Class_12 Feb 14 - Transforming Quadratic Functions

Wednesday, February 8, 2023 11:43 AM

Tonight's Class:

- **Questions?**
- **Chapter 3 Test**
- **Working through sections 4.1, 4.3, 4.4**
  - **Properties of a Quadratic Function**
  - **Transforming Graphs of Quadratic Functions**
- **Work on practice questions from worktext**

p200, #18b

$$\frac{1}{2}x^2 + \frac{9}{8}x + \frac{1}{4}$$

$$= \frac{1}{8}\cdot 8\left(\frac{1}{2}x^2 + \frac{9}{8}x + \frac{1}{4}\right)$$

$$= \frac{1}{8}\left(4x^2 + 9x + 2\right)$$

$$= \frac{1}{8}\left(4x^2 + 1x + 8x + 2\right)$$

$$= \frac{1}{8}\left(x(4x+1) + 2(4x+1)\right)$$

$$= \frac{1}{8}(4x+1)(x+2)$$

$\frac{1}{8}\cdot 8 = \frac{8}{8} = 1$

$AC = 4\cdot 2 = 8$, mult $= 8$, add $= 9$: 1, 8

12 a) ii)

$$9(x-3)^2 - 4(2y+1)^2$$

$\boxed{}^2 - \triangle^2$

$$= 9A^2 - 4B^2$$

$A = x-3$

$B = 2y+1$

$$= (3A - 2B)(3A + 2B)$$

$$= (3(x-3) - 2(2y+1))(3(x-3) + 2(2y+1))$$

$$= (3x - 9 - 4y - 2)(3x - 9 + 4y + 2)$$

$$= (3x - 4y - 11)(3x + 4y - 7)$$

p212 #13

$$6\left[\frac{x^2}{2} + \frac{7x}{6} = 1\right]$$

$$\frac{6x^2}{2} + \frac{42x}{6} = 6$$

$$3x^2 + 7x = 6$$

$$3x^2 + 7x - 6 = 0$$

$$3x^2 + 9x - 2x - 6 = 0$$

$$3x(x+3) - 2(x+3) = 0$$

$$(3x-2)(x+3) = 0$$

$AC = -18$, add $+7$: 9, −2

Solve

$3x(x+3) - 2(x+3) = 0$

$(3x-2)(x+3) = 0$

$x = -3$

$3x - 2 = 0$

$+2 \quad +2$

$\frac{3x}{3} = \frac{2}{3}$

$x = 2/3$

4.1 Properties of a Quadratic Function

Focus: determine the characteristics of a quadratic function and sketch its graph

Graph a Quadratic Equation and Find its Characteristics

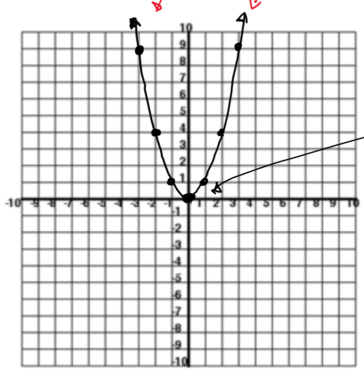
Simplest Quadratic Function

$y = x^2$

x	y
−3	9
−2	4
−1	1
0	0
1	1
2	4
3	9

table of key points

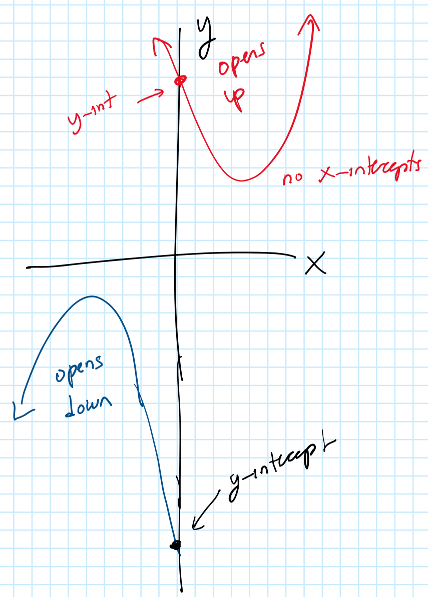
include arrows on the ends; because the graph continues



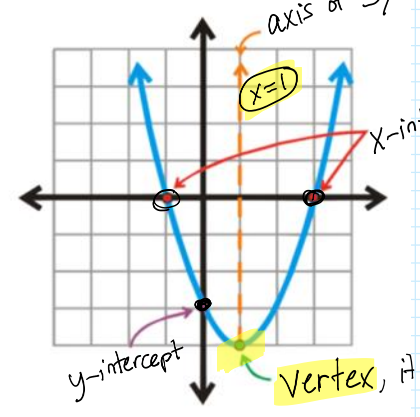
x-intercept
y-intercept

domain: $x \in \mathbb{R}$
(all the x-values)

range: $y \ge 0$
(all the y-values)



Key Characteristics for $y = ax^2 + bx + c$, $a \neq 0$



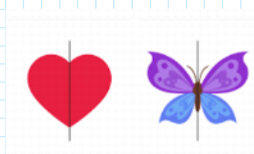
axis of symmetry (vertical line, goes through the vertex, and it "chops the parabola in half")

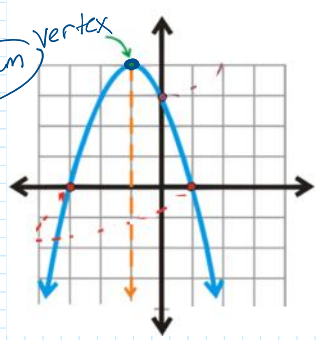
$\boxed{x = \text{number}}$

x-intercepts

y-intercept

Vertex, it's always either a minimum or a maximum (lowest/highest y-value)





maximum

vertex

Hand-out - front side only

Characteristics of Quadratic Functions

$y = ax^2 + bx + c$

Graph 1

Opens up/down? up

Maximum or Minimum? If asked for the value of the minimum, give the y-value $= -8$

Vertex give as an ordered pair $(-1, -8)$ x y

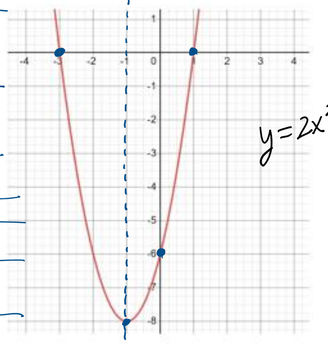
Axis of symmetry equation $x = -1$

Domain $x \in \mathbb{R}$

Range $y \geq -8$

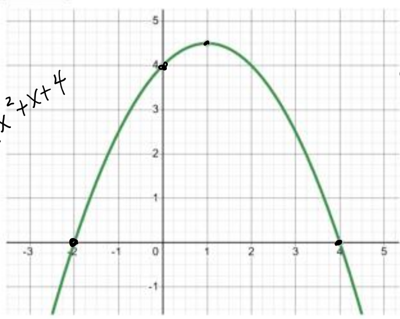
x-intercepts give as ordered pairs $(-3,0)$ $(1,0)$

y-intercept $(0,-6)$



$y = 2x^2 + 4x - 6$

Graph 2



$y = -\frac{1}{2}x^2 + x + 4$

Characteristics

Opens up/down? down

Maximum or Minimum? What is its value? 4.5

Vertex $(1, 4.5)$

Axis of symmetry $x = 1$

Domain $x \in \mathbb{R}$

Range $y \leq 4.5$

x-intercepts $(-2,0)$ and $(4,0)$

y-intercept $(0,4)$

First graph's equation: $y = 2x^2 + 4x - 6$	Second graph's equation: $y = -\frac{1}{2}x^2 + x + 4$

Notice:

- if $a > 0$ graph will open up, has a minimum
- if $a < 0$ graph will open down, has a maximum

Soon we'll learn how to change the form of the function equation **(Section 4.5)**

$y = ax^2 + bx + c$

General Form

$y = a(x - h)^2 + k$

Vertex Form

Vertex form is better for discovering key characteristics of the graph.

#4 Where is the y-intercept? (look at c-value, the constant)

#5 max/min, look at a-value

Try page 277:4-6

#5c answer should say minimum

#6 vertical intercept = y-intercept.
Give as ordered pair.

4.2 - Omitting this section (requires graphing calculators)

4.3 Transforming a Quadratic Function's Graph

Focus: exploring three transformations of the graph of a quadratic function

What are transformations?

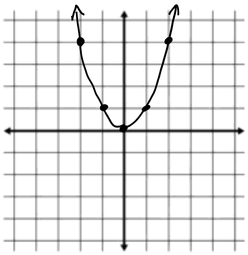
changes made to the equation, that result in specific changes to its graph

1) translations (moving L/R, U/D)
2) vertical stretches
3) reflection

Translations (moving the shape)

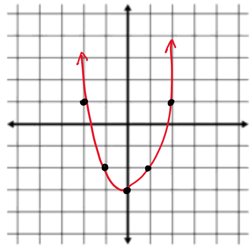
Hand-out with grids

$\boxed{y = x^2}$ (base graph) vertex at $(0,0)$



x	y
−2	4
−1	1
0	0
1	1
2	4

$y = x^2 - 3$



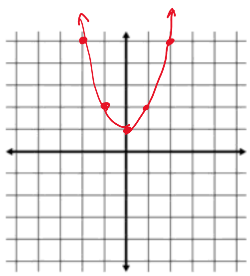
x	y	
−2	1	$(-2)^2 - 3$
−1	−2	$(-1)^2 - 3$
0	−3	$(0)^2 - 3$
1	−2	$(1)^2 - 3$
2	1	$(2)^2 - 3$

What happened to the base graph?

$y = x^2$ base graph

$y = x^2 + k$ moves up/down "k" units

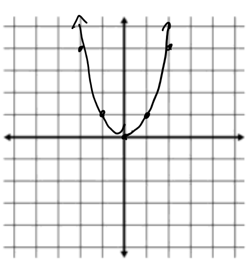
$y = x^2 + 1$



x	y	
−2	5	$(-2)^2 + 1$
−1	2	$(-1)^2 + 1$
0	1	$(0)^2 + 1$
1	2	$(1)^2 + 1$
2	5	$(2)^2 + 1$

Function	Vertex	Transformation
$y = x^2$	$(0,0)$	
$y = x^2 - 3$	$(0,-3)$	down 3
$y = x^2 + 1$	$(0,1)$	up 1

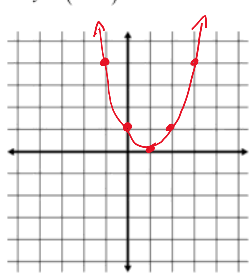
$y = x^2$ (base graph)



x	y
−2	4
−1	1
0	0
1	1
2	4

$y = (x-1)^2$

hold y-values steady



x	y
−1	4
0	1
1	0
2	1
3	4

right 1 unit

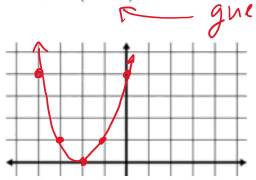
$y = x^2$ base

$y = (x+h)^2$ left/right "h" units

$y = (x+2)^2$

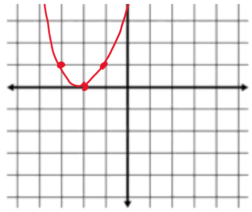
guess, what will happen

left 2 units



x	y
−4	4
−3	1
−2	0

$y = (x + h)^2$ left/right "h" units



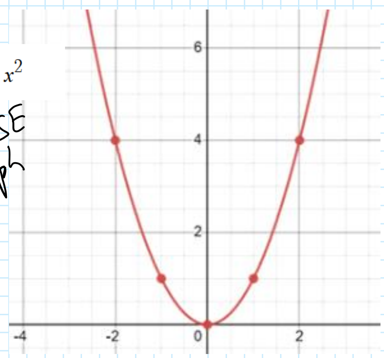
x	y
−4	4
−3	1
−2	0
−1	1
0	4

Function	Vertex	Transformation
$y = x^2$	(0, 0)	
$y = (x-1)^2$	(1, 0)	right 1
$y = (x+2)^2$	(−2, 0)	left 2

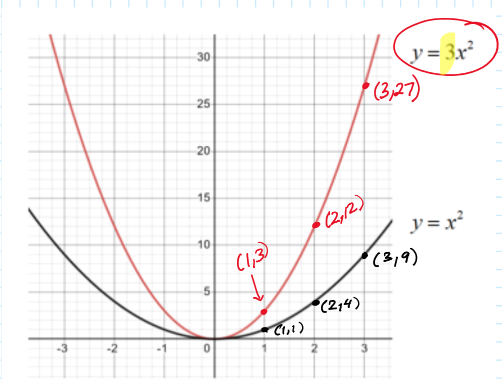
Stretches (vertical expansions and compressions)

$y = x^2$

BASE graph



When there's a number in front of the x^2 term, the graph gets vertically expanded or compressed. The graph's shape is changed (not its position)

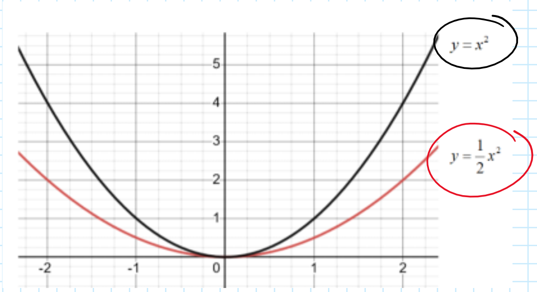


y-values multiplied by 3

(1,1) → (1,3)

(2,4) → (2,12)

(3,9) → (3,27)



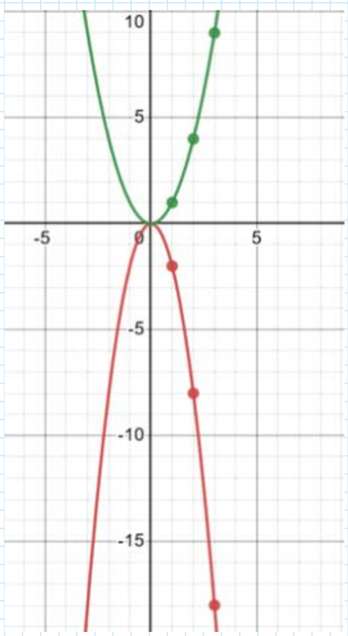
New points are ?

$(0,0) \rightarrow (0,0)$
$(1,1) \rightarrow (1, \frac{1}{2})$
$(2,4) \rightarrow (2,2)$

y-values get multiplied by $\frac{1}{2}$

Reflections

If "a" is negative, graph reflects and opens downward.



For next class

- **Work on these worktext questions for 4.1, 4.3**
  - **4.1, #4-6, 8**
  - **4.3, #1, 2ab, 3, 4**