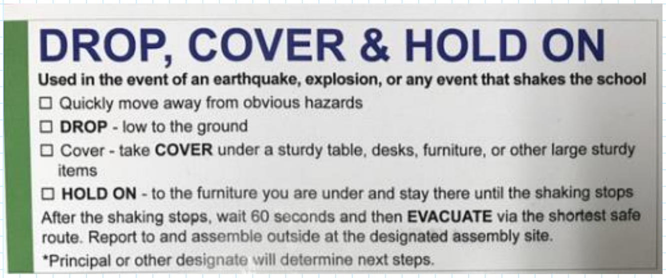
Class_13 Oct 20 Trig Identities

Sunday, October 16, 2022 4:05 PM

Tonight's Class:

- **Review of Earthquake procedures**
- **Making trig equations - partner activity**
- **6.1 Trigonometric Identities**
- **Chapter 5 Test next Tuesday, Oct 25**



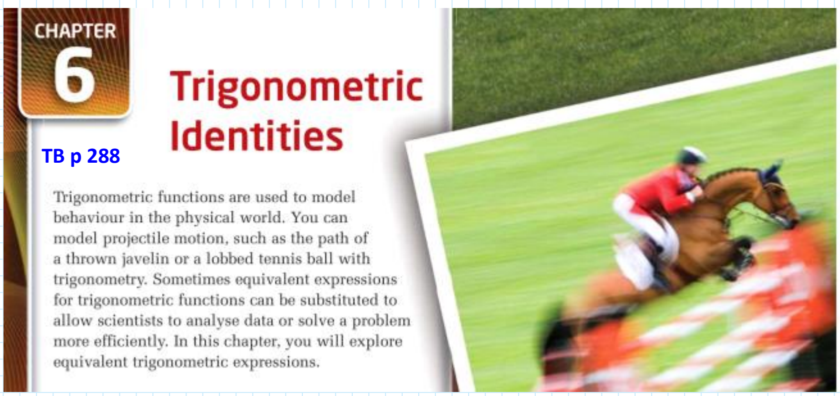
DROP, COVER & HOLD ON

Used in the event of an earthquake, explosion, or any event that shakes the school

- ☐ Quickly move away from obvious hazards
- ☐ **DROP** - low to the ground
- ☐ Cover - take **COVER** under a sturdy table, desks, furniture, or other large sturdy items
- ☐ **HOLD ON** - to the furniture you are under and stay there until the shaking stops

After the shaking stops, wait 60 seconds and then **EVACUATE** via the shortest safe route. Report to and assemble outside at the designated assembly site.

*Principal or other designate will determine next steps.



CHAPTER 6

Trigonometric Identities

TB p 288

Trigonometric functions are used to model behaviour in the physical world. You can model projectile motion, such as the path of a thrown javelin or a lobbed tennis ball with trigonometry. Sometimes equivalent expressions for trigonometric functions can be substituted to allow scientists to analyse data or solve a problem more efficiently. In this chapter, you will explore equivalent trigonometric expressions.

Chapter 6: Trigonometric Identities

6.1 Trigonometric Identities

In this chapter we talk about trigonometric ***identities***. Trigonometric identities look like trigonometric equations, but there's a difference.

identities

$2x + 4 = 2(x + 2)$

$2x+4 = 2x+4$

Always true, for any x-value

$\frac{(x^2 - 9)}{x + 3} = x - 3$

$\frac{\cancel{(x+3)}(x-3)}{\cancel{x+3}} = x-3$

$x-3 = x-3$

true all the time

EXCEPT $x = -3$

$\csc x = \frac{1}{\sin x}$

equations

$2x + 3 = 9$

$-3 \quad -3$

$2x = 6$

$x = 3$

$x^2 + 5x = -6$

$x^2+5x+6 = 0$

$(x+2)(x+3) = 0$

$x = -2$ $\quad$ $x = -3$

$\sin x = 0.53$

This can be solved:

$x = \sin^{-1}(0.53)$

$x \doteq 32^\circ$ or $180^\circ - 32^\circ = 148^\circ$

general solutions

$32^\circ + 360^\circ n,\ n \in I$

$148^\circ + 360^\circ n,\ n \in I$

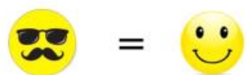
Identity - an equation that is true for **ALL** permissible values

don't try to divide by 0!!

When we are given an identity to prove, we see different expressions on the left-side and right-side of the equation. Proving the identity means we must ***change*** the expressions so that we end up with the SAME expression on both sides of the equation.

Our tools to do this are:

- algebra skills (getting common denominator, combining terms, factoring)
- basic identity substitutions



Verity (verdad) } true

We will be verifying and proving trigonometric identities.

- *Verifying* an identity means we show it *seems* true. Done by:
  - *substituting* in a specific value and confirming that, for that value, the left and right sides of the identity are equal
  - *graphing* the left and right sides of the identity separately, and confirming that the graphs are exactly the same in that window
- ***Proving*** an identity means using algebra and/or Basic Identities to change the form of one or both sides of the identity, until the two sides are exactly the same.

use radian mode

Example

Verify the given identity, for the value $x = \frac{\pi}{5}$ $\qquad \sec x = \frac{\tan x}{\sin x}$

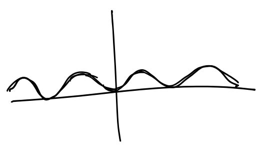
$\sec(\pi/5)$	$\frac{\tan(\pi/5)}{\sin(\pi/5)}$
$\frac{1}{\cos(\pi/5)}$	
✓ 1.236067977	1.236067977

Example

Verify graphically: $\qquad \frac{\sin^3 x}{\sin x} = \sin^2 x$

$Y_1 = \sin^3(x)/\sin(x)$

$Y_2 = \sin^2(x)$



They look like they are the same.

We will use these a lot in this chapter!

Basic Identities

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

we can re-arrange it:

$\sin^2\theta = 1 - \cos^2\theta$

$\cos^2\theta = 1 - \sin^2\theta$

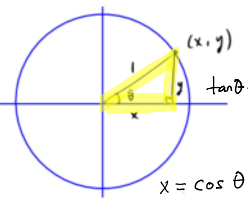
$x^2 + y^2 = 1^2$

$x^2 + y^2 = 1$

Substitute in

$(\cos\theta)^2 + (\sin\theta)^2 = 1$

$\cos^2\theta + \sin^2\theta = 1$



$\tan\theta = \frac{opp}{adj} = \frac{y}{x}$

$x = \cos\theta$

$y = \sin\theta$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

Reciprocal Identities

$\csc\theta = \dfrac{1}{\sin\theta}$ $\qquad$ $\sec\theta = \dfrac{1}{\cos\theta}$ $\qquad$ $\cot\theta = \dfrac{1}{\tan\theta}$

Quotient Identities

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\qquad$ $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Addition Identities

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\qquad$ $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$ $\qquad$ $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

Double Angle Identities

$\sin(2\theta) = 2\sin\theta\cos\theta$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$\cos(2\theta) = 2\cos^2\theta - 1$

$\cos(2\theta) = 1 - 2\sin^2\theta$

$\tan(2\theta) = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

Practice Using Basic Identities

Match the expressions on the left with those on the right-hand column. Put the letter of the expression that matches in the blank provided. Each gets used exactly once.

G	1. $\frac{\sin B}{\cos B}$	A. 1
H	2. $\frac{1}{\cos B}$	B. $\sin^2 B$
C	3. $\csc^2 B$	C. $\cot^2 B + 1$
A	4. $\sin^2 B + \cos^2 B = 1$	D. $\cos^2 B$
J	5. $\cot B \sin B = \frac{\cos B}{\cancel{\sin B}} \cdot \cancel{\sin B} = \cos B$	E. $1 + \tan^2 B$
F	6. $\frac{\cos B}{\sin B}$	F. $\cot B$
B	7. $1 - \cos^2 B$	G. $\tan B$
K	8. $\frac{\cos^2 B}{1+\sin B} = \frac{1-\sin^2 B}{1+\sin B} = \frac{\cancel{(1+\sin B)}(1-\sin B)}{\cancel{1+\sin B}} = 1-\sin B$	H. $\sec B$
E	9. $\sec^2 B$	I. $\csc B$
I	10. $\frac{1}{\sin B}$	J. $\cos B$
L	11. $\frac{\cos B}{\cot B} = \frac{\cos B}{\frac{\cos B}{\sin B}} = \cos B \div \frac{\cos B}{\sin B} = \frac{\cancel{\cos B}}{1} \cdot \frac{\sin B}{\cancel{\cos B}} = \sin B$	K. $1 - \sin B$
D	12. $1 - \sin^2 B$ (re-arrangements)	L. $\sin B$

$\sin^2\theta + \cos^2\theta = 1$

perfect square − perfect square

$x^2 - 9$
$= (x+3)(x-3)$

difference of squares

$4 - y^2$
$= (y+2)(y-2)$
$= y^2 - \cancel{2y} + \cancel{2y} - 4$
$= y^2 - 4$

$(2-y)(2+y)$
$= 4 + \cancel{2y} - \cancel{2y} - y^2$

$(-y+2)(y+2)$
$= -y^2 - \cancel{2y} + \cancel{2y} + 4$
$= -y^2 + 4$

Factor:

1) $\sin^2\theta - 1 = (\sin\theta + 1)(\sin\theta - 1)$

2) $1 - \cot^2\theta = (1 + \cot\theta)(1 - \cot\theta)$

$\sin^2\theta + \cos^2\theta = 1$

More Practice

Simplify each expression below. Look for substitutions you can make, using basic identities. Your final answer should contain no more than one trigonometric function.

1. $\frac{\cos^2\theta}{\sin^2\theta} = \left(\frac{\cos\theta}{\sin\theta}\right)^2 = (\cot\theta)^2 = \cot^2\theta$

$\frac{\cos\theta}{\sin\theta} = \cot\theta$

2. $\tan\theta\sec\theta\cos\theta = \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\cancel{\cos\theta}} \cdot \frac{\cancel{\cos\theta}}{1} = \frac{\sin\theta}{\cos\theta} = \tan\theta$

3. $1 - \cos^2\theta = \sin^2\theta$

$1 - \cos^2\theta = \cancel{\sin^2\theta + \cos^2\theta} - \cancel{\cos^2\theta} = \sin^2\theta$

$\sin^2\theta + \cos^2\theta = 1$

4. $\cos^2\theta - 1$
$= \cos^2\theta - (\sin^2\theta + \cos^2\theta)$
$= \cancel{\cos^2\theta} - \sin^2\theta - \cancel{\cos^2\theta}$
$= -\sin^2\theta$

5. $1 + \tan^2\theta = \sec^2\theta$

6. $\underbrace{\sin^2\theta + \cos^2\theta}_{} + 1$
$= 1 + 1$
$= 2$

$\frac{(2)(5)}{(2)(10)} = \frac{10}{20} = \frac{1}{2}$

$\frac{\cancel{(2)}(5)}{} = \frac{5}{} = \frac{1}{}$

$= \cancel{\cos^2\theta} - \sin^2\theta - \cancel{\cos^2\theta}$

$= -\sin^2\theta$

$= 2$

7. $\csc^2\theta - \cot^2\theta$ $= 1$

$1 + \cancel{\cot^2\theta} = \csc^2\theta$
$-\cancel{\cot^2\theta}$ $-\cot^2\theta$
$1 = \csc^2\theta - \cot^2\theta$

8. $\sin^2\theta + \cos^2\theta + \tan^2\theta$

$= 1 + \tan^2\theta$

$= \sec^2\theta$

9. $\dfrac{\sin^2\theta + \sin\theta}{\cos\theta + \cos\theta\sin\theta}$

$= \dfrac{\sin\theta\,(\cancel{\sin\theta + 1})}{\cos\theta\,(\cancel{1 + \sin\theta})}$

$= \dfrac{\sin\theta}{\cos\theta} = \tan\theta$

10. $\dfrac{\sqrt{1-\sin^2 x}}{\sqrt{1+\tan^2 x}}$

$= \dfrac{\sqrt{\cos^2 x}}{\sqrt{\sec^2 x}}$

$= \dfrac{\cos x}{\sec x}$

$= \dfrac{\cos x}{\frac{1}{\cos x}}$

$= \cos x \div \dfrac{1}{\cos x}$

$= \dfrac{\cos x}{1} \cdot \dfrac{\cos x}{1} = \cos^2 x$

11. $1 - \sec^2 x$

$= -\tan^2 x$

12. $\sec^2 x - 1$

$\tan^2 x$ ✓

$1 + \tan^2\theta = \sec^2\theta$
-1 -1
$\tan^2\theta = \sec^2\theta - 1$

$\dfrac{(2)(5)}{(2)(10)} = \dfrac{10}{20} = \dfrac{1}{2}$

$\dfrac{\cancel{(2)}(5)}{\cancel{(2)}(10)} = \dfrac{5}{10} = \dfrac{1}{2}$

$\dfrac{2+5}{2+10} = \dfrac{7}{12}$

$\cancel{\dfrac{\cancel{2}+5}{\cancel{2}+10} = \dfrac{5}{10}}$

$\sqrt{9+16} = \sqrt{25} = 5$

$\cancel{3+4}$

$\sqrt{x^2+y^2} \neq x+y$

6.0 Algebra Skills Used in Chapter 6

Multiplying Trigonometric Expressions

1. $\sin x(2\sin x - 1)$ $= 2\sin^2 x - \sin x$

2. $(\cos x + 2)(\cos x - 7)$ $= \cos^2 x - 7\cos x + 2\cos x - 14$

$= \cos^2 x - 5\cos x - 14$

3. $(\cos x - 3)^2$ $= (\cos x - 3)(\cos x - 3)$

$= \cos^2 x - 3\cos x - 3\cos x + 9$

$= \cos^2 x - 6\cos x + 9$

Factoring Trigonometric Expressions

Greatest Common Factor

1. $\sin^2 x - 3\sin x$ $= \sin x\,(\sin x - 3)$

2. $5\tan^2 x + 15\tan x$ $= 5\tan x\,(\tan x + 3)$

Difference of Perfect Squares

1. $\sin^2 x - 1$ $= (\sin x + 1)(\sin x - 1)$

2. $1 - \tan^2 x$ $= (1 + \tan x)(1 - \tan x)$

Trinomials

1. $2\cos^2 x + \cos x - 1$

$(2\cos x - 1)(\cos x + 1)$

check: $2\cos^2 x + 2\cos x - \cos x - 1$
$= 2\cos^2 x + \cos x - 1$

$2b^2 + 1b - 1$
$= (2b - 1)(b + 1)$

$AC = (2)(-1)$
$= -2$
mult to -2
add to 1
$-1, 2$

$\underline{2b^2 - 1b} + \underline{2b - 1}$
$= b(2b-1) + 1(2b-1)$
$= (2b-1)(b+1)$

2. $3\sin^2 x + 2\sin x - 1$

$= 3\sin^2 x + 3\sin x - \sin x - 1$
$= 3\sin x(\sin x + 1) - 1(\sin x + 1)$
$= (\sin x + 1)(3\sin x - 1)$

$AC = -3$
sum $= 2$
prod $= -3$
$3, -1$

Adding/Subtracting Trigonometric Terms

We can only add ***like terms***

- Terms must contain the same angle
- Terms must use the same trigonometric function

Which of these terms can be combined?

$\cancel{4\sin x + 3\cos x}$ $\qquad$ $\cancel{2\sin x + 5\sin 2x}$

$3\sin x + 4\sin x$ $\qquad$ $2\sin^2 \cancel{x + 4}\sin x$

$= 7\sin x$

Errors to Avoid

Omitting the angle

$\cos + 4\cos$ $\qquad$ These terms contain no angle - they don't mean anything!

$\cos x + 4\cos x = 5\cos x$

Incorrect cancelling

You can never "cancel" the angle, or any part of the angle, in a trigonometric expression.

$\dfrac{\cos x}{x}$ $\qquad$ $\dfrac{\tan 2x}{2}$

This one <u>can</u> be reduced:

$\dfrac{\cancel{3}\sin x}{\cancel{3}x} = \dfrac{\sin x}{x}$

More incorrect cancelling
You can NEVER cancel just a portion of a factor.

$$\frac{\cos x + 1}{\cos x} =$$

You **CANNOT** cancel the "$\cos x$" on the top with the "$\cos x$" on the bottom!

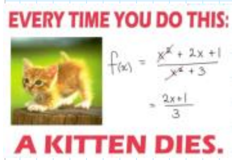
$$\frac{(\cancel{\cos x+1})(\cos x - 1)}{(\cancel{\cos x+1})} = \cos x - 1$$

You **CAN** cancel the "$\cos x + 1$" factors

To CORRECTLY simplify a rational expression, factor it completely.
If the numerator and the denominator contain the same factor, you can reduce.

For example: $$\frac{\sin^2 x + 8\sin x + 12}{\sin^2 x + \sin x - 30} = \frac{(\cancel{\sin x + 6})(\sin x + 2)}{(\sin x - 5)(\cancel{\sin x + 6})} = \boxed{\frac{\sin x + 2}{\sin x - 5}}$$

Notice that we **cannot** reduce $\frac{\sin x + 2}{\sin x - 5}$ by canceling the $\sin x$'s, because $\sin x$ is NOT a factor of the numerator and the denominator. $\frac{\sin x + 2}{\sin x - 5} \neq \frac{2}{-5}$



Distributing when you can't:

$\cos(x + y)$ This does **NOT** equal $\cos x + \cos y$!
We are not multiplying "cos" with $(x + y)$.

What this expression DOES mean is the cosine of the angle "$x + y$"

For example, consider what happens if $x = 15^\circ$ and $y = 28^\circ$

$\cos(15^\circ + 28^\circ) = \cos 43^\circ = 0.73135$

$\cos(15^\circ) + \cos(28^\circ) = 1.84887$

not equal. Can't do this!

This shows us that $\cos(x + y) \neq \cos x + \cos y$

Practice
(6.1) TB p 296: 3, 4, 5a, 6b, 10, 14-16 **Don't have to identify NPVs

Important Things

- **Chapter 5 hand-in assignment due Tuesday, Oct 25.**
  Understanding the concepts in this assignment can help you prepare for the test.

- **Chapter 5 Test next class - Tuesday, Oct 25**
  Includes a NO-calculator section
  - **Given a sinusoidal equation, be able to sketch it without using technology.**
  - **Given a sinusoidal graph, be able to figure out its equation without using technology.**
  - **Know how to find period, phase shift, amplitude, vertical displacement, spacing between key points, coordinates of key points.**
  - **Know the period and domain for tangent, and how to adjust them if there's a horizontal stretch.**
  - **Know how to algebraically solve an equation similar to the example in the notes, page 36. (Or, like #12b in the hand-in assignment)**
  - **Understand the method for graphically solving trigonometric equations**
  - **Be able to create a circular motion equation and solve it. (similar to TB p 279, #19)**