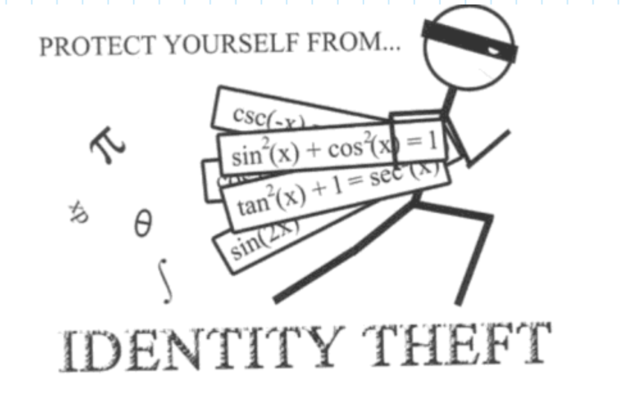
Class_12 May 24 Trig Identities

Plan For Today:

1. Question about anything from last class? 6.1
   - 6.1 Review
2. Start Chapter 6: Trig Identities & Solving Equations
   - 6.1: Reciprocal, Quotient & Pythagorean Identities
     - (Simplifying & Proving)
   - **6.2: Sum, Difference & Double-Angle Identities**
   - **6.3: Proving Identities**
   - 6.4: Solve Trig Equations with Identities
3. Work on practice questions from Textbook

Page 306: #1ade, 2ac, 4ace, 5, 8ce, 10, 11, 16, 20acd
Page 314: #2, 3ac, 5, 7, 10c, 11a, 12a, 15, 18



Plan Going Forward:

1. Finish working through textbook question from 6.1-6.3 and continue working on Chapter 6 Assignment.

2. You will practice 6.3 proofs on Thursday (tomorrow) and possibly start 6.4 if time. You will finish Ch6 with 6.4 on Monday after Test 4.

- **TEST 4 ON MONDAY, MAY 29TH (ON 6.1-6.3)**
- **CHAPTER 6 ASSIGNMENT DUE TUESDAY, MAY 30TH OR WEDNESDAY, MAY 31ST**

Please let us know if you have any questions or concerns about your progress in this course. The notes from today will be posted at egolfmath.weebly.com after class.
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Your Turn   p.293

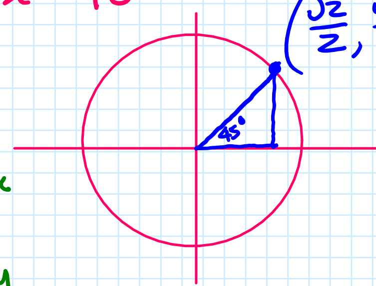
a) Determine the non-permissible va

$\cot x = \dfrac{\cos x}{\sin x}$.

b) Verify that $x = 45°$ and $x = \dfrac{\pi}{6}$ are

$x = 45°$

$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ or $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$



$\cot\theta = \frac{x}{y}$

$$\cot x = \frac{\cos x}{\sin x}$$

$\cot(45°)$	$\dfrac{\cos 45°}{\sin 45°}$
$\dfrac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}$	$\dfrac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}$
1	

$\leftarrow \cos\theta = x$

$\leftarrow \sin\theta = y$

$1 = 1$ ✓ verified

Try doing the rest of this page!

$\sin^2\theta + \cos^2\theta = 1$

More Practice

Simplify each expression below. Look for substitutions you can make, using basic identities. Your final answer should contain no more than one trigonometric function.

1. $\frac{\cos^2\theta}{\sin^2\theta} = \left(\frac{\cos\theta}{\sin\theta}\right)^2 = (\cot\theta)^2 = \cot^2\theta$

2. $\tan\theta\sec\theta\cos\theta = \frac{\sin\theta}{\cos\theta}\cdot\frac{1}{\cancel{\cos\theta}}\cdot\frac{\cancel{\cos\theta}}{1} = \frac{\sin\theta}{\cos\theta} = \boxed{\tan\theta}$

3. $1-\cos^2\theta = \sin^2\theta$

or $(\sin^2\theta + \cancel{\cos^2\theta}) - \cancel{\cos^2\theta} = \sin^2\theta$

4. $\cos^2\theta - 1 = -\sin^2\theta$

$\sin^2\theta + \cos^2\theta = 1$
$\quad -1 \quad -1$

$\sin^2\theta + \underline{\cos^2\theta - 1} = 0$
$-\sin^2\theta \qquad -\sin^2\theta$

$\cos^2\theta - 1 = \boxed{-\sin^2\theta}$

5. $1+\tan^2\theta = \sec^2\theta$

6. $\underbrace{\sin^2\theta + \cos^2\theta}_{} + 1 = 1 + 1 = 2$

7. $\csc^2\theta - \cot^2\theta = \boxed{1}$

$1 + \cot^2\theta = \csc^2\theta$
$-\cot^2\theta \qquad -\cot^2\theta$

$\boxed{1} = \csc^2\theta - \cot^2\theta$

8. $\underbrace{\sin^2\theta + \cos^2\theta}_{=1} + \tan^2\theta$

2 steps
① $\underbrace{1 + \tan^2\theta}$
② $\boxed{\sec^2\theta}$

$\sec 2x \neq \sec^2 x$

9. $\frac{\sin^2\theta + \sin\theta}{\cos\theta + \cos\theta\sin\theta}$ ← common factor (GCF) $= \sin\theta$
GCF $= \cos\theta$

$\frac{\sin\theta(\cancel{\sin\theta + 1})}{\cos\theta(\cancel{1+\sin\theta})} = \frac{\sin\theta}{\cos\theta} = \boxed{\tan\theta}$

10. $\frac{\sqrt{1-\sin^2 x}}{\sqrt{1+\tan^2 x}}$ ← single term 1st

$\sqrt{16-4} \neq \cancel{4-2}$
$= \sqrt{12}$
$= 2\sqrt{3}$

$\sqrt{x^2} = x$

$\sin^2 x + \cos^2 x = 1$
$\cos^2 x = 1 - \sin^2 x$

$\frac{\sqrt{\cos^2 x}}{\sqrt{\sec^2 x}} = \frac{\cos x}{\sec x} = \frac{\cos x}{\frac{1}{\cos x}}$ reciprocal

$\frac{1}{\cos x}$

$\cos x \times \frac{\cos x}{1} = \boxed{\cos^2 x}$

11. $1-\sec^2 x = \boxed{-\tan^2 x}$

$1+\tan^2\theta = \sec^2\theta$
$1 - \sec^2\theta = -\tan^2\theta$

$1 + \tan^2\theta = \sec^2\theta$
$\qquad\qquad -1$
$\tan^2\theta = \sec^2\theta - 1$

12. $\sec^2 x - 1 = \boxed{\tan^2 x}$

6.0 Algebra Skills Used in Chapter 6

Multiplying Trigonometric Expressions

1. $\sin x(2\sin x-1)$

FOIL

2. $(\cos x+2)(\cos x-7)$

3. $(\cos x-3)^2$ $= (\cos x-3)(\cos x-3)$

short-cut : $(\cos x)^2 - 2(3)(\cos x) + (-3)^2$

Factoring Trigonometric Expressions

Greatest Common Factor

1. $\sin^2 x-3\sin x$ $\quad \sin x(\qquad)$

2. $5\tan^2 x+15\tan x$ $\quad 5\tan x(\qquad)$

Difference of Perfect Squares

1. $\sin^2 x-1$ $\quad (\sin x+1)(\sin x-1)$

2. $1-\tan^2 x$ $\quad (1+\tan x)(1-\tan x)$

Trinomials

1. $2\cos^2 x + \cos x - 1$

A B C

$AC = -2$

$+2, -1 = 1 \ (B)$

$2\cos^2 x + 2\cos x - \cos x - 1$

$2\cos x(\cos x + 1) - (\cos x + 1)$

$\boxed{(2\cos x - 1)(\cos x + 1)}$

2. $3\sin^2 x + 2\sin x - 1$

$\boxed{(3\sin x - 1)(\sin x + 1)}$

Adding/Subtracting Trigonometric Terms

We can only add ***like terms***

- Terms must contain the same angle
- Terms must use the same trigonometric function

Which of these terms can be combined?

$4\sin x + 3\cos x$ ✗ $2\sin x + 5\sin 2x$ ✗

$3\sin x + 4\sin x = \boxed{7\sin x}$ ✓ $2\sin^2 x + 4\sin x$ ✗

Errors to Avoid

Omitting the angle

$\cos + 4\cos$ These terms contain no angle - they don't mean anything!

Incorrect cancelling

You can never "cancel" the angle, or any part of the angle, in a trigonometric expression.

$\frac{\cos \cancel{x}}{\cancel{x}}$ wrong $\frac{\tan \cancel{2}x}{\cancel{2}}$ wrong

$\frac{\cancel{\cos x}}{\cancel{\cos x}} = 1$ ✓ $\frac{\cancel{2}\tan x}{\cancel{2}} = \tan x$ ✓

More incorrect cancelling

You can NEVER cancel just a portion of a factor.

$\frac{\cancel{\cos x}+1}{\cancel{\cos x}} =$ wrong

You **CANNOT** cancel the "$\cos x$" on the top with the "$\cos x$" on the bottom!

$\frac{\cancel{(\cos x+1)}(\cos x-1)}{\cancel{(\cos x+1)}} = \cos x - 1$ ✓

You **CAN** cancel the "$\cos x+1$" factors

To CORRECTLY simplify a rational expression, factor it completely.
If the numerator and the denominator contain the same factor, you can reduce.

For example: $\frac{\sin^2 x+8\sin x+12}{\sin^2 x+\sin x-30} = \frac{\cancel{(\sin x+6)}(\sin x+2)}{\cancel{(\sin x+6)}(\sin x-5)} = \boxed{\frac{\sin x+2}{\sin x-5}}$ Final Answer

Notice that we **cannot** reduce $\frac{\sin x+2}{\sin x-5}$ by canceling the $\sin x$'s, because $\sin x$ is NOT a factor of the numerator and the denominator. $\frac{\sin x+2}{\sin x-5} \neq \frac{2}{-5}$

Distributing when you can't:

$\cos(x+y)$

cos is not a number

This does **NOT** equal $\cos x+\cos y$!
We are not multiplying "cos" with $(x+y)$.

What this expression DOES mean is the cosine of the angle "$x+y$"

$\cos x(x+y)$
term
$x\cos x + y\cos x$ ✓

For example, consider what happens if $x=15^\circ$ and $y=28^\circ$ Degree Mode

$\cos(15^\circ+28^\circ) =$ 0.73

$\cos(15^\circ)+\cos(28^\circ) =$ 1.85

$\neq$

This shows us that $\cos(x+y)\neq\cos x+\cos y$

1-2 of ch 6 Assign.

Trig_Identities Cross...

#1 $\frac{\tan x}{\sec x} = \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}}$ flip

$= \frac{\sin x}{\cancel{\cos x}} \times \frac{\cancel{\cos x}}{1}$

$\boxed{}$

c) $\frac{1-\cot x}{\tan x-1} = \frac{1-\frac{1}{\tan x}}{\tan x-1}$

$= \frac{\frac{\tan x}{\tan x}-\frac{1}{\tan x}}{\tan x-1}$

$= \frac{\frac{\tan x-1}{\tan x}}{\tan x-1}$ flip

$\cancel{\tan x-1} \times \frac{}{} \quad 1$

$1-\frac{1}{2}$

$\frac{2}{2}-\frac{1}{2}=\frac{2-1}{2}$

$=\frac{1}{2}$

* multiply by reciprocal rule

reciprocal rule

$\tan x - 1$ flip

$$\frac{\cancel{\tan x - 1}}{\tan x} \times \frac{1}{\cancel{\tan x - 1}}$$

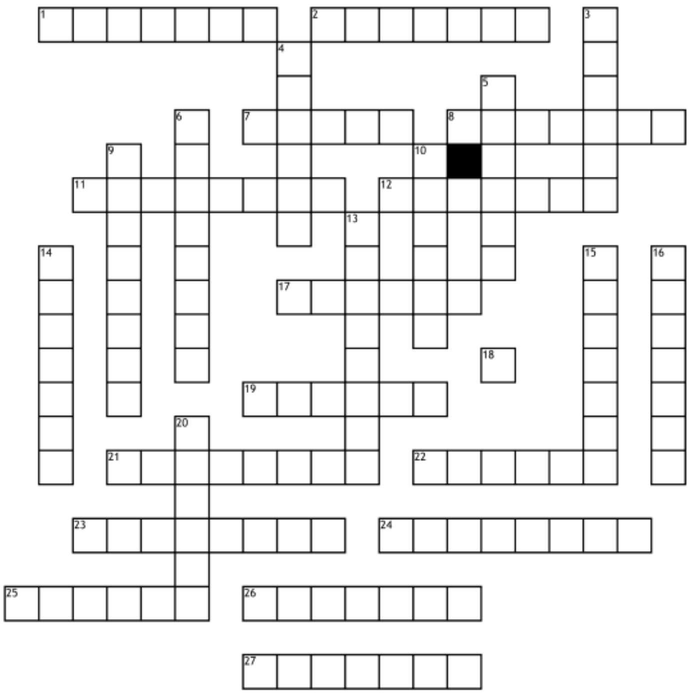
$$= \frac{1}{\tan x}$$

$$= \boxed{\cot x}$$

$= \sin x$

Name: ____ Date: ________

Trig Identities



Across

1. 1 - sin^2(x)=
2. 2sin(u)cos(u)=
7. cot^2(x) + 1=
8. cos^2(x) - sin^2(x)=
11. sin(x)=
12. cos(u)cos(x) + sin(u)sin(v)=
17. sin(pi/2 -x)=
19. sec(-x)=
21. cos(u)cos(x) - sin(u)sin(v)=
22. sin(-x)=
23. sin(u)cos(v) + cos(u)sin(v)=
24. 1 + tan^2(x)=
25. sec(pi/2 -x)=
26. sin(u)cos(v) - cos(u)sin(v)=
27. cot(x)=

Down

3. tan(-x)=
4. cos(-x)=
5. tan(pi/2 -x)=
6. cos(x)=
9. sec(x)=
10. cot(-x)=
13. csc(x)=
14. tan(x)=
15. 2tan(x)/1 - tan^2(x)=
16. 2cos^2(x) - 1=
18. cos^2 + sin^2=
20. csc(-x)=

Word Bank

1/sin(x)	1/cos(x)	cos/sin	1/csc(x)	1/sec(x)	sin/cos	1
sec^2(x)	csc^2	cos(x)	cot(x)	csc(x)	-sin(x)	-csc(x)
cos(x)	sec(x)	-tan(x)	cot(x)	cos(u+v)	cos(u-v)	sin(u+v)
sin(u-v)	sin2(x)	cos2(x)	cos2(x)	cos2(x)	tan2(x)	

6.2 Sum, Difference & Addition Identities

https://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=160

Addition Identities

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta \qquad \cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta \qquad \cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

https://slideplayer.com/slide/12573563/

Verify this identity for: $a=\frac{\pi}{4},\quad b=\frac{\pi}{2}$

$$\sin(a+b) = \sin a\cos b + \cos a\sin b$$

Left Side	Right Side
$\sin\left(\frac{\pi}{4}+\frac{\pi}{2}\right)$	$\sin\frac{\pi}{4}\cos\frac{\pi}{2} + \cos\frac{\pi}{4}\sin\frac{\pi}{2}$
$\sin\frac{3\pi}{4}$	$\left(\frac{1}{\sqrt{2}}\right)(0) + \left(\frac{1}{\sqrt{2}}\right)(1)$
$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$

Left Side = Right Side

Example 2 – *Proving a Cofunction Identity*

Prove the cofunction identity $\cos\left(\frac{\pi}{2}-u\right)=\sin u$.

Solution:

By the Subtraction Formula for Cosine we have

$$\cos\left(\frac{\pi}{2}-u\right)=\cos\frac{\pi}{2}\cos u+\sin\frac{\pi}{2}\sin u$$

$$=0\cdot\cos u+1\cdot\sin u$$

$$=\sin u$$

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Example — VERIFYING AN IDENTITY USING SUM AND DIFFERENCE IDENTITIES

- Verify that the equation is an identity.

$$\sin\left(\frac{\pi}{6}+\theta\right)+\cos\left(\frac{\pi}{3}+\theta\right)=\cos\theta$$

$$\sin\left(\frac{\pi}{6}+\theta\right)+\cos\left(\frac{\pi}{3}+\theta\right)$$

$$=\left(\sin\frac{\pi}{6}\cos\theta+\sin\theta\cos\frac{\pi}{6}\right)+\left(\cos\frac{\pi}{3}\cos\theta-\sin\frac{\pi}{3}\sin\theta\right)$$

$$=\left(\frac{1}{2}\cos\theta+\frac{\sqrt{3}}{2}\sin\theta\right)+\left(\frac{1}{2}\cos\theta-\frac{\sqrt{3}}{2}\sin\theta\right)$$

$$=\frac{1}{2}\cos\theta+\frac{1}{2}\cos\theta=\cos\theta$$

Slide 1-52

EXAMPLE 5 Using the Sum Formula for Sine

Find the exact value of

$$\sin 63^\circ\cos 27^\circ+\cos 63^\circ\sin 27^\circ$$

without using a calculator.

Solution

This expression is the right side of the sum identity for $\sin(u+v)$, where $u=63^\circ$ and $v=27^\circ$.

$$\sin 63^\circ\cos 27^\circ+\cos 63^\circ\sin 27^\circ=\sin(63^\circ+27^\circ)$$

$$=\sin(90^\circ)$$

$$=1$$

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EXERCISE 3 Using a Sum-of-Two-Angles Formula

Find the exact value of $\sin\frac{7\pi}{12}$.

SOLUTION

Since $\frac{\pi}{3}+\frac{\pi}{4}=\frac{4\pi}{12}+\frac{3\pi}{12}=\frac{7\pi}{12}$, use the formula for $\sin(A+B)$

EXAMPLE 1 Using the Difference Identity for Cosine

Find the exact value of $\cos\frac{\pi}{12}$ by using $\frac{\pi}{12}=\frac{\pi}{3}-\frac{\pi}{4}$.

SOLUTION

Since $\frac{\pi}{3}+\frac{\pi}{4}=\frac{4\pi}{12}+\frac{3\pi}{12}=\frac{7\pi}{12}$, use the formula for $\sin(A+B)$ where $A=\frac{\pi}{3}$ and $B=\frac{\pi}{4}$:

$$\begin{aligned}\sin(A+B) &= \sin A\cos B+\cos A\sin B\\ &= \left(\sin\frac{\pi}{3}\right)\left(\cos\frac{\pi}{4}\right)+\left(\cos\frac{\pi}{3}\right)\left(\sin\frac{\pi}{4}\right)\\ &= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)+\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)\\ &= \frac{\sqrt{6}}{4}+\frac{\sqrt{2}}{4}\\ &= \frac{\sqrt{6}+\sqrt{2}}{4}\end{aligned}$$

Find the exact value of $\cos\frac{\pi}{12}$ by using $\frac{\pi}{12}=\frac{\pi}{3}-\frac{\pi}{4}$.

Solution

$$\begin{aligned}\cos\frac{\pi}{12} &= \cos\left(\frac{\pi}{3}-\frac{\pi}{4}\right)=\cos\frac{\pi}{3}\cos\frac{\pi}{4}+\sin\frac{\pi}{3}\sin\frac{\pi}{4}\\ &= \frac{1}{2}\times\frac{\sqrt{2}}{2}+\frac{\sqrt{3}}{2}\times\frac{\sqrt{2}}{2}\\ &= \frac{\sqrt{2}}{4}+\frac{\sqrt{6}}{4}=\frac{\sqrt{2}+\sqrt{6}}{4}\end{aligned}$$

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6.3 Example Using Sine and Tangent Sum or Difference Formulas

(b) $\tan\frac{7\pi}{12}=\tan\left(\frac{\pi}{3}+\frac{\pi}{4}\right)=\dfrac{\tan\frac{\pi}{3}+\tan\frac{\pi}{4}}{1-\tan\frac{\pi}{3}\tan\frac{\pi}{4}}$

$$=\frac{\sqrt{3}+1}{1-\sqrt{3}\cdot 1}$$

$=-2-\sqrt{3}$ Rationalize the denominator and simplify.

(c) $\sin 40^\circ\cos 160^\circ-\cos 40^\circ\sin 160^\circ=\sin(40^\circ-160^\circ)$
$=\sin(-120^\circ)$
$=-\frac{\sqrt{3}}{2}$

 Slide 9-9

Using your trig tables and the sum/difference identities Find

$\sin 75^\circ \quad =\sin(45^\circ+30^\circ)$

$\sin 15^\circ \quad =\sin(45^\circ-30^\circ)$

$\cos 75^\circ \quad =\cos(45^\circ+30^\circ)$

$\cos 15^\circ \quad =\cos(45^\circ-30^\circ)$

Double Angle Identities:

https://slidetodoc.com/section-6-3-doubleangle-and-halfangle-identities-objectives/

Double Angle Identities

$\sin(2\theta)=2\sin\theta\cos\theta$

$\cos(2\theta)=\cos^2\theta-\sin^2\theta$

$\cos(2\theta)=2\cos^2\theta-1$

$\cos(2\theta)=1-2\sin^2\theta$

$\tan(2\theta)=\dfrac{2\tan\theta}{1-\tan^2\theta}$

Example 1– *A Triple-Angle Formula*

Write $\cos 3x$ in terms of $\cos x$.

Solution:

$\cos 3x = \cos(2x + x)$

$= \cos 2x \cos x - \sin 2x \sin x$ — Addition formula

$= (2\cos^2 x - 1)\cos x - (2\sin x \cos x)\sin x$ — Double-Angle Formulas

$= 2\cos^3 x - \cos x - 2\sin^2 x \cos x$ — Expand

$= 2\cos^3 x - \cos x - 2\cos x(1 - \cos^2 x)$ — Pythagorean identity

$= 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x$ — Expand

$= 4\cos^3 x - 3\cos x$ — Simplify

Verify the identity $\sin 3x = 3\sin x - 4\sin^3 x$.

Solution

$$\begin{aligned}
\sin 3x &= \sin(2x + x)\\
&= \sin 2x \cos x + \cos 2x \sin x\\
&= (2\sin x \cos x)\cos x + (1 - 2\sin^2 x)\sin x\\
&= 2\sin x \cos^2 x + \sin x - 2\sin^3 x\\
&= 2\sin x(1 - \sin^2 x) + \sin x - 2\sin^3 x\\
&= 2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x\\
&= 3\sin x - 4\sin^3 x
\end{aligned}$$

Evaluate $\sin 2\theta$, where $\cos\theta = -\frac{2}{5}$ with θ in Quadrant II.

Solution :

We first sketch the angle θ in standard position with terminal side in Quadrant II as in Figure 2.

Since $\cos\theta = x/r = -\frac{2}{5}$, we can label a side and the hypotenuse of the triangle in Figure 2.

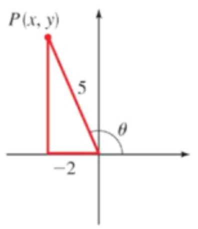

Figure 2

To find the remaining side, we use the Pythagorean Theorem.

$x^2 + y^2 = r^2$	Pythagorean Theorem
$(-2)^2 + y^2 = 5^2$	$x = -2$, $r = 5$
$y = \pm\sqrt{21}$	Solve for y^2
$y = +\sqrt{21}$	Because $y > 0$

We can now use the Double-Angle Formula for Sine.

$\sin 2\theta = 2\sin\theta\cos\theta$	Double-Angle Formula
$= 2\left(\frac{\sqrt{21}}{5}\right)\left(-\frac{2}{5}\right)$	From the triangle
$= -\frac{4\sqrt{21}}{25}$	Simplify

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EXAMPLE 1 Using Double-Angle Identities

If $\cos\theta = -\frac{3}{5}$ and θ is in quadrant II, find the exact value of each expression.

a. $\sin 2\theta$ b. $\cos 2\theta$ c. $\tan 2\theta$

Solution

First, we use identities to find $\sin\theta$ and $\tan\theta$.

$$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{9}{25}} = \frac{4}{5}$$

θ is in QII so $\sin > 0$.

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{4/5}{-3/5} = -\frac{4}{3}$$

a. $$\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{4}{5}\right)\left(-\frac{3}{5}\right) = -\frac{24}{25}$$

b. $$\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(-\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2 = \frac{9}{25} - \frac{16}{25} = -\frac{7}{25}$$

c. $$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\left(-\frac{4}{3}\right)}{1-\left(-\frac{4}{3}\right)^2}$$

$$= \frac{-\frac{8}{3}}{1-\frac{16}{9}} = \frac{-\frac{8}{3}}{-\frac{7}{9}} = \left(-\frac{8}{3}\right)\left(-\frac{9}{7}\right) = \frac{24}{7}$$

6.2 Sum, Difference, and Double-Angle Identities

Sum/Difference Identities

$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$ $\quad$ $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$ $\quad$ $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

Double Angle Identities

$\sin(2\theta)=2\sin\theta\cos\theta$ $\quad$ $\cos(2\theta)=\cos^2\theta-\sin^2\theta$ $\quad$ $\tan(2\theta)=\dfrac{2\tan\theta}{1-\tan^2\theta}$

$\cos(2\theta)=2\cos^2\theta-1$

$\cos(2\theta)=1-2\sin^2\theta$

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*Ex4 p.304

Examples

Find the exact value of the following expressions.

* when you have an angle which is NOT special, convert it into the sum or difference of two special angles.

a) $\cos 195^\circ =$

$\cos(45^\circ+150^\circ) \rightarrow \cos45^\circ\cos150^\circ-\sin45^\circ\sin150^\circ$

$\left(\frac{\sqrt2}{2}\right)\left(-\frac{\sqrt3}{2}\right)-\left(\frac{\sqrt2}{2}\right)\left(\frac12\right)$

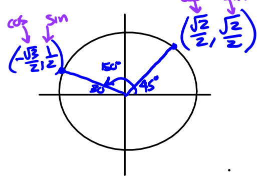
$-\frac{\sqrt6}{4}-\frac{\sqrt2}{4}$

$=\boxed{\frac{-\sqrt6-\sqrt2}{4}}$

or $-\frac{\sqrt6+\sqrt2}{4}$

or $\cos(60^\circ+135^\circ)$

or $\cos(240^\circ-45^\circ)$



b) $\sin\left(\frac{5\pi}{12}\right)=$

$\sin\left(\frac{3\pi}{12}+\frac{2\pi}{12}\right)=\sin\left(\frac{\pi}{4}+\frac{\pi}{6}\right)=\sin\frac{\pi}{4}\cos\frac{\pi}{6}+\cos\frac{\pi}{4}\sin\frac{\pi}{6}$

$\left(\frac{\sqrt2}{2}\right)\left(\frac{\sqrt3}{2}\right)+\left(\frac{\sqrt2}{2}\right)\left(\frac12\right)$

$=\boxed{\frac{\sqrt6+\sqrt2}{4}}$

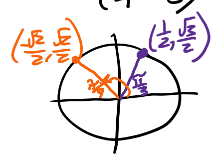
or $\sin\left(\frac{9\pi}{12}-\frac{4\pi}{12}\right)$

$=\sin\left(\frac{3\pi}{4}-\frac{\pi}{3}\right)=\sin\frac{3\pi}{4}\cos\frac{\pi}{3}-\cos\frac{3\pi}{4}\sin\frac{\pi}{3}$

$\left(\frac{\sqrt2}{2}\right)\left(\frac12\right)-\left(-\frac{\sqrt2}{2}\right)\left(\frac{\sqrt3}{2}\right)$

$\frac{\sqrt2}{4}+\frac{\sqrt6}{4}$

$=\boxed{\frac{\sqrt2+\sqrt6}{4}}$



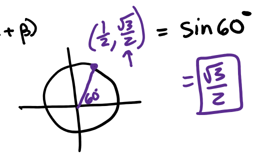
c) $\underbrace{\overset{\alpha}{\sin 50^\circ}\,\overset{\beta}{\cos10^\circ}+\cos50^\circ\sin10^\circ}_{\sin(\alpha+\beta)} = \sin(50^\circ+10^\circ)$

* 50° + 10° not special ∴ can't get exact values

60° is special ∴ get exact value

$\left(\frac12,\frac{\sqrt3}{2}\right) = \sin 60^\circ$

$=\boxed{\frac{\sqrt3}{2}}$



Write each expression in a simpler form, using identities. Give exact value, if possible.

a) $2\sin 15^\circ \cos 15^\circ = \sin(2(15^\circ))$

$\sin 2\theta = 2\sin\theta\cos\theta$

$= \sin 30^\circ$ special ✓

$= \boxed{\frac{1}{2}}$

b) $\cos^2(\pi/8) - \sin^2(\pi/8) = \cos 2\left(\frac{\pi}{8}\right)$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \cos\frac{\pi}{4}$

$= \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$

c) $1 - 2\sin^2(8) = \cos 2(8)$

$= \boxed{\cos 16}$ not special

d) $10\cos^2(x) - 5 =$

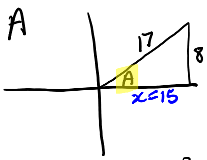
GCF

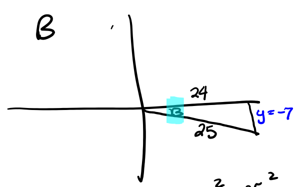
$5(2\cos^2 x - 1)$

$= \boxed{5\cos 2x}$

Given that $\sin A = \frac{8}{17}$ (8: opp $= y$; 17: hyp $= r$), where A is a Q1 angle; and $\cos B = \frac{24}{25}$, where B is a Q4 angle.

a) Draw two coordinate systems. Sketch a reference triangle for angle A on one of the systems, and one for angle B on the other.





$x^2 + 8^2 = 17^2$ -64

$\sqrt{x^2} = \sqrt{225}$

$x = \pm 15$

$y^2 + 24^2 = 25^2$ -24^2

$\sqrt{y^2} = \sqrt{49}$

$y = \pm 7$

b) Use identities to find the exact value of:

$\sin(A - B) = \sin A\cos B - \cos A\sin B$

$\left(\frac{8}{17}\right)\left(\frac{24}{25}\right) - \left(\frac{15}{17}\right)\left(\frac{-7}{25}\right)$

$\frac{192}{425} + \frac{105}{425}$

$= \boxed{\frac{297}{425}}$

$\tan(2A) = \dfrac{2\tan A}{1 - \tan^2 A}$

$\tan A = \frac{opp}{adj} = \frac{8}{15}$

$= \dfrac{2\left(\frac{8}{15}\right)}{1 - \left(\frac{8}{15}\right)^2} \rightarrow \dfrac{\frac{16}{15}}{\frac{225}{225} - \frac{64}{225}}$

$= \dfrac{\frac{16}{15}}{\frac{161}{225}} \rightarrow \frac{16}{\cancel{15}_1} \times \frac{\cancel{225}^{15}}{161}$

$= \boxed{\frac{240}{161}}$

$\sin 2B = 2\sin B\cos B$

$2\left(\frac{-7}{25}\right)\left(\frac{24}{25}\right)$

$= \boxed{-\frac{336}{625}}$

$\cos 2A$ any identity will work.

$= 2\cos^2 A - 1$

$= 2\left(\frac{15}{17}\right)^2 - 1$

$= \frac{450}{289} - \frac{289}{289} \rightarrow \boxed{\frac{161}{289}}$

$\cos^2 A - \sin^2 A$

$\left(\frac{15}{17}\right)^2 - \left(\frac{8}{17}\right)^2$

$\frac{225}{289} - \frac{64}{289}$

$= \boxed{\frac{161}{289}}$

Do ch 6 Assign up to #5 (try #6)