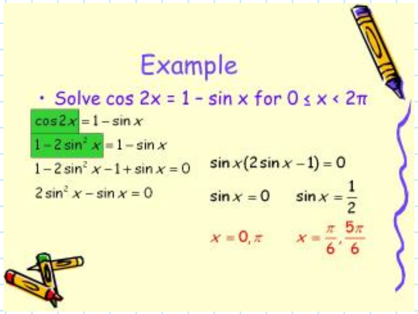
Class_14 May 29 Solving Trig Equations with Identities

Plan For Today:

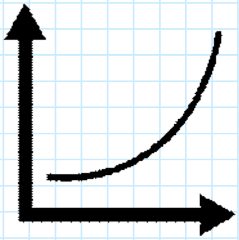
1. Question about anything from last week? 6.1-6.3
   - **DO TEST 4 ON 6.1-6.3**
     - *Approximately 60-75 minutes?*
     - *After test try some intro stuff for 6.4*
     - *Will be marked for tomorrow's class*
2. Finish Chapter 6: Trig Identities & Solving Equations
   - ✓ 6.1: Reciprocal, Quotient & Pythagorean Identities
     - ✓ (Simplifying & Proving)
   - ✓ 6.2: Sum, Difference & Double-Angle Identities
   - ✓ 6.3: Proving Identities
   - **6.4: Solve Trig Equations with Identities**



3. Work on practice questions from Textbook

   Page 320:
   #1bc, 2bc, 3abc, 4, 5, 10, 14

4. Possibly start Chapter 7: Exponential Functions
   - **7.1: Characteristics of Exponential Functions**
   - 7.2: Transformations of Exponential Functions
   - 7.3: Solving Exponential Equations



5. Work on practice questions from Textbook

   Page 342:
   #1, 3-8

Plan Going Forward:

1. Finish working through textbook question from 6.4 and finish working on Chapter 6 Assignment.
   - **CHAPTER 6 ASSIGNMENT DUE TOMORROW, TUESDAY, MAY 30TH**

2. You will go over 7.1-7.2 Exponential functions and graphing transformations tomorrow. Please review transformations.

Please let us know if you have any questions or concerns about your progress in this course.
The notes from today will be posted at egolfmath.weebly.com after class.

Anurita Dhiman = adhiman@sd35.bc.ca
Susana Egolf = segolf@sd35.bc.ca

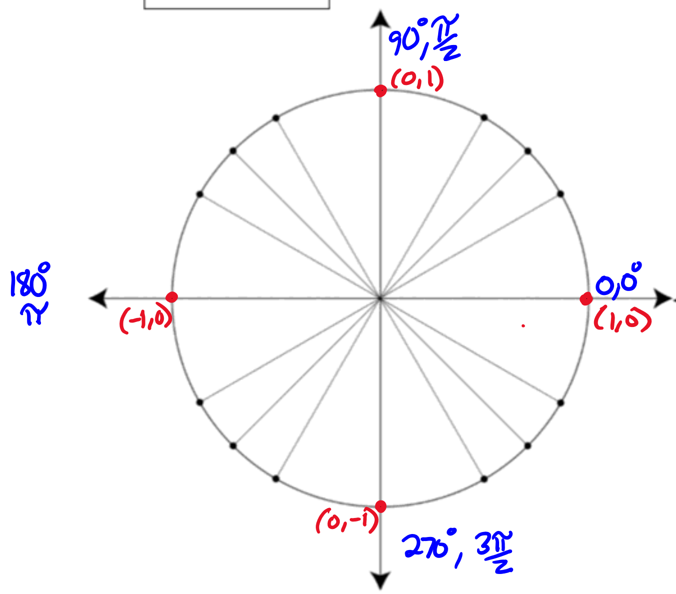
Review

4.4 Review for 6.4 lesson May2023

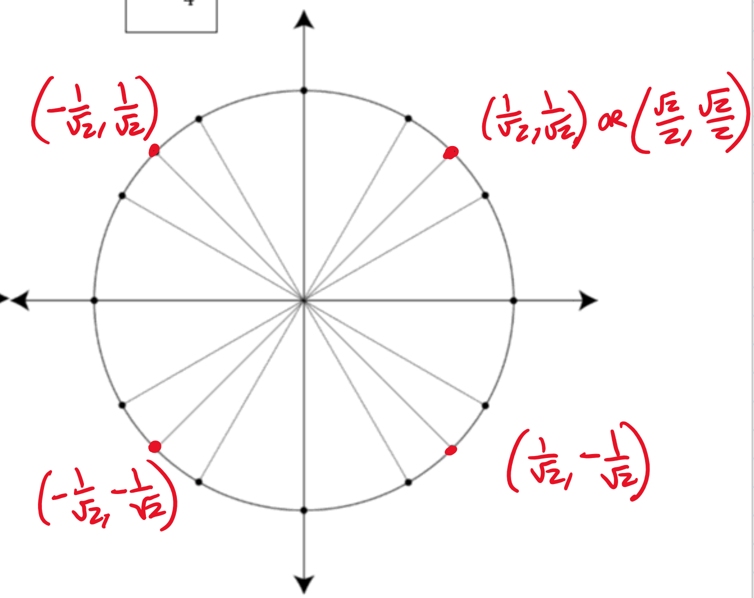
Section 4.4 Trigonometric Equations Review to Prepare for 6.4

1. Show your special angles and exact coordinates for each of the following:

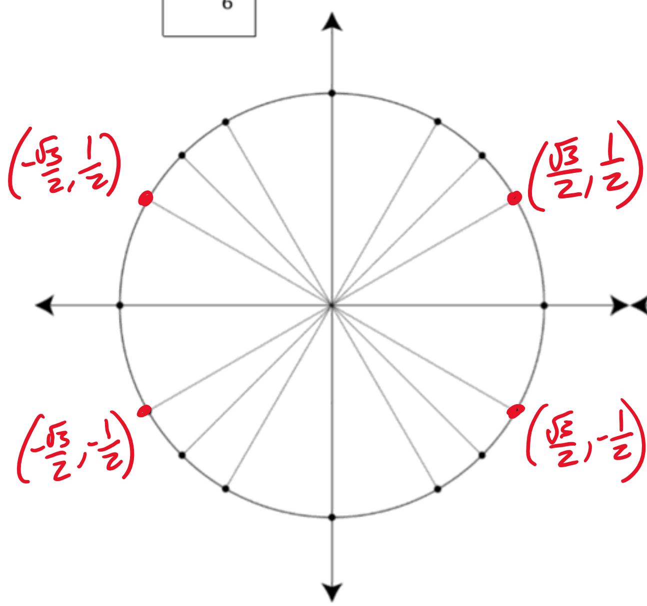
Quadrant Angles



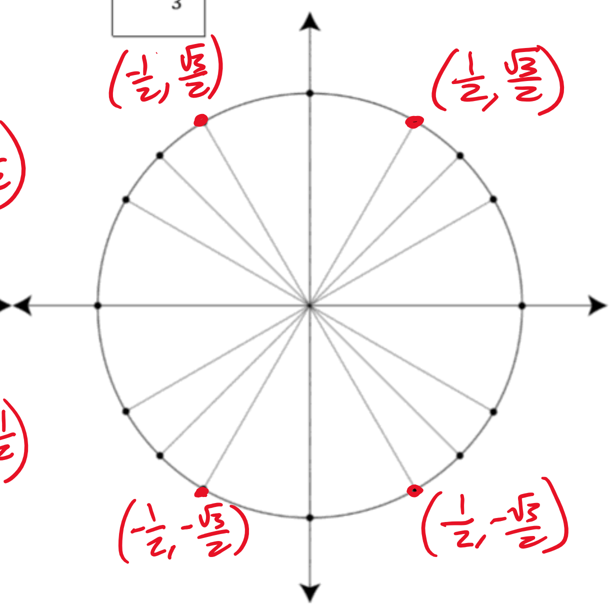
$45°, \frac{\pi}{4}$



$30°, \frac{\pi}{6}$



$60°, \frac{\pi}{3}$



2. Solve each of the following algebraically providing exact answers where possible.

a) $\sin x - 1 = 0$ for $0 \le x < 2\pi$

$\hookrightarrow +1$

$\sin x = 1$

$\downarrow$

y-coord = 1

$x = \frac{\pi}{2}$

b) $2\cos x = 1$ for $0^\circ \le x < 360^\circ$

$\cos x = \frac{1}{2}$

$\downarrow$

x-coord $+\frac{1}{2}$

$QI = x = 60^\circ$

$QIV = x = 300^\circ$

c) $4\sin^2\theta - 4 = -1$ for $0 \le \theta < 2\pi$

$\hookrightarrow +4$

$4\sin^2\theta = 3$ (÷4)

$\sqrt{\sin^2\theta} = \sqrt{\frac{3}{4}}$

$\sin\theta = \pm\frac{\sqrt{3}}{2}$

$\downarrow$

y-coord $= \pm\frac{\sqrt{3}}{2}$

QI $\theta = \frac{\pi}{3}$

QII $\theta = \frac{2\pi}{3}$

QIII $\theta = \frac{4\pi}{3}$

QIV $\theta = \frac{5\pi}{3}$

d) $\sin^2\theta = \sin\theta\cos\theta$ for $0^\circ \le \theta < 360^\circ$

$\sin^2\theta - \sin\theta\cos\theta = 0$

$\sin\theta(\sin\theta - \cos\theta) = 0$

$\sin\theta = 0$ $\quad$ $\sin\theta - \cos\theta = 0$

$\theta = 0^\circ, 180^\circ$

$\sin\theta = \cos\theta$

y-coor $\frac{\sqrt{2}}{2}$ $\quad$ x-coord $\frac{\sqrt{2}}{2}$

QI + + 45°

QIII − − 225°

$\theta = 45^\circ, 225^\circ$

$\boxed{\theta = 0^\circ, 45^\circ, 180^\circ, 225^\circ}$

e) $2\cos^2\theta - 3\cos\theta + 1 = 0$ for $0 \le x < 2\pi$

$2x^2 - 3x + 1 = 0$

AC = 2

−2, −1 = −3

$2\cos^2\theta - 2\cos\theta - \cos\theta + 1 = 0$

$2\cos\theta(\cos\theta - 1) - (\cos\theta - 1) = 0$

$(2\cos\theta - 1)(\cos\theta - 1) = 0$

$\cos\theta = \frac{1}{2}$ $\quad$ $\cos\theta = 1$

x-coord

$QI = \frac{\pi}{3}$ $\quad$ $\theta = 0$

$QIV = \frac{5\pi}{3}$

$\boxed{\theta = 0, \frac{\pi}{3}, \frac{5\pi}{3}}$

f) $\sin^2\theta + \sin\theta - 2 = 0$ for $0^\circ \le x < 360^\circ$

+2, −1 = 1

$(\sin\theta + 2)(\sin\theta - 1) = 0$

$\sin\theta = -2$ $\quad$ $\sin\theta = 1$

not special $\quad$ $\theta = 90^\circ$

$\theta = \sin^{-1}(-2)$

$\theta =$ no solution

$\boxed{\theta = 90^\circ}$

6.4 Solving Trig Equations (with and without identities)

Steps:

1. Isolate the trig ratio

2. Determine where the ratio is positive or negative on the unit circle

3. Determine the reference (special) angle that matches the ratio

- ***If it is not a special angle exact value, use inverse of the trig ratio to solve for the first angle.***

4. Determine the angles in standard position to solve the equation

Solving a Trigonometric Equation by Linear Methods

Example Solve $2 \sin x - 1 = 0$ over the interval $[0, 2\pi)$.

Analytic Solution Since this equation involves the first power of $\sin x$, it is linear in $\sin x$.

$$2\sin x - 1 = 0$$
$$2\sin x = 1$$
$$\sin x = \frac{1}{2}$$

Two x values that satisfy $\sin x = ½$ for $0 \le x < 2\pi$ are $x = \frac{\pi}{6}$ and $x = \frac{5\pi}{6}$

However, if we do not restrict the domain there will be an infinite amount of answers since:

$$x = \frac{\pi}{6} + 2k\pi \text{ and } x = \frac{5\pi}{6} + 2k\pi$$

for k any integer.

Example: Linear Method

- Solve 2 $\cos^2 x - 1 = 0$
- Solution: First, solve for cos x on the unit circle.

$$2\cos^2 x - 1 = 0$$
$$2\cos^2 x = 1$$
$$\cos^2 x = \frac{1}{2}$$
$$\cos x = \pm\sqrt{\frac{1}{2}}$$
$$\cos x = \pm\frac{\sqrt{2}}{2}$$

$$x = 45^\circ, 135^\circ, 225^\circ, 315^\circ$$
$$or\ \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

Slide 6-4

Equations Solvable by Factoring

Example Solve sin x tan x = sin x.

Solution

$$\sin x \tan x = \sin x$$
$$\sin x \tan x - \sin x = 0$$
$$\sin x(\tan x - 1) = 0$$

$$\sin x = 0 \quad \text{or} \quad \tan x - 1 = 0$$
$$\tan x = 1$$

$$x = 0 \quad \text{or} \quad x = \pi \qquad x = \frac{\pi}{4}, \quad \text{or} \quad x = \frac{5\pi}{4}.$$

The solutions are $x = k\pi$ and $x = \frac{\pi}{4} + k\pi$ for any integer k

Caution Avoid dividing both sides by sin x. The two solutions that make sin x = 0 would not appear.

$$2\sin^2 x + \sin x - 1 = 0$$

$$(2\sin x - 1)(\sin x + 1) = 0 \quad \text{(factor)}$$

$$2\sin x - 1 = 0 \qquad \text{or} \qquad \sin x + 1 = 0$$

$$\sin x = \frac{1}{2} \qquad\qquad \sin x = -1$$

$$x = \frac{\pi}{6} \text{ or } \frac{5\pi}{6} \qquad\qquad x = \frac{3\pi}{2}$$

$$x = 30^\circ,\ 150^\circ \qquad\qquad x = 270^\circ$$

Solving an Equation Using a Double-Number Identity

Example Solve cos 2*x* = cos *x* over the interval [0, 2π).

Analytic Solution

$$\cos 2x = \cos x$$

$$2\cos^2 x - 1 = \cos x$$

$$2\cos^2 x - \cos x - 1 = 0$$

$$(2\cos x + 1)(\cos x - 1) = 0$$

$$2\cos x + 1 = 0 \qquad \text{or} \qquad \cos x - 1 = 0$$

$$\cos x = -\frac{1}{2} \qquad \text{or} \qquad \cos x = 1$$

Solving each equation yields the solution set $\{0, \frac{2\pi}{3}, \frac{4\pi}{3}\}$.

Example Solve $4 \sin x \cos x = \sqrt{3}$ over the interval $[0, 2\pi)$.

Solution

$$4\sin x\cos x = \sqrt{3}$$

$$2(2\sin x\cos x) = \sqrt{3}$$

$$2\sin 2x = \sqrt{3}$$ Since $2 \sin x \cos x = \sin 2x$.

$$\sin 2x = \frac{\sqrt{3}}{2}$$

From the given domain for x, $0 \le x < 2\pi$, the domain for $2x$ is $0 \le 2x < 4\pi$.

$$\sin 2x = \frac{\sqrt{3}}{2}$$

$$2x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}$$

$$x = \frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}$$

6.4 Solving Trigonometric Equations Using Identities

Some trigonometric equations cannot be solved until they are re-written in a different form, using trigonometric identities.

Example

Algebraically solve this equation, giving the ***general solution***, in radian measure.

$\theta + 360°n, n \in I$

$\theta + 2\pi n, n \in I$

$$2\sin x = 9 - 4\csc x$$

$\frac{1}{\sin x}$

When there is a fraction, multiply the whole equation by the common denominator

$$2\sin x = 9 - 4\left(\frac{1}{\sin x}\right)$$

$$\sin x\left[2\sin x = 9 - \frac{4}{\sin x}\right]$$

$$2\sin^2 x = 9\sin x - 4$$

AC
8
$-8, -1$ $= -9$

$$2\sin^2 x - 9\sin x + 4 = 0$$

$$2\sin^2 x - 8\sin x - \sin x + 4 = 0$$

$$2\sin x(\sin x - 4) - (\sin x - 4) = 0$$

$$(2\sin x - 1)(\sin x - 4) = 0$$

$\sin x = \frac{1}{2}$

+ y-coord

QI + QII

$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}}$

$\sin x = 4$

$x = \sin^{-1}(4)$

$x =$ no solution

general solutions $= \boxed{\left.\begin{array}{l} x = \frac{\pi}{6} + 2\pi n \\ x = \frac{5\pi}{6} + 2\pi n \end{array}\right\} n \in I}$

Example

Algebraically solve this equation

$$\cos 2x + \cos x = -1 \text{ , for } 0° \le x < 360°$$

3 choices
= choose easiest to keep trig ratios the same

$$2\cos^2 x - 1 + \cos x = -1$$

$+1$

$$2\cos^2 x + \cos x = 0$$

GCF
$\cos x$

$$\cos x(2\cos x + 1) = 0$$

$\cos x = 0$ $\quad$ $\cos x = -\frac{1}{2}$

x-coord=0 $\quad$ x-coord $-\frac{1}{2}$ QII QIII

$x = 90°, 270°$ $\quad$ $x = 120°, 240°$

$$\boxed{x = 90°, 120°, 240°, 270°}$$

Try

Algebraically solve this equation

$$2\cos x + 1 - \sin^2 x = 3 \text{ , for } 0 \le x < 2\pi$$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$\cos^2\theta = 1 - \sin^2\theta$

$2\cos x + \cos^2 x = 3$

$2\cos x + 1 - (1 - \cos^2 x) = 3$

$2\cos x \underline{+1 - 1} + \cos^2 x = 3$

$2\cos x + \cos^2 x = 3$

$\cos^2 x + 2\cos x - 3 = 0$

AC
-3
$+3, -1 = 2$

$(\cos x + 3)(\cos x - 1) = 0$

$\cos x = -3$ $\quad$ $\cos x = 1$

$x =$ no solution $\quad$ $\boxed{x = 0}$

if ? general solution

$x = 0 + 2\pi n$

$\boxed{x = 2\pi n, n \in I}$