Class_13 Feb 16 - Different Forms of Quadratic Equations

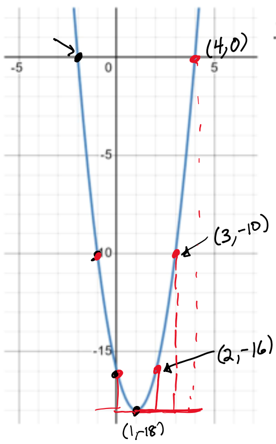
Monday, February 13, 2023 3:21 PM

Tonight's Class:

- **Returning Chapter 3 Test**
- **Working through sections 4.4-4.6**
  - **Transforming Quadratic Graphs (continued)**
  - **Changing Quadratics from General to Vertex Form**
  - **Changing Quadratics from General to Factored Form**
- **Work on practice questions from worktext**

Hand-out (back of sheet)

Finding Key Points Using Vertical Stretches

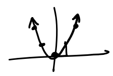


$y = 2x^2 - 4x - 16$

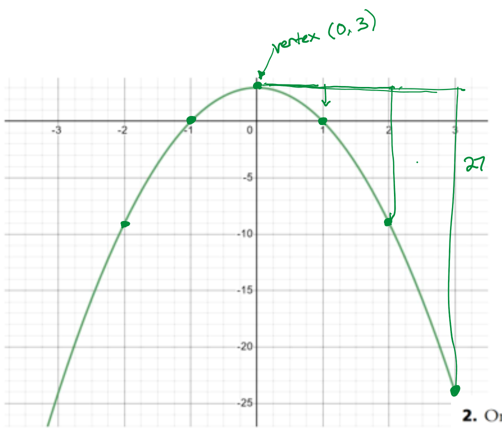
this tells us that the usual pattern for $y = x^2$ is multipled by 2.

1) find vertex
$(1, -18)$

usually
start vertex :



$y = x^2$		$y = 2x^2$...
1 right/L	1up	2 up
2 right/L	4up	8 up
3 right/L	9up	18 up



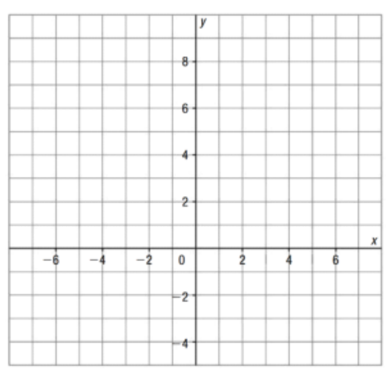
$y = -3x^2 + 3$

What will this parabola do?

usual		
1 L/R	1 up	-3 (down 3 from the vertex height)
2 L/R	4 up	4(-3) = -12 (down 12)
3 L/R	9 up	9(-3) = -27

2. On grid paper, graph $y = x^2$. Graph each quadratic function without using a table of values or graphing technology. Explain your strategy each time.

a) $y = x^2 + 5$ $\quad y = x^2 - 4$



Summary

Translations - move left/right, up/down

Vertical expansions/compressions – multiply every y-val

Reflections – when "a" is negative, the graph is reflected upside-down

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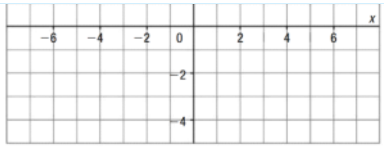

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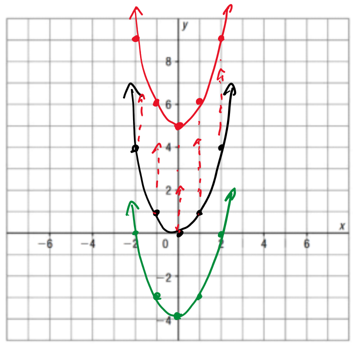
2. On grid paper, graph $y = x^2$. Graph each quadratic function without using a table of values or graphing technology. Explain your strategy each time.

a) $y = x^2 + 5$ $y = x^2 - 4$

5 up

4 down



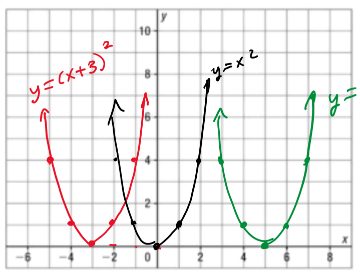
$y=x^2$

x	y
-2	4
-1	1
0	0
1	1
2	4

b) $y = (x + 3)^2$ $y = (x - 5)^2$

move 3 left

5 right

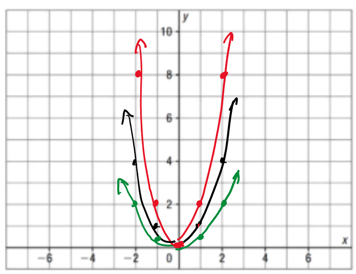


Vertical expansion 2

c) $y = 2x^2$ $y = \frac{1}{2}x^2$

Vertical compression by $\frac{1}{2}$

x	y
-2	4
-1	1
0	0
1	1
2	4



4.4 Properties of a Quadratic Function in Vertex/Standard Form

Focus: determine the characteristics of a quadratic function and sketch its graph

$$y = a(x - h)^2 + k$$

Vertex: (h, k)

Axis of symmetry: x = h

VERTICAL LINE

If a is positive, then it opens up.

If a is negative, then it opens down.

right 6

$y = 2(x-6)^2 + 1$ $V=(6,1)$

Vertical stretch by 2

up 1

$y = x^2$

$V=(0,0)$

VERTICAL LINE

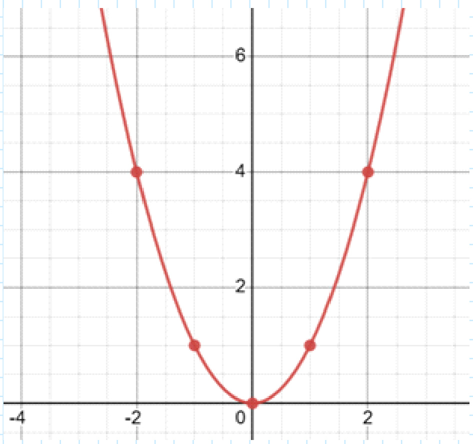
by 2

If **a** is positive, then it opens up.

If **a** is negative, then it opens down.

$y = x^2$ $V=(0,0)$

$y = -3(x+1)^2 - 5$	$y = \frac{1}{2}(x-4)^2 + 6$	$y = -5(x+\frac{1}{4})^2 + 17$
$V = (-1, -5)$	$V = (4, 6)$	$V = (-\frac{1}{4}, 17)$
A/S equation is $x = -1$	A/S eq $x = 4$	A/S eq $x = -\frac{1}{4}$
open down	open up	opens down
maximum, value is -5	minimum, value of 6	maximum, value 17



A parabola

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Example 2 **Graphing a Quadratic Function with Its Equation in Standard Form** / Vertex Form

For the quadratic function $y = -2(x+2)^2 - 3$

a) Identify:

i) the direction of opening downward

ii) the vertex $(-2, -3)$

iii) the equation of the axis of symmetry $x = -2$

iv) the intercepts

v) the domain and range of the function

b) Sketch a graph.

$y = -2x^2$

x	y
0	0
1	1
2	4

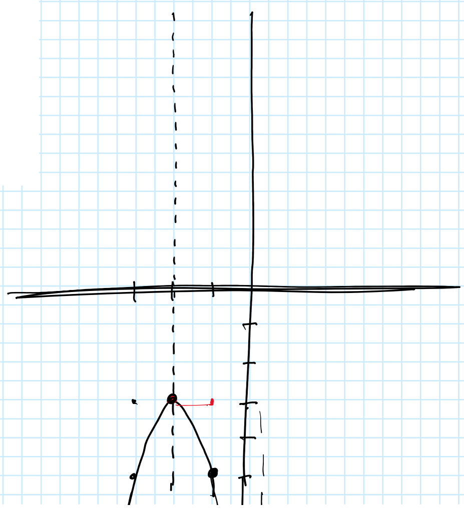
2 down

8 down

X-intercepts

none

y-intercept



2	4	8 down

y-intercept

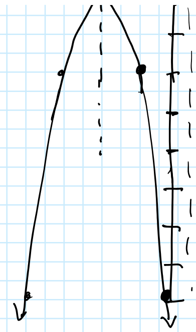
(0, −11)

domain

$x \in \mathbb{R}$

range

$y \le -3$



$$y = -2(x+2)^2 - 3$$

We can get the intercepts before making a graph.

y-intercept; let $x = 0$ in the equation, and solve:

$$y = -2(x+2)^2 - 3$$
$$y = -2(0+2)^2 - 3$$
$$y = -2(2)^2 - 3$$
$$y = -2(4) - 3$$
$$y = -8 - 3$$
$$y = -11$$

(0, −11)

---

x-intercepts, let $y = 0$ in the equation and solve

$$y = -2(x+2)^2 - 3$$
$$0 = -2(x+2)^2 - 3$$
$$+3 \qquad\qquad +3$$
$$\frac{3}{-2} = \frac{\cancel{-2}(x+2)^2}{\cancel{-2}}$$
$$\pm\sqrt{-\frac{3}{2}} = \sqrt{(x+2)^2}$$
$$\pm\sqrt{\frac{-3}{2}} = x+2$$

?

no x-intercepts.

$$\pm\sqrt{\frac{-3}{2}} = x+2$$

nope.

No solution, no x-intercepts.

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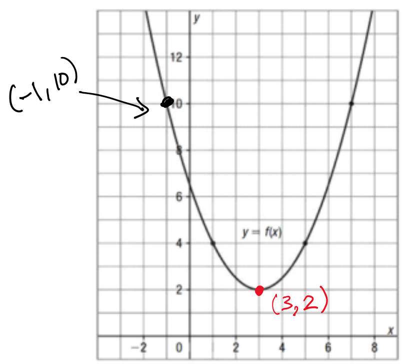
Example 1 **Determining an Equation of a Quadratic Function from Its Graph**

Determine an equation of this graph of a quadratic function.



1) find vertex, put it into the form

$$y = a(x-h)^2 + k$$

$$y = a(x-3)^2 + 2$$

2) pick any other point that's on the parabola → not the vertex!

$(-1, 10)$ — x, y

substitute its x-value and y-value into equation

(-1, 10) — x, y

$$y = a(x-3)^2 + 2$$
$$10 = a(-1-3)^2 + 2$$
$$10 = a(-4)^2 + 2$$
$$10 = a(16) + 2$$
$$-2 \qquad -2$$
$$\frac{8}{16} = \frac{\cancel{16}a}{\cancel{16}}, \quad a = \frac{8}{16}$$

reduce it, if you can

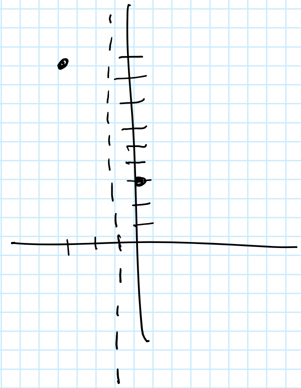
$$a = \frac{1}{2}$$

3) Substitute "a" value into the final equation:

$$y = \frac{1}{2}(x-3)^2 + 2$$

Example 3 **Determining an Equation of a Quadratic Function Using Its Characteristics**

The equation of the axis of symmetry of the graph of a quadratic function is $x = -1$. The graph passes through the points A(0, 3) and B(−3, 9). Determine an equation of the function.



1) the axis of symmetry equation is $x = -1$

Vertex = $(-1, ?)$

$y = a(x-h)^2 + k$, since I know $x=-1$ is on the vertex, we get:

$y = a(x+1)^2 + k$

2) We have (0,3) and (−3,9) both on the graph.

$(0,3)$: $0 \to x$, $3 \to y$ $\qquad$ $(-3,9)$: $-3 \to x$, $9 \to y$

$y = a(x+1)^2 + k$

$3 = a(0+1)^2 + k$

$3 = a(1)^2 + k$

$3 = a + k$

$9 = a(-3+1)^2 + k$

$9 = a(-2)^2 + k$

$9 = a(4) + k$

$9 = 4a + k$

3) Isolate a variable in one of those equations, then we'll substitute into the other one:

$3 = a + k$

$-k \qquad -k$

$3 - k = a$

$a = 3 - k$

$9 = 4a + k$

$9 = 4(3-k) + k$

$9 = 12 - 4k + k$

$9 = 12 - 3k$

$-12 \qquad -12$

$-3 = -3k$

$\frac{-3}{-3} = \frac{-3k}{\cancel{-3}}$

$\boxed{k = 1}$

4) Now, substitute the k-value to figure out "a".

to figure out "a".

$\rightarrow a = 3 - 1$

$\boxed{a = 2}$

5) What's the final equation? $y = a(x+1)^2 + k$

$\boxed{y = 2(x+1)^2 + 1}$

4.5 Changing a Quadratic Function from General to Vertex Form

Focus: completing the square, to change a quadratic equation to vertex/standard form

$y = ax^2 + bx + c$

general form

Complete the Square

$y = 2x^2 + 12x + 13$

$y = 2(x^2 + 6x) + 13$ ← Divide the number in front of x^2 out of first 2 terms

$\left(\frac{b}{2}\right)^2 = \left(\frac{6}{2}\right)^2 = (3)^2 = 9$ ← Determine the perfect square number

$y = 2(x^2 + 6x + 9 - 9) + 13$ ← Add and subtract the number. Do this inside the brackets

$y = 2(x+3)^2 - 5$ ← Factor the first 3 terms. Add up the leftovers. In this case 2 x -9 + 13 = -5

Vertex is (-3,-5) ← Remember to switch the x-coordinate

WT p 313

Example 1 **Completing the Square for an Equation of the Form $y = ax^2 + bx + c$, $a > 0$**

Determine the coordinates of the vertex of the parabola with equation $y = 2x^2 + 16x + 24$.

factor out the 2, from the first two terms

$y = 2(x^2 + 8x) + 24$

figure out what number to add in, so we get a perfect square

$\left(\frac{b}{2}\right)^2 = \left(\frac{8}{2}\right)^2$

$= (4)^2$

$= 16$

$y = 2(x^2 + 8x + 16 - 16) + 24$

$y = 2(x^2 + 8x + 16) - 32 + 24$

factoring shortcut is $\left(x+\frac{b}{2}\right)^2$

$$\boxed{y = 2(x+4)^2 - 8}$$

vertex: $(-4, -8)$

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Example 2 **Completing the Square for an Equation of the Form $y = ax^2 + bx + c,\ a < 0$**

Determine the equation of the axis of symmetry of the parabola with equation $y = -4x^2 + 9x - 2$.

$b = -\frac{9}{4}$

$$y = -4\left(x^2 - \frac{9}{4}x \quad\right) - 2$$

$$y = -4\left(x^2 - \frac{9}{4}x + \frac{81}{64} - \frac{81}{64}\right) - 2$$

$$y = -4\left(\underbrace{x^2 - \frac{9}{4}x + \frac{81}{64}}\right) + \frac{81}{16} - 2$$

$$y = -4\left(x - \frac{9}{8}\right)^2 + \frac{81}{16} - \frac{2}{1}\cdot\frac{16}{16}$$

$$y = -4\left(x - \frac{9}{8}\right)^2 + \frac{81}{16} - \frac{32}{16}$$

$$y = -4\left(x - \frac{9}{8}\right)^2 + \frac{49}{16}$$

equation of axis of symmetry $\quad x = \frac{9}{8}$

$b/2$

half of b

$$\left(\frac{b}{2}\right)^2 = \left(\frac{1}{2}\cdot\frac{-9}{4}\right)^2 = \left(\frac{-9}{8}\right)^2 = \frac{81}{64}$$

$$\frac{9/4}{2}$$

$$\frac{9}{4} \div 2 = \frac{9}{4} \times \frac{1}{2} = \frac{9}{8}$$

$$\frac{-4}{1} \times \frac{-81}{64} \quad (\div 4) = \frac{-1 \times 81}{16} = +\frac{81}{16}$$

$$\boxed{\frac{-4}{1} \times \frac{-81}{64} = \frac{324 \div 4}{64 \div 4} = \frac{81}{16}}$$

Try:
WT p 316, #6a
WT p 318, #8b

4.6 Changing from General Form to Factored Form

Focus: changing a quadratic equation to factored form to help analyze it

Getting Information about the Graph from General Form

Stretch factor

General form

Stretch factor

General form

$$y = ax^2 + bx + c,\ a \neq 0$$

- $a > 0$ opens up and has a minimum
- $a < 0$ opens down, has a maximum

- To find the y-intercept, make x=0 and solve for y
- To find the x-intercepts, make y=0 and solve the quadratic equation for x

- To find the axis of symmetry, average the x-intercepts
  - $x = \frac{x_1 + x_2}{2}$
- To find the vertex
  - x-coordinate is same number as in the axis of symmetry
  - y-coordinate is found by substituting that x-value into the quadratic equation

Let's try this!

$y = 4x^2 + 4x - 15$ in general form

find its x-intercepts, let $y=0$ and solve for x

$0 = 4x^2 + 4x - 15$

$0 = 4x^2 + 10x - 6x - 15$

$0 = 2x(2x+5) - 3(2x+5)$

$0 = (2x+5)(2x-3)$

Factor:
$AC = 4(-15)$
$= -60$

ADD to +4

10, and -6

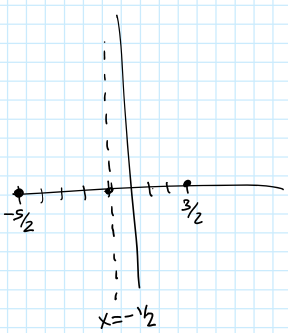
60	
1	60
2	30
3	20
4	15
5	12
6	10

$2x + 5 = 0$
$-5 \quad -5$
$\frac{2x}{2} = \frac{-5}{2}$
$x = -\frac{5}{2}$

$\left(-\frac{5}{2}, 0\right)$

$2x - 3 = 0$
$+3 \quad +3$
$\frac{2x}{2} = \frac{3}{2}$
$x = 3/2$

$\left(\frac{3}{2}, 0\right)$



Axis of symmetry is found by averaging these:

$$\frac{x_1 + x_2}{2}$$

$$\frac{-5/2 + 3/2}{2} = \frac{-2/2}{2} = \frac{-1}{2}$$

Vertex $= (-1/2, y)$

to get it, plug in x-values to the general form equation

$y = 4x^2 + 4x - 15$

$4(\ )^2 + 4(\ ) - 15$

vertex $\Rightarrow$ $y = 4\left(\frac{-1}{2}\right)^2 + \frac{4}{1}\left(\frac{-1}{2}\right) - 15$

$= \frac{4}{1}\left(+\frac{1}{4}\right) + \left(\frac{-4}{2}\right) - 15$

$= \frac{4}{4} + -2 - 15$

$\left(-\frac{1}{2}, -16\right)$ $= 1 + -2 - 15$

$= -16$

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Example 2 **Graphing a Quadratic Function from Its Equation in General Form**

Sketch a graph of each quadratic function.

a) $y = 4x^2 + 4x - 15$

opens up, and has a minimum

this is the y-intercept

y-intercept, let $x=0$

$y = 4(0)^2 + 4(0) - 15$

$y = -15$

X-intercepts, let $y=0$

(we did this, above)

$(-5/2, 0)$ and $(3/2, 0)$

A/S

also done above,

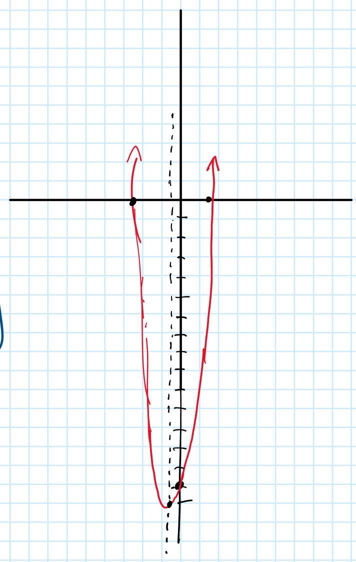
$\boxed{x = -1/2}$

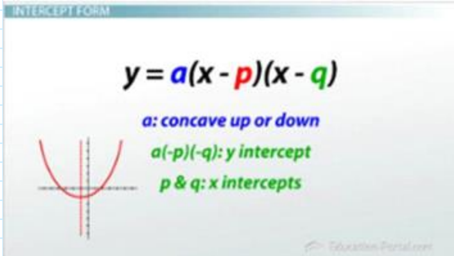
vertex, we find

$(-1/2, -16)$

$\boxed{y=(2x+5)(2x-3)}$

factored form





Since we get the x-intercepts from the factors, this is sometimes called INTERCEPT, or FACTORED form.

WT 323: Try "Check Your Understanding" 2b

For next class

- **NO class on Tuesday, February 21**
- **Work on the worktext questions for 4.1, 4.3-4.6**
  - **In section 4.4, there's a mistake in the answer given for #7. It should say: +1/4 in the equation, not -1/4**
- **Start on Chapter 4 Hand-in assignment**