Class_13 May 25 More Trig Identities

Thursday, May 25, 2023 9:55 AM

Tonight's Class:

- **6.3 Proving Identities**

6.3 Proving Identities

When we prove identities:

- Step by step, use algebra and/or Basic Identities to change the way either the left- hand side (LHS) or the right-hand side (RHS) looks.
- Think of the " = " sign separating the LHS and RHS as a barrier. Don't take terms from one side of the equals sign to the other.
- When the LHS and the RHS look exactly the same, the identity is proven.

Strategies for Proofs

- Write each step directly below the previous one.
- Don't skip steps – aim to be as CLEAR as possible.
- See if there's any factoring you can do, especially GCF or difference of squares.
- Don't cancel anything, unless you have identical factors on the top and bottom of an expression.
- If rational expressions are added /subtracted together, get a common denominator so you can combine the expressions and simplify.
- If possible, substitute known identities to simplify expressions.
- If the LHS and RHS look as below, where they are ***almost*** reciprocals of each other, multiply one side, top and bottom, by the conjugate of the binomial. Then use a Pythagorean identity to simplify further.

$$\boxed{\frac{\cos y}{1-\sin y}} = \frac{1+\sin y}{\cos y}\cdot\frac{(1-\sin y)}{(1-\sin y)}$$

$$\frac{1-\cancel{\sin y}+\cancel{\sin y}-\sin^2 y}{\cos y\,(1-\sin y)}$$

$$\frac{1-\sin^2 y}{\cos y\,(1-\sin y)}$$

$$\frac{\cos^{\cancel{2}} y}{\cancel{\cos y}\,(1-\sin y)}$$

$$\boxed{\frac{\cos y}{1-\sin y}}$$

Try these strategies to prove the identities on the following pages.

Prove

1. $(\sin x+\cos x)^2 = 1+2\sin x\cos x$

$$(\sin x+\cos x)(\sin x+\cos x)$$

$$\sin^2 x+\sin x\cos x+\cos x\sin x+\cos^2 x$$

$$\sin^2 x+2\sin x\cos x+\cos^2 x$$

$$1+2\sin x\cos x$$

2. $\tan^2 x\sin^2 x-\tan^2 x = -\sin^2 x$

$\tan x=\frac{\sin x}{\cos x}$

$\tan^2 x=\frac{\sin^2 x}{\cos^2 x}$

$$\tan^2 x(\sin^2 x-1)$$

$$\frac{\sin^2 x}{\cos^2 x}(\sin^2 x-1)$$

$$\frac{\sin^2 x}{\cos^2 x}(-\cos^2 x)$$

$$-\frac{\sin^2 x\,\cancel{\cos^2 x}}{\cancel{\cos^2 x}}$$

$$-\sin^2 x$$

3. $\sec^4 x = \tan^4 x+2\tan^2 x+1$

A C

1 1

$$\tan^4 x+1\tan^2 x+1\tan^2 x+1$$

$$\tan^2 x(\tan^2 x+1)+1(\tan^2 x+1)$$

$$(\tan^2 x+1)(\tan^2 x+1)$$

$$(\sec^2 x)(\sec^2 x)$$

$$\sec^4 x$$

4. $(3-3\sin x)(3+3\sin x) = 9\cos^2 x$

$$9+\cancel{9\sin x}-\cancel{9\sin x}-9\sin^2 x$$

$$9-9\sin^2 x$$

$$9(1-\sin^2 x)$$

$$(5xy)^2$$

$$=5^2x^2y^2$$

$$=25x^2y^2$$

$$(x+3)^2$$

$$\neq x^2+3^2$$

$$(4+3)^2=7^2$$

$$=49$$

$$4^2+3^2$$

$$=16+9\neq 49$$

$$\sin^2\theta+\cos^2\theta=1$$

$$\sin^2\theta-1+\cos^2\theta=0$$

$$\sin^2\theta-1=-\cos^2\theta$$

$9 - 9\sin^2 x$

$9(1 - \sin^2 x)$

$9\cos^2 x$

What can help us get more comfortable with math questions?
- **Do more of them**
- **Sleep on it**

TB, page 311

Your Turn

Prove that $\frac{\sin 2x}{\cos 2x + 1} = \tan x$ is an identity for all permissible values of x.

$$\frac{2\sin x\cos x}{2\cos^2 x \cancel{-1} \cancel{+1}} \qquad \frac{\sin x}{\cos x}$$

$$\frac{\cancel{2}\sin x \cancel{\cos x}}{\cancel{2}\cos^{\cancel{2}} x}$$

$$\frac{\sin x}{\cos x}$$

Remember how we get common denominators when we add or subtract fractions:

LCD = (5)(4)

$$\frac{4}{4}\cdot\frac{2}{5} + \frac{3}{4}\cdot\frac{5}{5}$$

$$= \frac{8}{20} + \frac{15}{20}$$

$$= \frac{23}{20}$$

We use the same method to simplify identities that are rational expressions. This is often helpful when we try to prove an identity.

Pre-Calc 12 – Unit 2
Page 50

5. $\frac{1}{1+\cos x} + \frac{1}{1-\cos x} = 2\csc^2 x$

$$\frac{(1-\cos x)}{(1-\cos x)}\cdot\frac{1}{(1+\cos x)} + \frac{1}{(1-\cos x)}\cdot\frac{(1+\cos x)}{(1+\cos x)} \qquad 2\cdot\frac{1}{\sin^2 x} \qquad \csc x = \frac{1}{\sin x}$$

$$\frac{1-\cancel{\cos x} + 1+\cancel{\cos x}}{(1-\cos x)(1+\cos x)} \qquad \boxed{\frac{2}{\sin^2 x}}$$

$$\frac{2}{1 + \cancel{\cos x} - \cancel{\cos x} - \cos^2 x}$$

$$\frac{2}{1-\cos^2 x}$$

$$\boxed{\frac{2}{\sin^2 x}}$$

What is a conjugate? How does multiplying by it help?

- **The conjugate of a binomial looks exactly like the binomial, except that the sign between the two terms is OPPOSITE from what it is in the original binomial.**
- **When we multiply an expression, top & bottom, by the conjugate of the binomial that is in the numerator or denominator, we are really multiplying by 1.**
- **The resulting expression can be written in a different form, by using a Pythagorean Identity.**

5. $\frac{1}{1+\cos x}+\frac{1}{1-\cos x} = 2\csc^2 x$

6. $\frac{\cos y}{1-\sin y} = \frac{1+\sin y}{\cos y}$

$\frac{\cos y}{1-\sin y}\cdot\frac{1+\sin y}{1+\sin y}$

$\frac{\cos y(1+\sin y)}{1+\cancel{\sin y}-\cancel{\sin y}-\sin^2 y}$

$\frac{\cos y(1+\sin y)}{1-\sin^2 y}$

$\frac{\cancel{\cos y}(1+\sin y)}{\cos^{\cancel{2}} y}$

$\frac{1+\sin y}{\cos y}$

7. $2\sec x = \frac{\cos x}{1+\sin x}+\frac{1+\sin x}{\cos x}$

$\frac{2}{1}\left(\frac{1}{\cos x}\right)$

$\frac{2}{\cos x}$

$\frac{\cos x}{1+\sin x}\cdot\frac{\cos x}{\cos x}+\frac{(1+\sin x)}{\cos x}\cdot\frac{(1+\sin x)}{1+\sin x}$

$\frac{\cos^2 x+(1+\sin x)(1+\sin x)}{(1+\sin x)(\cos x)}$

$\frac{\cos^2 x+1+\sin x+\sin x+\sin^2 x}{(1+\sin x)(\cos x)}$

$\frac{1+1+2\sin x}{(1+\sin x)(\cos x)} = \frac{2+2\sin x}{(1+\sin x)(\cos x)} = \frac{2(\cancel{1+\sin x})}{(\cancel{1+\sin x})(\cos x)} = \frac{2}{\cos x}$

8. $\frac{\sin x}{1+\cos x} = \frac{1-\cos x}{\sin x}$

$\frac{\sin x}{(1+\cos x)}\cdot\frac{(1-\cos x)}{(1-\cos x)}$

$\frac{\sin x(1-\cos x)}{1-\cancel{\cos x}+\cancel{\cos x}-\cos^2 x}$

$\frac{\sin x(1-\cos x)}{1-\cos^2 x}$

$\frac{\cancel{\sin x}(1-\cos x)}{\sin^{\cancel{2}} x}$

$\frac{1-\cos x}{\sin x}$