Class_14 Feb 23 - More Quadratics

Wednesday, February 15, 2023   3:03 PM

Jody Primeau 1:55 PM

LEC Staff Hello Everyone! I plan to visit classes next Wednesday and Thursday to talk to students about Scholarships and Graduation. Probably about 10 minutes per class. Mornings, I will be there sometime between 11:00 and 12:30, evenings between 6:30 and 7:30. If these times do not work well for you, please let me know. I am also attaching a PowerPoint. If you could please have that available for my visit. Thanks! Jody

Tonight's Class:

- **Scholarships/Graduation presentation, counselor**
- **#thatwellness thing - Wednesday, March 1**

- **Any questions from last class? (4.4-4.6)**
- **Quadratic Functions Group Activity**
- **Working through section 4.7**
    - **Modelling Quadratics (Max/Min questions)**
- **Work on practice questions from worktext**
- **Chapter 4 Test (4.1-4.7) on Tuesday, Feb 28**

p307 #8b

$y = -\frac{1}{4}(x+8)^2 - 1$

Vertex form

$y = a(x-h)^2 + k$
vertex: (h, k)

a) vertex $(-8, -1)$

domain $x \in \mathbb{R}$

range $y \leq -1$

direction of opening – downward ($-\frac{1}{4}$ tells us)

axis symmetry equation $\boxed{x = -8}$

intercepts

x-intercepts – none

y-intercept, process: let $x = 0$

$y = -\frac{1}{4}(x+8)^2 - 1$

$y = -\frac{1}{4}(0+8)^2 - 1$

$y = -\frac{1}{4}(8)^2 - 1$

$y = -\frac{1}{4}(64) - 1$

$y = -\frac{64}{4} - 1$

$y = -16 - 1$

$y = -17$

$(0, -17)$

x	y
-12	-5
-8	-1
-4	-5

$y = -\frac{1}{4}(x+8)^2 - 1$

$= -\frac{1}{4}(-4+8)^2 - 1$

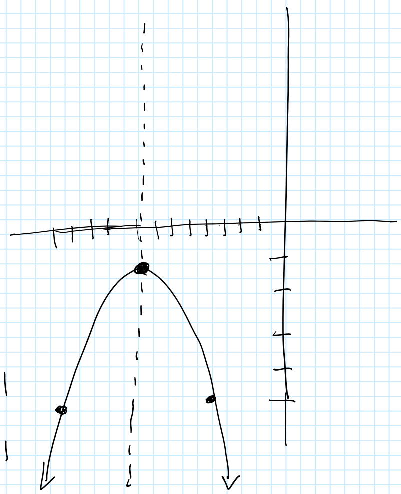
$= -\frac{1}{4}(4)^2 - 1$

$= -\frac{1}{4}(16) - 1$

$= -\frac{16}{4} - 1$

$= -4 - 1$

$= -5$



p317 8a)

$y = 2x^2 + 5x - 3$

coordinate vertex

1) factor constant out

p311 8a)

$y = 2x^2 + 5x - 3$

$y = 2\left(x^2 + \frac{5}{2}x + \frac{25}{16} - \frac{25}{16}\right) - 3$

$y = 2\left(x^2 + \frac{5}{2}x + \frac{25}{16}\right) \underbrace{- \frac{50}{16} - 3}$

$y = 2\left(x + \frac{5}{4}\right)^2 - \frac{50}{16} - 3$

$y = 2\left(x + \frac{5}{4}\right)^2 - \frac{25}{8} - \frac{3}{1} \cdot \frac{8}{8}$

$y = 2\left(x + \frac{5}{4}\right)^2 - \frac{25}{8} - \frac{24}{8}$

$y = 2\left(x + \frac{5}{4}\right)^2 - \frac{49}{8}$

Vertex: $\left(-\frac{5}{4}, -\frac{49}{8}\right)$

$\left(\frac{b}{2}\right)^2$

$\left(\cancel{\frac{\frac{5}{2}}{2}}\right)^2$

$\left[\frac{1}{2}\left(\frac{5}{2}\right)\right]^2$

$= \left(\frac{5}{4}\right)^2$

coordinate vertex

1) factor constant out of the first two term

2) "b" value, $\left(\frac{b}{2}\right)^2$ add in, and subtract off.

3) remove the last term in the brackets take it OUT of the bracket.

4) write the trinomial as $\left(x + \frac{b}{2}\right)^2$ & simplify the constant

OR $\left(x - \frac{b}{2}\right)^2$

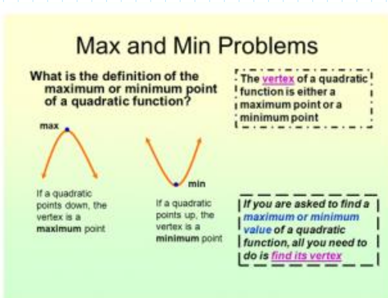
this sign matches the first sign in the trinomial

4.7 Modelling Problems with Quadratic Functions

Focus: write a quadratic function to model a problem, then solve the problem

In this section, you will solve application problems using quadratic equations.

These are called Max/Min problems because the quadratic equation is used to model the application and the vertex is used to answer questions about the maximum or minimum of the quadratic.

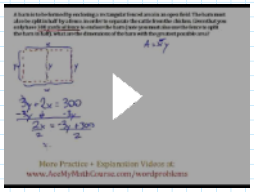


Steps:
1. When you have a word problem in this section, you will create two equations.
2. Use substitution to form one equation. This equation becomes the quadratic function that represents the word problem.
3. Convert the quadratic from general (trinomial) form to vertex form.
4. Use the vertex to answer the Max/Min question or use other operations with the equation to answer other questions.

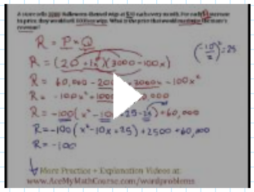
[Maximum Area Word Problem - Solved by Completing the Square]

[Maximizing Revenue Word Problem (Completing the Square): Straightforward Worked Example!]

[Maximum Area Word Problem - Solved by Completing the Square]



[Maximizing Revenue Word Problem (Completing the Square): Straightforward Worked Example!]



Page 340, Ex 1

Example 1 **Determining Maximum or Minimum Related to Operations with Numbers**

Two numbers have a sum of 20. Does the sum of their squares have a maximum or a minimum value? Determine this value and the two numbers.

sum = add to
product = multiply
difference = subtracted
quotient = division

let x = one of the numbers
let y = the other number

$\boxed{x + y = 20}$

$x^2 + y^2 = \text{value}$ min/max I don't know yet

1) isolate the "y" term in the first equation

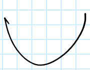
$x + y = 20$
$-x \qquad -x$
$y = 20 - x$

2) substitute that expression into the OTHER equation

$x^2 + y^2 = \text{value}$
$x^2 + (20 - x)^2 = \text{value}$

3) simplify the equation

$x^2 + (20-x)(20-x)^2 =$ sum value
$x^2 + 400 - 20x - 20x + x^2 =$ (sum value)
$2x^2 - 40x + 400 =$ minimum value sum



4) change to vertex form, to easily find the minimum

sum $= 2x^2 - 40x + 400$
$= 2(x^2 - 20x + 100 - 100) + 400$
$= 2(x^2 - 20x + 100) - 200 + 400$
$= 2(x - 10)^2 + 200$

$\left(\frac{-20}{2}\right)^2$
$= (-10)^2$
$= 100$

Vertex $= (10, 200)$
$x = 10$

When $x = 10$, we get a minimum
$y = 10$ also

Minimum value = 200
check: $10^2 + 10^2 = 200$

Page 340, Check your understanding, #1

Check Your Understanding

1. Two numbers have a difference of 18. Does their product have a maximum or a minimum value? Determine this value and the two numbers.

Let x = one number
let y = other number

$\boxed{x - y = 18}$

1) isolate y:

$x - y = 18$
$-x \qquad -x$
$-y = 18 - x$
$\div(-1) \quad \div(-1)$

and the two numbers.

$x - y = 18$

1) isolate y:

$x - y = 18$
$-x \quad\quad -x$
$-y = 18 - x$
$\div -1 \quad \div -1$
$y = -18 + x$
or $y = x - 18$

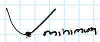
$xy = \text{product}$
$x(x-18) = \text{product}$

$\text{Product} = x(x-18)$
$P = x^2 - 18x$

Now, we put it in vertex form

$\left(\frac{b}{2}\right)^2 = \left(\frac{-18}{2}\right)^2 = (-9)^2 = 81$

$= (x^2 - 18x + 81 - 81)$
$= (x^2 - 18x + 81) - 81$
$= (x-9)^2 - 81$



$V = (9, -81)$

minimum value

What are the numbers?

$x = 9$
$y = x - 18$
$y = 9 - 18$
$y = -9$

Page 343, #5

RM, US

5. The sum of the length and width of a rectangle is 20 cm. Determine the dimensions that produce the maximum area.

let l = length
let w = width

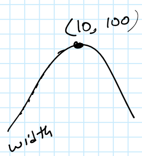
Area = lw
maximize

$l + w = 20$
$-w \quad\quad -w$
$l = 20 - w$

$A = lw$
$A = (20-w)(w)$
$A = 20w - w^2$
$A = -w^2 + 20w$
$A = -1(w^2 - 20w + 100 - 100)$
$A = -1(w^2 - 20w + 100) + 100$
$A = -1(w-10)^2 + 100$

$w = 10$ maximum area

$\left(\frac{b}{2}\right)^2$
$\left(\frac{-20}{2}\right)^2$
$= (-10)^2$
$= 100$

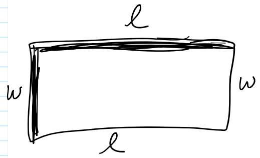


$l = 20 - w$
$l = 20 - 10$
$l = 10$

Dimensions:
$l = 10$ cm
$w = 10$ cm
Maximum area = $100\ \text{cm}^2$

Try p344, #7

7. A rectangular play area is to be bounded by 120 m of fencing. Determine the maximum area and the dimensions of this rectangle.



$2w + 2l = 120$
$\div 2$

$w + l = 60$
$-w \quad\quad -w$
$l = 60 - w$

isolate a variable, and substitute it into the other equation

Area = lw

$A = (60-w)(w)$

$A = 60w - w^2$

$A = -w^2 + 60w$

Now, change to vertex form.

$A = -(w^2 - 60w + 900 - 900)$

$\left(\frac{-60}{2}\right)^2 = (-30)^2 = 900$

$A = -(w^2 - 60w + 900) + 900$

$A = -(w-30)^2 + 900$

$w = 30$ m and $l = 30$ m

$900\ \text{m}^2$

$\boxed{w = 30\text{ m}}$ and $\boxed{l = 30\text{ m}}$

$\boxed{900\text{ m}^2 \text{ maximum area}}$

Page 341, Example 2 (Revenue Problem)

Example 2 **Modelling a Problem for Maximizing Profit**

A student parking pass costs \$20. At this price, 150 students will purchase passes. For every \$5 increase in price, 20 fewer students will purchase passes.

a) What is the price of a parking pass that will maximize the revenue?

b) What is the maximum revenue?

base

Revenue = (price)(number sold)

$R = pn$

$\boxed{x = \text{the number of increases in price}}$

price $= 20 + 5x$

number sold $= 150 - 20x$

$$R = (p)(n)$$
$$R = (20 + 5x)(150 - 20x)$$
$$R = 3000 - 400x + 750x - 100x^2$$
$$R = -100x^2 + 350x + 3000$$
$$R = -100(x^2 - 3.5x + 3.0625 - 3.0625) + 3000$$
$$R = -100(x^2 - 3.5x + 3.0625) + 306.25 + 3000$$
$$R = -100(x - 1.75)^2 + 3306.25$$

$\left(\frac{b}{2}\right)^2$

$= \left(\frac{3.5}{2}\right)^2$

$= 3.0625$

$(1.75)^2$

$x =$ number of \$5 increases, round to nearest whole number $\Rightarrow$ $x = 2$ increases

price $= \$20 + 5x$
$= 20 + 5(2)$
$= 20 + 10 = \boxed{\$30}$

number sold $= 150 - 20x$
$= 150 - 20(2)$
$= 150 - 40$
$= \boxed{110 \text{ sold}}$

maximum revenue $= (\$30)(110)$
$= \$3300$

For next class

- Finish worktext questions for 4.1, 4.3-4.7
- Complete the Chapter 4 Hand-in, #1-10 only, due next class
- Prepare for the Chapter 4 Test, next class
  - Test will cover 4.1, 4.3-4.7 only
  - Make sure you know how to change equations into vertex form