
- recognizing that more than one equation can be used to represent the graph of a sinusoidal function

The motion of a body attached to a suspended spring, the motion of the plucked string of a musical instrument, and the pendulum of a clock produce oscillatory motion that you can model with sinusoidal functions. To use the functions $y = \sin x$ and $y = \cos x$ in applied situations, such as these and the ones in the images shown, you need to be able to transform the functions.

Electric power and the light waves it generates are sinusoidal waveforms.

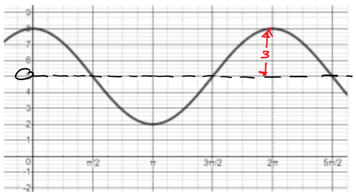
The pistons and connecting rods of a steam train drive the wheels with a motion that is sinusoidal.

Ocean waves created by the winds may be modelled by sinusoidal curves.

5.2 More Transformations of Sinusoidal Functions

Vertical Displacement

is the amount of vertical translation (up/down) a sinusoidal graph moves

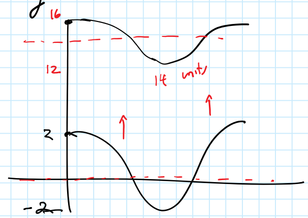


For $y = a\cos x + d$ or $y = a\sin x + d$

- vertical displacement, d units
- center line is located at $y = d$
- when we have no equation, we can figure out the vertical displacement from the graph:

$$vertical\ disp = \frac{\max + \min}{2}$$

$y = 2\cos\theta + 14$



Vertical displacement for this graph? $\frac{8+2}{2} = \frac{10}{2} = 5$

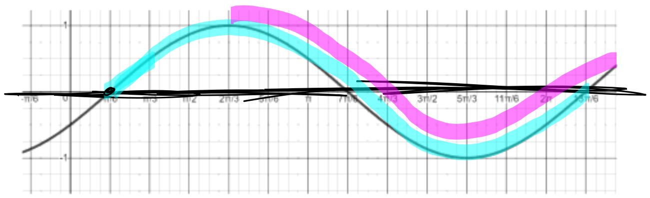
From graph, amplitude = 3

Equation of this graph?

$y = 3\cos(x) + 5$

Phase Shift

is the amount of horizontal translation (left/right) a sinusoidal graph moves



For $y = \cos(x - c)$ or $y = \sin(x - c)$

- phase shift, c units
- when we have no equation, we use the graph to find the phase shift. Choose a period of either sine or cosine that begins near the y-axis. Identify how much it has moved left/right compared to the basic (untransformed) graph.

For the graph above, find its equation in the form: $y = \sin(x - c)$ $y = \sin(x - \pi/6)$

For the graph above, find its equation in the form: $y = \cos(x - c)$ $y = \cos(x - 2\pi/3)$

Summary

affects the period
Hor. stretch by $\frac{1}{b}$
period $= \frac{2\pi}{b}$ or $\frac{360^\circ}{b}$

$$y = a\sin b(x-c)+d$$

Vertical displacement
(up/down, vertical translation)

Vertical stretch
amplitude $= |a|$

phase shift
(horizontal translation L/R)

Try
Determine the amplitude, period, phase shift and vertical displacement.

a) $y = 5\cos\left(3\left(x+\frac{\pi}{4}\right)\right)+2$

amp = 5
period = $\frac{2\pi}{b} = \frac{2\pi}{3}$
p.shift = $\pi/4$ left
v disp = 2 up

HC by $\frac{1}{3}$
$\left(2\pi \times \frac{1}{3} = \frac{2\pi}{3}\right)$

b) $y = \frac{1}{2}\cos(x+80^\circ)+5$

amp = $\frac{1}{2}$
period = 360°
p shift = 80° left
v disp = 5 up

c) $y = -2\sin\left(5x-\frac{\pi}{3}\right)+4$

$y = -2\sin\left[5\left(x-\frac{\pi}{15}\right)\right]+4$

FACTOR

amp = $|-2| = 2$
period = $\frac{2\pi}{5}$
p shift = $\pi/15$ right
v disp = 4 up

In-class Worksheet: Graphing Sinusoidal Functions (front page)

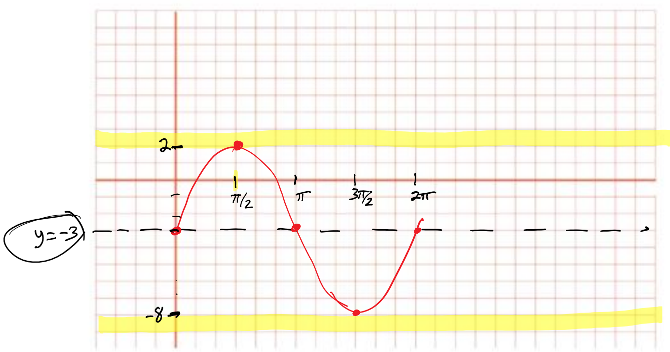
Graphing Sinusoidal Functions – two methods

$y = 5\sin\theta - 3$

Key Point Method

basic shape	vertical displacement	amplitude
(sketch of sine curve)	3 down	5
equation of center line	**maximum**	**minimum**
$y = -3$	$-3 + 5 = 2$	$-3 - 5 = -8$

1) plot center line
2) label Max and min on y-axis
3) label x-axis
4) make a table of key points
5) plot & connect the key points



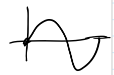
$y = \sin\theta$ Mapping Method

x	y
0	0
$\pi/2$	1
π	0
$3\pi/2$	-1
2π	0

x	5y-3
0	-3
$\pi/2$	2
π	-3
$3\pi/2$	-8
2π	-3

$(x, y) \rightarrow (x, 5y-3)$

1) create mapping
2) create BASE table and transformed table
3) label x and y-axis
4) plot & connect the key points



x	y	
0	-3	CL
$\pi/2$	2	M
π	-3	CL
$3\pi/2$	-8	m
2π	-3	CL

More Practice

- **In-class worksheet: look at the back of the worksheet**
- **Worksheet: Transformed to Try - try the two graphs on the first page of this one**
- **(5.2) TB p 250: 1-7, 10, 14, 15ac, 16ac**