Class_12 Oct 18 Trig Graphs and Applications

Monday, October 17, 2022 3:00 PM

Tonight's Class:

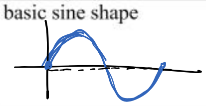
- **Chapter 4 Test Return**
  - **Compare your Chapter 4 test with your Chapter 4 hand-in Assignment. Circle assignment questions that connect to ideas you should work more with before the Unit 2 Test.**
- **5.2 Transforming Trig Graphs (continued)**
- **5.3 Tangent Graph**
- **5.4 Trig Applications**

Sketching a Sinusoidal Graph

Consider the equation: $\boxed{y = 3\sin\left(\frac{2\pi}{12}(x+5)\right) - 1}$

amp HE by $\frac{12}{2\pi}$ v.disp. p.shift

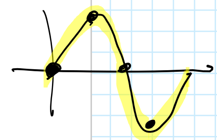
a) Key features:

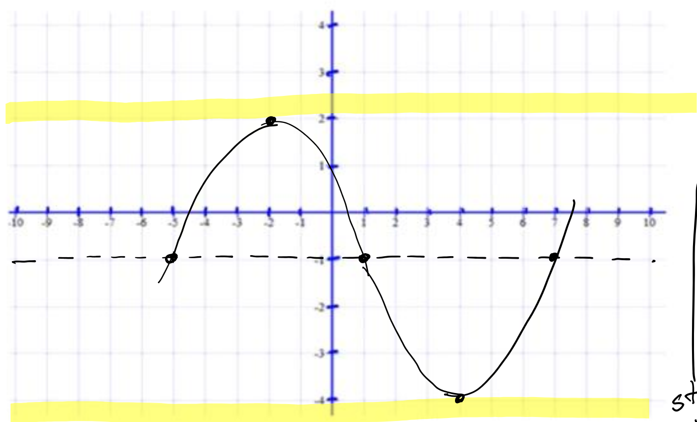
basic sine shape	vertical displacement	equation of center line
	down 1	$y = -1$
amplitude	**maximum**	**minimum**
3	$-1+3 = 2$	$-1-3 = -4$
period	**spacing** (period ÷ 4)	**phase shift**
$\frac{12}{\cancel{2\pi}} \times (\cancel{2\pi}) = 12$	$= \frac{12}{4} = 3$	5 left

Horizontal stretch factor

normal period

b) Accurately sketch one period of the graph. Give the coordinates of 5 key points. Include the center line on your sketch.





x	y	
-5	-1	CL
-2	2	Max
1	-1	CL
4	-4	min
7	-1	CL

from the "key features"

start where the phase shift tells you to.

use spacing to get the other x-values

In-class Worksheet: Graphing Sinusoidal Functions (question on back)

Finding the Equation of a Graph

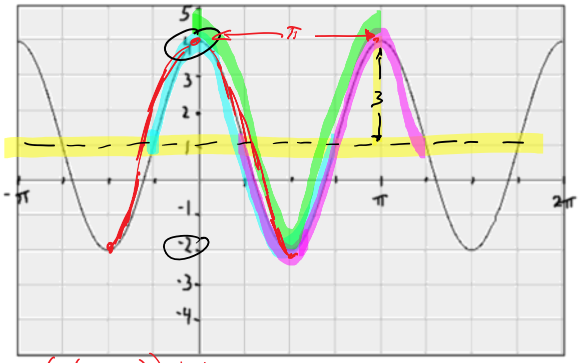
Sine and cosine graphs are both called ***sinusoidal graphs***.

- For any sinusoidal graph, it is possible to write a sine equation that creates that graph, and a cosine equation that creates that same graph.
- There are many different equations that generate the same sinusoidal graph.

$y = a\sin(b(x-c)) + d$

Example

Give two different equations that create this graph.



Maximum: 4

Minimum: −2

Center line: $\frac{4+-2}{2} = \frac{2}{2} = 1$, $y = 1$

Vertical displacement: up 1

Amplitude: 3

Period: π

b value: $\frac{2\pi}{\pi} = 2$

remember, $b = \frac{\text{normal period}}{\text{graph's period}} = \frac{2\pi}{\text{graph's period}}$

$y = -3\cos\left(2\left(x + \frac{\pi}{2}\right)\right) + 1$

Possible sine equation:

$y = 3\sin\left(2\left(x + \pi/4\right)\right) + 1$

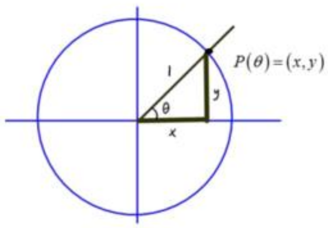
$y = -3\sin\left(2\left(x - \pi/4\right)\right) + 1$

Possible cosine equation: $y = 3\cos(2x) + 1$

A BRIEF look at section 5.3 (pages 33-34)

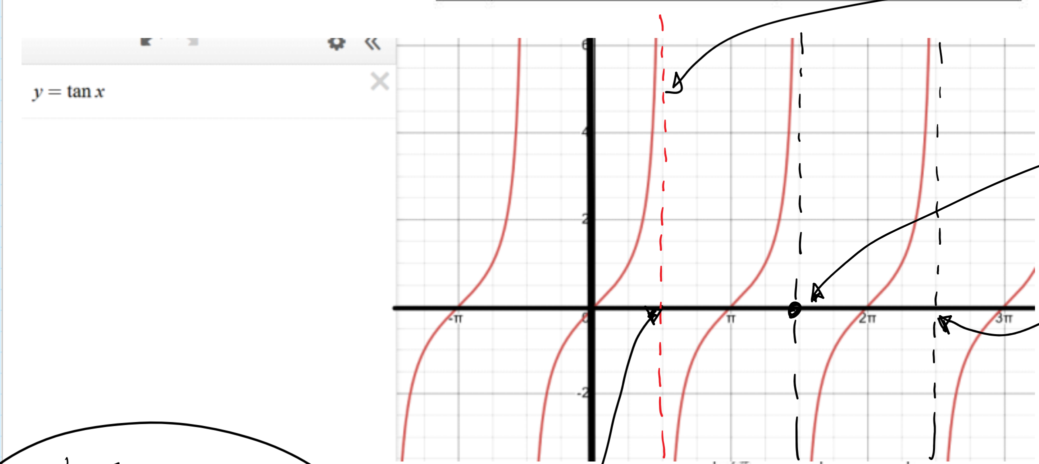
5.3 The Tangent Function

Let's track what happens to the values of $y = \tan\theta$ as θ, a standard-position angle, gets larger.



		$y = \tan\theta$
Q1	As θ increases from 0 to $\frac{\pi}{2}$	tangent values (y/x)
Q2	As θ increases from $\frac{\pi}{2}$ to π	tangent values (y/x)
Q3	As θ increases from π to $\frac{3\pi}{2}$	tangent values (y/x)
Q4	As θ increases from $\frac{3\pi}{2}$ to 2π	tangent values (y/x)

asymptote



$3\pi/2$

$5\pi/2$

$\frac{7\pi}{4}$	
$\frac{8\pi}{4} = 2\pi$	

$x = \pi/2$

$x \neq \frac{\pi}{2} + n\pi,\ n \in I$

Domain: x-intercepts: Asymptote equations:

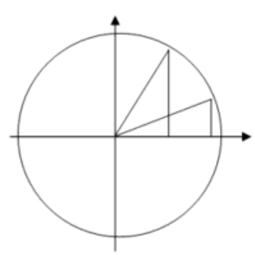
Period: Range:

π

The tangent graph shows how the ***slope*** of the terminal arm of a standard-position angle θ changes, as the angle increases in size.

$$\tan\theta = \frac{y}{x}$$

$$slope = \frac{change\ in\ y}{change\ in\ x} = \frac{rise}{run}$$



Try

1) Sketch the graph of $y = \tan\theta$. Try to figure out points on the graph yourself (rather than just using your calculator), by using the slope idea discussed above.

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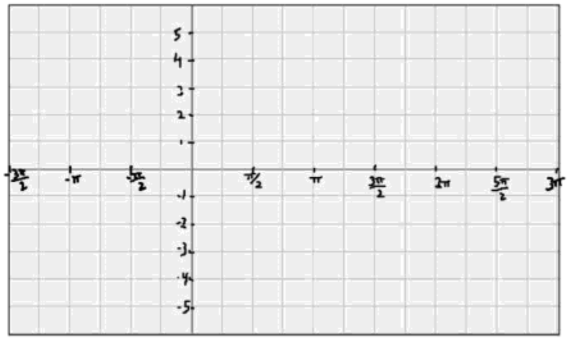


Period:

x-intercepts:

Domain:

Asymptote equations:

2) For the graph of $y = \tan(3\theta)$

a) what does the "3" do? HC by $\frac{1}{3}$

b) period of $y = \tan(3\theta)$ $\quad \frac{1}{3} \times \pi = \frac{\pi}{3}$

c) asymptote equations for $y = \tan(3\theta)$

$$X = \frac{1}{3}\left(\frac{\pi}{2} + n\pi\right), \; n \in I$$

$$\boxed{X = \frac{\pi}{6} + \frac{n\pi}{3}, \; n \in I}$$

Know how to do this. (Ch 5 test)

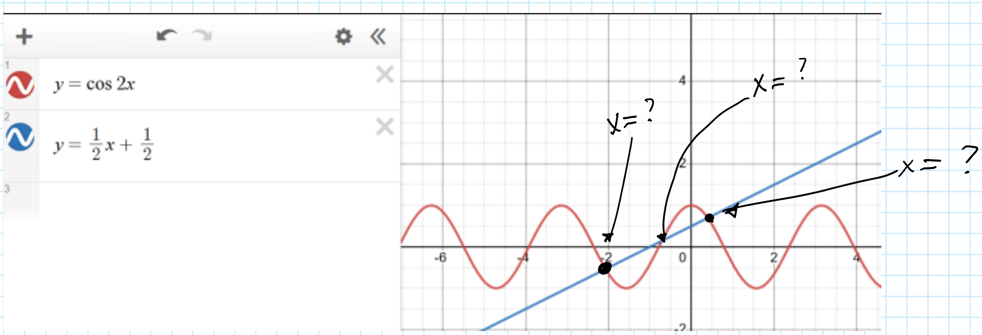
domain: $X \neq \frac{\pi}{6} + \frac{n\pi}{3}, \; n \in I$

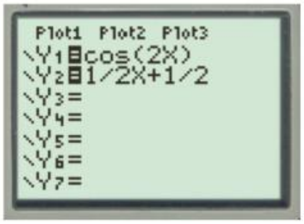
5.4 Trig Equations and Application Questions

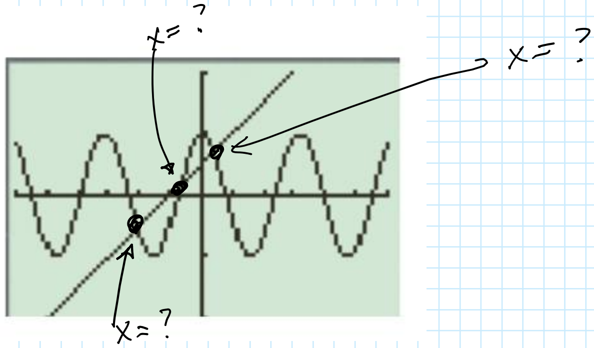
Some equations cannot be solved algebraically. For this reason, we want to understand how to solve equations graphically.

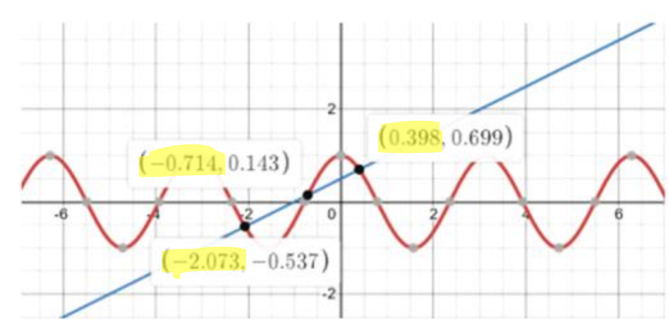
Example: Solve: $\cos 2x = \frac{1}{2}x + \frac{1}{2}$, for $0 \leq x < 2\pi$.

graph graph









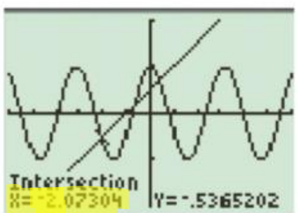
Find the solutions to the equation

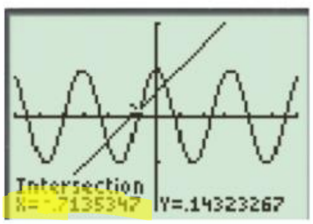
$$\cos(2x) = \frac{1}{2}x + \frac{1}{2}, \text{ for } 0 \le x < 2\pi$$

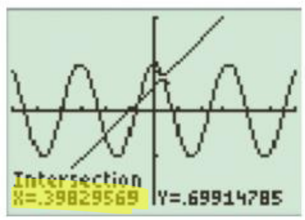
$x = -2.073$

$x = -0.714$

$x = 0.398$

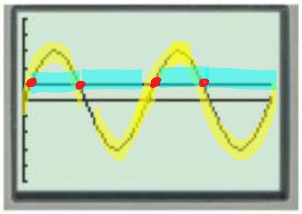


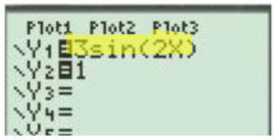


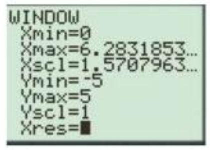


5.4 Equations and Graphs of Trigonometric Functions

Below we see how we can solve a trigonometric equation, ***graphically***.







a) What is the equation that is being solved? $3\sin(2x) = 1$

b) The window has been restricted to match the domain for this question. What is that domain?

$0 \le x < 2\pi$

c) How many solutions are there, in this domain? Mark them on the calculator graph screenshot, shown above.

4 solutions

Remember, there are two ways to solve equations GRAPHICALLY

Intersection Method
1) Enter the LHS of the equation as Y_1
2) Enter the RHS of the equation as Y_2
3) Set the *Xmin* and *Xmax* values using the given domain.
4) Use the "intersect" feature to find each place where the LHS = RHS.
The *x*-values of the intersections are the equation's solutions.

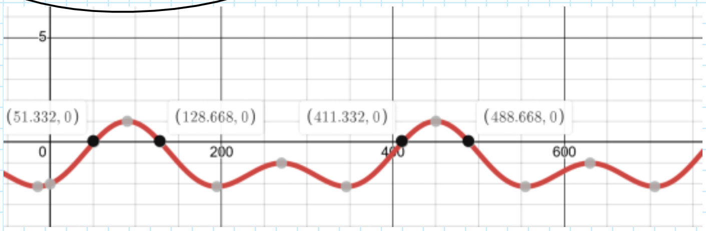
X-intercepts (zeroes) Method
1) Collect all terms of the equation on one side of the equals sign, so it looks like
$\boxed{} = 0$.
2) Enter the equation as Y_1
3) Set the *Xmin* and *Xmax* values using the given domain.
4) Use the "Zero" feature to find each *x*-intercept. These *x*-values are the equation's solutions.

Try
Solve ***graphically***, correct to 1 decimal place. Include a sketch of the graph with the solutions marked on it.

$2\sin^2 x + \sin x - 2 = 0$, for $0^\circ \le x \le 720^\circ$

$x = 51.3^\circ$
128.7°
411.3°
488.7°

Pay attention to
- **Mode**
- **Window values**



Try

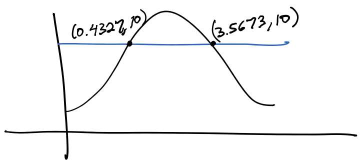
Solve ***graphically***, correct to 1 decimal place. Include a sketch of the graph with the solutions marked on it.

$$2\sin^2 x + \sin x - 2 = 0, \text{ for } 0^\circ \le x \le 720^\circ$$

Example

Consider the trigonometric equation $6\sin\left(\frac{\pi}{4}x\right)+8=10$

a) Solve ***graphically*** for $0 \le x < 2\pi$, correct to 4 decimal places. Include a sketch of the graph with the solutions marked on it.



$x = 0.4327$

$x = 3.5673$

b) Find the general solution, ***algebraically***, correct to 4 decimal places.

Solve:

1) isolate ✓
2) decide: calculator ✓
3) ref. angle
4) solve

$$6\sin\left(\frac{\pi}{4}x\right) + 8 = 10$$
$$-8 \quad -8$$

$$\frac{\cancel{6}\sin\left(\frac{\pi}{4}x\right)}{\cancel{6}} = \frac{2}{6}$$

$$\sin\left(\frac{\pi}{4}x\right) = \frac{1}{3}$$

$\theta_R = \sin^{-1}\left(\frac{1}{3}\right)$ use radian mode

$= 0.3398\ldots$ don't lose any of these digits. Keep all of them!

Q_1: $\dfrac{\frac{\pi}{4}x}{(\pi/4)} = \dfrac{0.3398\ldots}{(\pi/4)}$

$x = 0.4326937919$

$\boxed{x \doteq 0.4327}$

Q_2: $\frac{\pi}{4}x = \pi - \theta_R$

$\dfrac{\frac{\pi}{4}x}{(\pi/4)} = \dfrac{\pi - 0.3398\ldots}{(\pi/4)}$

$\boxed{x \doteq 3.5673}$

$(\pi - \sin^{-1}(1/3))/(\pi/4)$

General solution – must add on the period $\times\, n$

period = ? $6\sin\left(\frac{\pi}{4}x\right) + 8 = 10$

this changes the period

normal period

$\frac{4}{\cancel{\pi}} \cdot \frac{2\cancel{\pi}}{1} = 8$

$$\boxed{\left.\begin{array}{l} x = 0.4327 + 8n \\ \quad\ 3.5673 + 8n \end{array}\right\} n \in I}$$

$$\boxed{\left.\begin{aligned} x &= 0.4327 + 8n \\ & 3.5673 + 8n \end{aligned}\right\} n \in I}$$

Example of an application question

The depth of water, h meters, at a certain port, at time t hours, is given by this equation, where $t = 0$ represents midnight:

$$h(t) = 1.4\sin\left(\frac{2\pi}{12.2}(t-0.8)\right) + 2.7$$

use $t = 14$

How deep will the water be at 2:00 AM? At 2:00 PM?

use $t = 2$, use radians

$$h(2) = 1.4\sin\left(\frac{2\pi}{12.2}(2-0.8)\right) + 2.7$$

$\doteq 3.5m$

$$h(14) = 1.4\sin\left(\frac{2\pi}{12.2}(14-0.8)\right) + 2.7$$

$\doteq 3.4m$

WB - Creating a Sinusoidal Graph and Equation

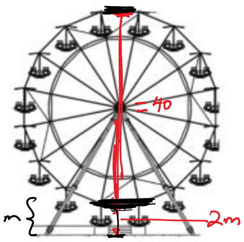
Example

Suppose the pictured Ferris wheel has diameter 40 meters, and the height of the seat where you first get on is 2 meters above the ground. This wheel takes 2 minutes to rotate and travels at a constant speed.

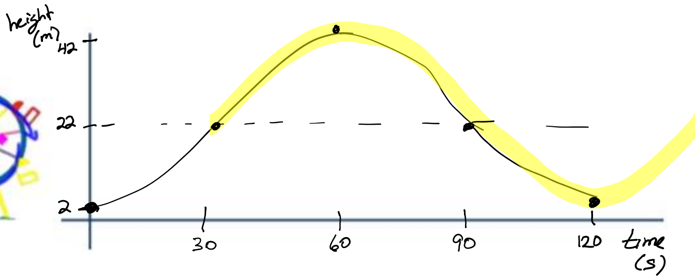
2 minutes = 120 seconds

- Minimum height? 2 m
- Maximum height? 42 m
- Center line height? $\frac{2+42}{2} = \frac{44}{2} = 22\,m$
- Period length in seconds? 120

- amplitude — 20



a) Sketch a complete period of the graph, showing the height of a passenger above the ground as a function of time, in seconds. Give the coordinates of 5 key points.



t	h
0	2
30	22
60	42
90	22
120	2

$$\boxed{b = \frac{2\pi}{120}}$$

b) Create a sinusoidal equation for this graph.

amplitude = radius of the wheel

$$h = -20\cos\left(\frac{2\pi}{120}t\right) + 22$$

OR

$$h = 20\sin\left(\frac{2\pi}{120}(t-30)\right) + 22$$

c) How high above the ground is a passenger 12 seconds after getting on, correct to one decimal place?

let $t = 12$

$$h = -20\cos\left(\frac{2\pi}{120}(12)\right) + 22$$

$$\boxed{h \doteq 5.8\ m}$$

d) During the first rotation of the Ferris wheel, what is the first time that the passenger reaches a height of 30 meters above the ground? Solve ~~this *graphically*~~. algebraically

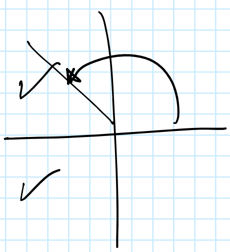
$$30 = -20\cos\left(\frac{2\pi t}{120}\right) + 22$$

$$-22 \qquad\qquad -22$$

$$8 = -20\cos\left(\frac{2\pi t}{120}\right)$$

$$\frac{-20\cos\left(\frac{2\pi t}{120}\right)}{-20} = \frac{8}{-20}$$

$$\cos\left(\frac{2\pi t}{120}\right) = -0.4$$



$$\theta_R = \cos^{-1}(+0.4)$$

$$= 1.159\ldots\ldots$$

$$\pi - \theta_R = \frac{2\pi t}{120}$$

Q2 answer:

George Washington Gale Ferris Jr.

Born	February 14, 1859 Galesburg, Illinois
Died	November 22, 1896 (aged 37) Pittsburgh, Pennsylvania
Cause of death	Typhoid fever
Education	Rensselaer Polytechnic Institute (1881)
Known for	The original Chicago Ferris Wheel and the Ferris wheel concept

Q2 answer :

$$\pi - \theta_R = \frac{2\pi t}{120}$$

$$\frac{(\pi - 1.159...)}{\left(\frac{2\pi}{120}\right)} = \frac{\frac{\cancel{2\pi} t}{\cancel{120}}}{\left(\cancel{\frac{2\pi}{120}}\right)}$$

$$\underset{\text{seconds}}{37.9} = t$$

p 275: 1, 4ac, 8b, 9, 10, 18, 19

More TB practice

- (5.2) TB p 250:3-7, 10, 14, 15ac, 16ac
- (5.4) TB p 275: 1, 4ac, 8b, 9, 10, 18, 19

Start working on the Chapter 5 Hand-in assignment, due Oct 25.
Partial solutions posted.

The Chapter 5 Test will be on Tuesday, Oct 25.